Introductory Photometry

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Light

This note introduces the concepts and methods used to measure light. *Radiant transfer* of energy happens whenever photons travel between objects. When the photons can be seen by humans the radiant energy is called *light*. A nice definition of light is "visually evaluated radiant energy"[1] because the measurement methods for light were developed to match human perception. The basic concepts surrounding light predate much modern technology, including electricity. Therefore, in the field of photometry (measurement of light), many of the terms[2] come from the era when light could only be measured by the human eye. Some vestiges of old usage that still persist will be mentioned later.[3]

The study of light or radiant energy has been a major focus[4] of physics research for the past several hundred years. No attempt will be made to summarize this entire body of work. Only the parts that are directly applicable to machine vision will be discussed.

Light can be modelled as photons or as waves. The distinction is most significant when diffraction effects are involved (for example, when imaging objects that are small compared to the wavelength of light), or when the quantity of light is very low (because photon statistics become important) or very high (because multiple-photon effects in materials become important). None of these cases describes normal uses of light to illuminate objects for vision - which corresponds to most applications in image processing. Therefore we will use a geometric model that considers light to be made of continuous rays with no diffraction effects.

In vacuum light travels at a constant speed, 3x108m/sec. According to our current theories of physics, this is the fastest that anything can travel. Inside materials light slows down. The ratio

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[1] JF Snell, "Radiometry and Photometry", in WG Driscoll, **Handbook of Optics** (1978, McGraw-Hill Book Company, New York), p. 1-2.

[2] An excellent dictionary of lighting terminology can be found in Section 1 of JE Kaufman, ed. **IES Lighting Handbook**, vol 1 (1981, Illuminating Engineering Society of North America, New York). The definitive dictionary is the **International Lighting Vocabulary** published by the Commission Internationale de l´Éclairage (CIE-1.1.).

[3] The conversion from perceptual (visual) measurement of light to technical measurement did not progress directly to electronic measurement. Photochemical measurement using photographic materials was the first step. The primary battles between people who favored perception versus those who favored technology happened about one hundred years ago. One account of this struggle is contained in W.B. Ferguson, ed., **The Photographic Researches of Ferdinand Hurter and Vero C Driffield** (1974, Morgan and Morgan, Dobbs Ferry NY).

[4] The reader is asked to begin noticing how visual terms pervade our language. In his book **The Origins of Knowledge and Imagination** (1978, Yale University Press, New Haven), J Bronowski claims this is because much human knowledge is vision-based. The words imagine, imaginary, visionary, etc. show this connection clearly. As a result, humans have difficulty in objectively evaluating light and images. Learning to consider one's own vision system as an instrument with technical strengths and weaknesses which must be studied and understood before it is used as a measurement tool is an important step toward success in image processing.

of the vacuum speed to the internal speed is called the index of refraction. It is always greater than one. Indices of refraction for a few materials are given in Table 1. Notice that speed of light is almost the same in air as in vacuum.

Table 1. Refractive Index for some Materials

glasses	1.5 - 2.0
transparent plastics	1.5
water	1.3
air	1.0003

Light travels in straight lines, called <u>rays</u>, in regions of constant refractive index. When a light ray strikes a boundary separating regions with different indices of refraction, two thing happen. First, part of the light is reflected[5]. The amount of reflection, which depends on polarization of the light, absorption within the materials, and direction of incidence, is on the order of:

$$R = \left(\frac{n-1}{n+1}\right) \quad (1)$$

where $n=n_2/n_1$ is the ratio of the two indices of refraction with n_1 and n_2 selected so that $n \geq 1$. For typical glasses, the fraction reflected is on the order of 4% to 10%, for the worst combinations of angle and polarization.

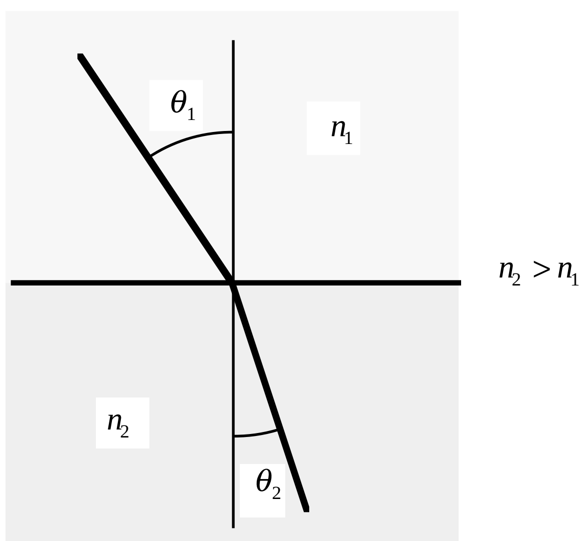

Figure 1. Light Ray deflection at a change in I.

[5] A compendium of information on the interactions between light and materials can be found in AK Stenius, “Nomenclature and Definitions Applicable to Radiometric and Photometric Characteristics of Matter”, **ASTM Special Technical Publication #475**, American Society for Testing and Materials, 1970.

The second thing that happens at boundaries is that the light ray is deflected. Its direction of travel obeys Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{2}$$

where θ_1 and θ_2 are, respectively, the angles the light ray makes with respect to the normal on each side of the boundary, Figure 1. When the angles are small θ, $\sin\theta$, and $\tan\theta$ are all about equal, which leads to a small-angle approximation of Equation (2):

$$n_1\theta_1 \approx n_2\theta_2 \tag{3}$$

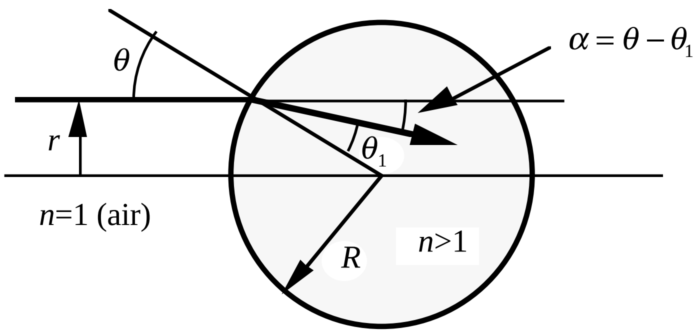

Figure 2. Light Ray striking a glass sphere.

At curved boundaries Snell's law is applied locally wherever a ray strikes the boundary. Consider a glass sphere with radius R. A ray strikes the glass at a distance r away from the axis of the sphere, Figure 2.

Using the small angle approximation,.the angle of incidence is $\theta = r/R$. From equation (2), the refracted angle is $\theta_1 = r/nR$. The ray entered the glass with slope r'=0 and inside the glass it has slope:

$$r' = -\tan^{-1}\alpha \approx -\alpha = -(\theta - \theta_1) = \frac{-r}{R}\left(\frac{n-1}{n}\right)$$

An object which causes light rays to change slope in proportion to the radius at which they strike it is called a <u>lens</u>. Curved reflectors are also lenses. The small angle approximation avoided having to deal with the fact that a sphere is not a perfect lens (it has a geometrical aberration: spherical aberration). Further discussion of lenses is beyond the scope of these notes, but many texts books are available on the subject[6].

[6] See, for example, WHA Fincham and MH Freeman, **Optics**, 9th edition (1980, Butterworths, London). The

2. Measurement of light

Historically, light was measured by visual comparison. Unknown sources were compared with standard light sources by juxtaposition in space (neighboring patches were compared to see if their intensities could be distinguished) or in time (part of a visual field was rapidly switched between sources to see if the flicker were visible). *Photometry* is the art of making visual comparisons of light. It is historically distinct from *radiometry*, which is the measurement of radiant energy in terms of standard SI units[7]. The connection between photometry and radiometry originally involved standard sources - physical artifacts were constructed and maintained in standards laboratories throughout the world. Their light output defined photometric units[8]. In the past ten years the measurement systems have been combined: photometric units are now defined in terms of radiometric units. Photometry currently means radiometry with sensors whose spectral sensitivity match the response of the human eye, which varies by several orders of magnitude over the 380nm to 780nm wavelength range of visible light.

Of the two measurement systems for radiant energy, why do we use the older one? Why are these notes called *Introductory Photometry* instead of *Introductory Radiometry*? At first glance radiometry seems to make more sense. Its methods and units have always been connected with modern physics and there are no ancient units or concepts to explain. In situations involving monochromatic energy, such as laser illumination, or invisible radiant energy, radiometry is preferable. However, when the illumination is broadband and visible photometry is used for several reasons. First, most environmental sources of light (the sun, incandescent and fluorescent lamps, campfires, television screens, squashed fireflies, rotting wood, lightning

spherical glass lens is an example in chapter 5.

[7] The traditional instrument for measuring radiant energy is the *bolometer*, a kind of calorimeter in which the temperature rise in an irradiated target of known heat capacity measures radiant energy transfer.

[8] The original artifacts were candles that burned controlled amounts of sperm whale oil. The photometric unit of luminous intensity is still called the *candela*; formerly it was called the *candle power*. Because of disagreements on how to construct oil candles, different countries had different definitions of the candela. From 1909 to 1948 banks of vacuum lamps with carbon filaments operating at controlled currents were used as photometric standards. Between 1948 and 1979 the candela was defined as 1/60 the luminous intensity of 100 square millimeters of glowing ceramic at the melting temperature of platinum. Instrumentation errors in this definition of the candela were several percent. In 1979 the photometric and radiometric standards were merged and luminous intensity is now specified using standard SI units, the meter and the Watt.

flashes, broken lifesaver candies, etc.) emit broadband radiant energy. It is easier to discuss them using photometry, which is inherently broadband (remember, photometry was developed with the human eye as the sensor). Second, although cameras do not see as humans, cameras are used to observe things that humans find interesting. Photometry provides simple answers to typical human questions like "Which fabric sample is more reflective, the red one or the blue one?". That question is difficult to answer using radiometric measurements. Third, most broadband detectors of visible radiant energy are designed to mimic the human visual response so photometric measurements apply directly to them. For example, television cameras are designed with the same light response as humans so that television images appear to be natural. Photometric measurements show directly how an object will appear to a camera. For these reasons, photometry is useful in applied machine vision. In some cases (eg, monochromatic or invisible radiation, thermal radiation) radiometry may be appropriate so equivalent radiometric concepts and units will be mentioned throughout these notes.

3. Luminous Flux

The basic quantity of light, the *lumen* (abbreviated lm), is a *luminous flux*. It corresponds to a flux of light which in principle represents some number of photons per second; the radiometric analog is *radiant flux*, whose units are W. The actual number of photons depends on the spectral content of the light, loosely called color. A Watt of 550nm (green) monochromatic light produces roughly twice the visual sensation of a watt of 610nm (orange) light: its *spectral luminous efficiency* is twice as large[9]. This function was experimentally determined by averaging the responses of a large number of humans. It forms the basis of the conversion between radiometry and photometry. To find the lumen content of some flux of light in principle one multiplies its spectral energy distribution (Watts per nm of wavelength evaluated as a function of wavelength) by the spectral luminous efficiency, which is a tabulated function, and integrates over the range 380 to 780nm. In fact, the total flux is rarely measured this way - it is too difficult and can be highly inaccurate.[10] Flux is measured by putting the unknown source and a standard source inside an *integrating sphere*[11], a large hollow sphere painted matte white

[9] The spectral luminous efficiency peaks at 555nm where its value is 683 lm/W. Its average value over the range 380 to 780nm is 187 lm/W.

[10] A report of errors greater than 20% when this method is applied to fluorescent lamps (which contain both broadband and spectral line radiation) is contained in B Steiner, "The Present State of Radiometry and Photometry", **NBS Technical Note 594-6**, 1974).

[11] EB Rosa and AH Taylor, "Theory, Construction and Use of the Photometric Integrating Sphere", **Scientific**

inside. Multiple reflections cause the light to spread out evenly along the sphere's inner surface, even if the source does not emit uniformly. An electronic sensor with filters to match its sensitivity to the spectral luminous efficiency function measures the light passing through a small hole in the sphere. This flux is proportional to the total flux from the source. The flux from the unknown source is measured by comparing it with the flux from the standard source. At the same time, the *luminous efficiency* of the source is obtained by dividing the light output by the electrical power input; its units are lm/W.

4. Luminous Intensity

The *luminous intensity* of a light source is the derivative of its flux with respect to solid angle:

$$I = \frac{d\Phi}{d\Omega} \tag{4}$$

The unit of intensity is the candela (abbreviated cd) - lm/sr. The total flux through a point is intensity integrated over all solid angles at that point. The radiometric analog is *radiant intensity*, whose units are W/sr.

There are two ways to measure the intensity of a point source. First, one can measure the flux Φ from the source which crosses a projected area A at a distance r The intensity is just $I = \Phi r^2 / A$. This is the same as measuring the flux into a solid angle and dividing by the solid angle. Second, one can use an integrating sphere to measure the total source flux and divide by the total solid angle: $I = \Phi / 4\pi$. This second method averages the intensity and results in the *mean luminous intensity*. A source which has the same intensity in all directions is isotropic. A small isotropic source is called a *point source*. It is “small” if its size is much less than the distance to the nearest object it illuminates. The intensity of a point source in any direction is the same as its mean intensity.

A standard 75W incandescent light bulb produces 1170 lm when operated at 120VAC. Its luminous efficiency is 1170/75=15.6 lm/W and its mean luminous intensity is 1170/4π=93cd. The actual intensity is lower than the mean beneath the bulb where the screw-in connector blocks the light, so it must be a little higher above the bulb.

Papers of the Bureau of Standards, **#447**, **18**, 280-325, 1922.

Some light sources are more anisotropic than the 75W bulb. Automobile headlamps are designed to put all of the flux into a narrow beam. The intensity of this type of source is specified in *beam candelas*. Either the *peak candelas* are specified within a given cone or a diagram of intensity in beam candelas versus angle is provided. An aircraft landing lamp rated at 400,000 beam candelas produces a total flux of 1000 lm. Therefore it emits into a solid angle of $\Omega = \Phi / I_{beam} = 0.003\mathrm{sr}$. This corresponds to a cone half-angle of $\alpha = (\Omega/\pi)^{1/2} = 0.03\mathrm{r} = 1.6°$. Figure 3 contains a diagram showing conversion between cone angle and solid angle. Aircraft landing light alignment is clearly very critical.

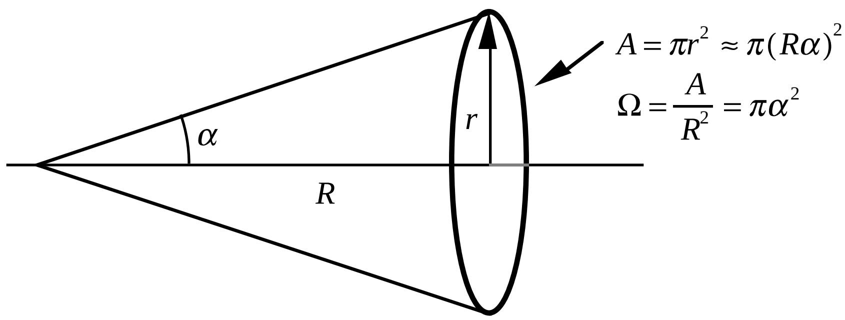

Figure 3. Steradian equivalent of a small cone angle.

5. Luminance

Most light sources are not infinitesimal point sources, they are *extended sources* or *finite sources*. That is because light flux is related to power and nature seems to place maximum limits on power densities. Extended sources are described by the second derivative of flux with respect to area and solid angle:

$$L = \frac{d^2\Phi}{dA\,d\Omega} \tag{5}$$

The units of luminance are cd/m^2. The total flux through a surface is the double integral of the luminance over the surface area and over all solid angles at each point in the area. The radiometric analog is *radiance*, whose units are W/(m^{2}sr).

Many other units of luminance persist in the literature, although their use is not currently fashionable. Some of them are the nit (nt) - also cd/m^2, the stilb (sb) - cd/cm^2, the apostilb (asb) - cd/πm^2, the Lambert (L) - cd/πcm^2, and the footLambert (fL) - cd/πft^2.

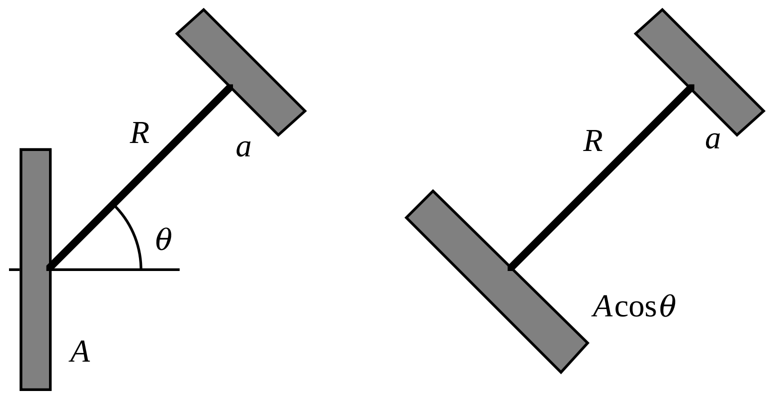

Figure 4. Test surface for studying luminance.

Luminance is a directional quantity; it is necessary to define the direction in which the light is emitted or measured. Consider a planar surface of area A whose luminance is some function of the polar angle with respect to the surface normal, $L = L(\theta)$. It is not assumed that L $L(\theta)$ is constant, merely that it is independent of the azimuthal angle. This describes most surfaces observed in machine vision applications. The total flux through the test area, a, located a distance R away is $\Phi_a = L(\theta)aA/R^2$, see the left side of Figure 4. Note that the test surface normal has been placed along the radial line connecting the two surfaces. Viewed from a point on the test surface the luminous surface appears to have area A and to be tilted at an angle θ, its projected area is $A\cos\theta$. We can replace the surface A by one with area $A\cos\theta$ whose normal points directly to the test surface, see the right side of Figure 4. The two cases are completely equivalent; there is no measurement that can be made from the point on the test surface that distinguishes between them. The flux through the test area in the second case is $\Phi_a = L'(\theta)aA/R^2$, where $L'(\theta)$ is the luminance of the new surface. We can equate the two fluxes to find $L'(\theta) = L(\theta)/\cos\theta$. Thus, if the original surface is rotated, the apparent luminance varies. We can deduce the angular dependence of the original surface's luminance $L(\theta)$ by measuring the apparent luminance $L'(\theta)$ and multiplying by the cosine of the rotation angle.

This is a good point to stop and look around. Look at curved objects that are either self-luminous or that have matte finishes: the sun, an egg, the moon, a light bulb, your arm, etc. All of these objects appear uniform all of the way to their edges. That means the apparent luminance

$L'(\theta) = L(\theta)/\cos\theta$ is constant with angle, which can only happen if an object's luminance varies as the cosine of the observation angle. If $L(\theta)$ fell off more slowly than $\cos\theta$, the very edge would be infinitely bright. If $L(\theta)$ fell off more rapidly than $\cos\theta$, the edge would be totally black. The general absence of *limb brightening* or *limb darkening* is a demonstration of *Lambert's law*, which says that an object's luminance is proportional to the cosine of the observation angle:

$$L = L_0 \cos\theta. \tag{6}$$

Surfaces that obey this law are called *diffuse*. Most surfaces that are not shiny are diffuse.

From the analysis surrounding Figure 4, the intensity of a luminous source with area A that is far enough away to ignore the changes in r and θ when moving from one edge of the source to another is:

$$I = L_0 A \cos\theta \tag{7}$$

Notice that the intensity is not isotropic. Viewed on edge, the source intensity drops to zero because the projected area falls off as $\cos\theta$. This is not a consequence of Lambert's law; it is simple geometric projection.

6. Illuminance

The *illuminance* at a surface is the derivative of the flux with respect to surface area:

$$E = \frac{d\Phi}{dA} \tag{8}$$

The units of illuminance are lux (abbreviated lx) - lm/m^2. The total flux through a surface is the integral of illuminance over its entire area. Unfashionable units for illuminance include the foot-candle (fc) - lm/ft^2. The concept of illuminance does not distinguish between light that leaves or enters a surface, but two terms that appear in the literature do: *exitance* is illuminance leaving a surface (such as a television screen) and *incidance* is illuminance entering a surface (such as the grass on a golf course). The radiometric analog is *irradiance*, whose units are W/m^2.

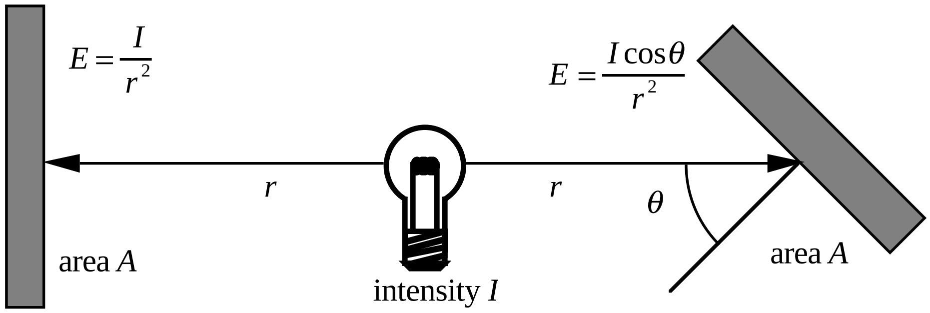

Figure 5. Illuminance due to a Point Source.

Illuminance is a directional quantity so it is necessary to specify the direction of the test surface. For example, a horizontal test surface on the earth at noon in the summer in New England receives an illuminance of about 10^5 lx; a vertical test surface facing south receives about $4x10^4$lx.

The illuminance at a surface facing a point source with intensity I located a distance *r* away is $E = I/r^2$. This is the famous *inverse square law* for illuminance. If the surface is inclined at an angle ø as shown in Figure 5 the illuminance is:

$$E = \frac{I\cos\theta}{r^2} \tag{9}$$

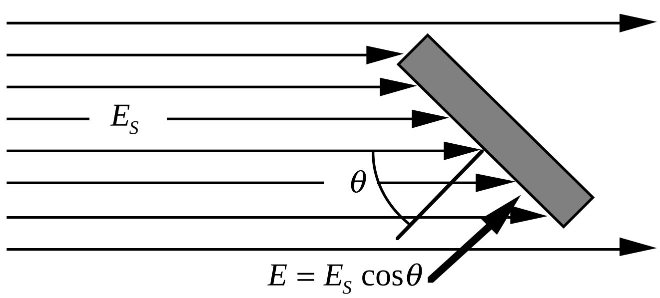

Figure 6. Illuminance from a Distant or Collimated source**.**

The $\cos\theta$ is due to geometric projection, not Lambert's law. Note that illuminance is a differential quantity measured at a point. On an extended surface, *r* will in general be different from point to point, so *E* will also vary across the surface. There are cases where *r* and θ are not significantly different from point to point: a source that is far away or a well-collimated (low divergence angle) source, such as a laser beam. In those cases it is appropriate to model the source as providing an illuminance E_S and to calculate the illuminance it produces on a target using $E = E_S \cos\theta$, Figure 6.

Calculate the illuminance at a point on a diffuse surface with axial luminance L_0. This is just the integral of the luminance over all solid angles in a hemisphere:

$$E = \int_0^{2\pi} L(\Omega) d\Omega \tag{10}$$

Use the fact that $d\Omega = dA/r^2$, where dA is the differential area on the surface of a hemisphere with radius r. This integral is easiest to perform in polar coordinates, see Figure 7:

$$E = L_0 \int\int_A \cos\theta \frac{dA}{r^2} = \frac{L_0 r^2}{r^2} \int_0^{2\pi} d\varphi \int_0^{\pi/2} \cos\theta \sin\theta d\theta = \pi L_0 \tag{11}$$

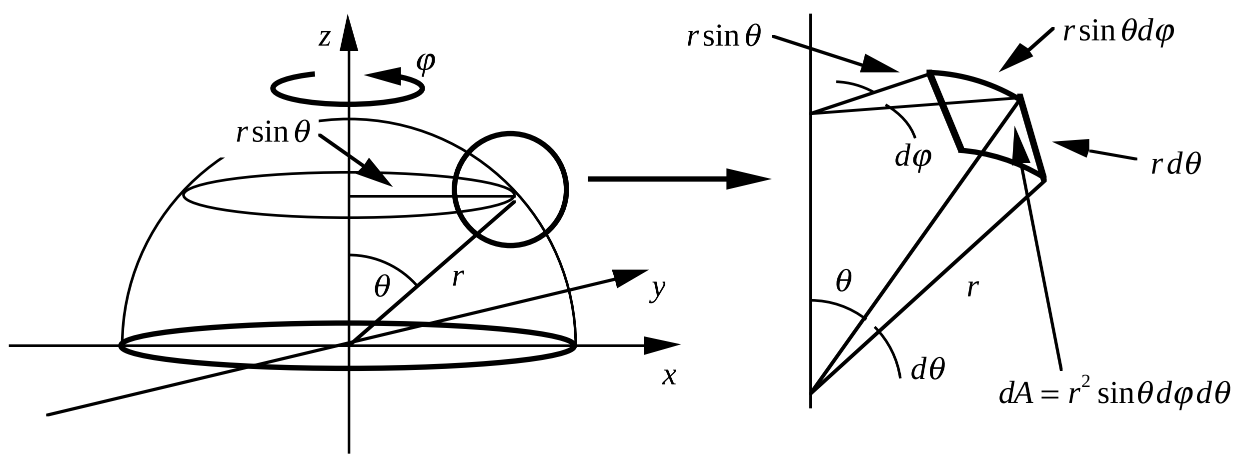

Figure 7. Illuminance of a Diffuse Object.

The Illuminance of a diffuse object is π times its luminance[12]. The 75W incandescent lamp with 1170 lm flux can be modelled as a sphere 60mm in diameter. Its surface area is $A=\pi(0.060)^2=0.011\text{m}^2$ so its surface illuminance is 1170/0.011 =103klx and its surface luminance is $103{,}000/\pi$=33kcd/m^2. Note this relationship is only true for diffuse objects: frosted lamps are highly diffuse. In the general case Equation (10) is used to relate luminance to illuminance.

[12] This is the origin of the pi in luminance units such as the Lambert. It makes the luminance and illuminance of a diffuse object numerically equal.

The Illuminance produced by an extended source at a point on a test surface is found by calculating the differential illuminance caused by a small patch at a point P on the source. Assume the patch is a diffuse emitter with luminance L_0, area dA, and it is located a distance r away from the test surface point, Figure 8. Then the differential illuminance is:

$$d^2E = \frac{L_0 \cos\theta_1 \cos\theta_2}{r^2} dA \tag{12}$$

This is just the combination of Equations (7) and (9). Using a differential area allowed us to model the light from the point P as being an intensity. To find the total luminance at the test point, integrate equation (12) over the entire surface area of the source:

$$E = L_0 \int\int_A \frac{L_0 \cos\theta_1 \cos\theta_2}{r^2} dA \tag{13}$$

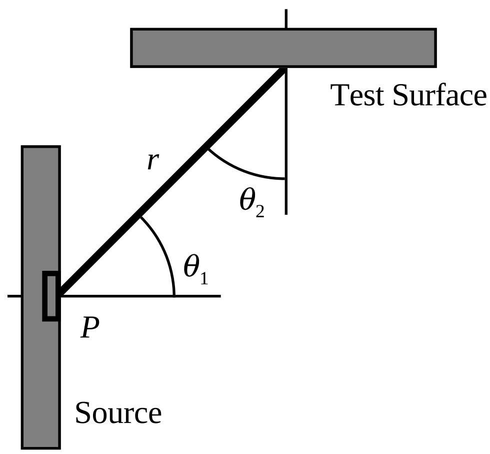

Figure 8. Illuminance due to an Extended Source.

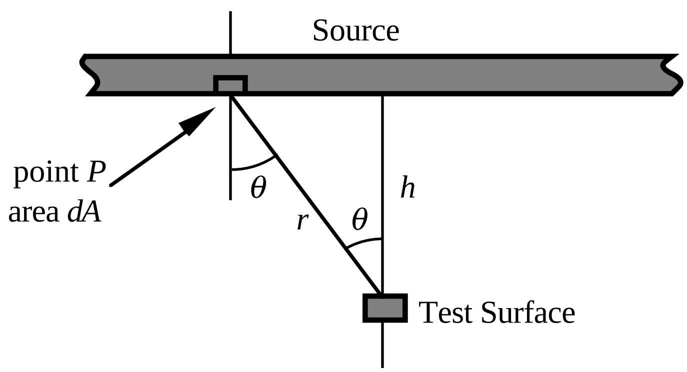

Figure 9. Illuminance when Source is Parallel to Test Surface.

As an example consider the illumination due to a diffuse planar source a distance h away from and parallel to the test surface, Figure 9. The radial distance from P on the source to the test surface is $r = h/\cos\theta$. From Equation (13), the illuminance is:

$$E = L_0 \int\int_A \cos^4\theta dA \tag{14}$$

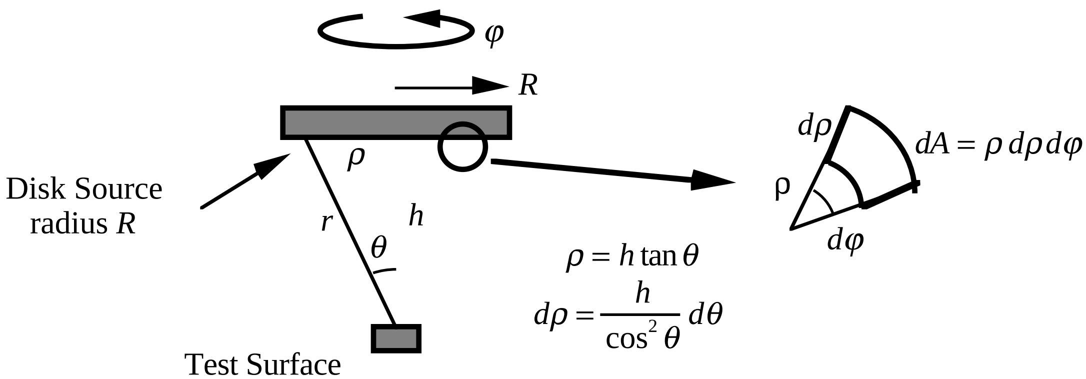

Figure 10. Illuminance when the Source is a Disk.

If the source is a disk of radius R, Figure 10, the illuminance is:

$$E = \frac{L_0}{h^2}\int_0^{2\pi} d\varphi \int_0^R \rho\cos^4\theta d\rho = \frac{2\pi L_0 h^2}{h^2}\int_0^{\tan^{-1}\frac{R}{H}} \sin\theta\cos\theta d\theta = \pi L_0 \frac{R^2}{R^2+h^2}$$

Note that in the limit as $R \to \infty$, the illuminance reduces to $E = \pi L_0$, which is just the illuminance of the source. Objects facing large planar sources receive the same illuminance as the source emits. The method used for calculating the illuminance caused by a disk can be extended to sources with other geometries.[13]

7. Reflections

So far, we have considered only self-luminous objects. Most objects are luminous because they reflect light. The term reflection covers a large range of phenomena. We are primarily interested in two cases: specular reflection and diffuse reflection. *Specular reflection* is mirror-

[13] Calculations for a number of source geometries are contained in Z Yamanouti, "Geometrical Calculations of Illumination due to Light from Luminous Sources of Simple Forms", **Res. Electrotech. Lab. (Jpn)**, **148**, 1924, and "Further Study of Geometrical Calculation of Illumination due to light from Luminous Surface Sources of Simple Forms", **ibid**., **194**, 1927.

like reflection in which a light ray reflects from the surface without diffusion. The incident and reflected rays lie in the same plane as the normal to the surface and they form equal angles with it at the point of incidence. *Diffuse reflection* is the case in which the reflected light is independent of illumination direction. No materials fit either of these cases exactly although often many materials can be adequately modelled by a combination of specular and diffuse reflections.[14] Other descriptions of reflection, which will be excluded from these notes, include: *bloom* - reflection near the specular angle due to deposits on the surface, haze - which is like bloom except that the scattering materials are part of the surface and cannot be removed, *sheen* - specular reflection at large angles of incidence for an otherwise matte specimen, and specular reflection distinctness - the sharpness with which outlines are reflected by the surface. Also, the reflection models used in computer graphics to make images that "look good" (eg., models which reduce the attention the eye places on ridges where the planar patches that form three-dimensional images meet) have been omitted[15]. Such models are beyond the scope of these notes.

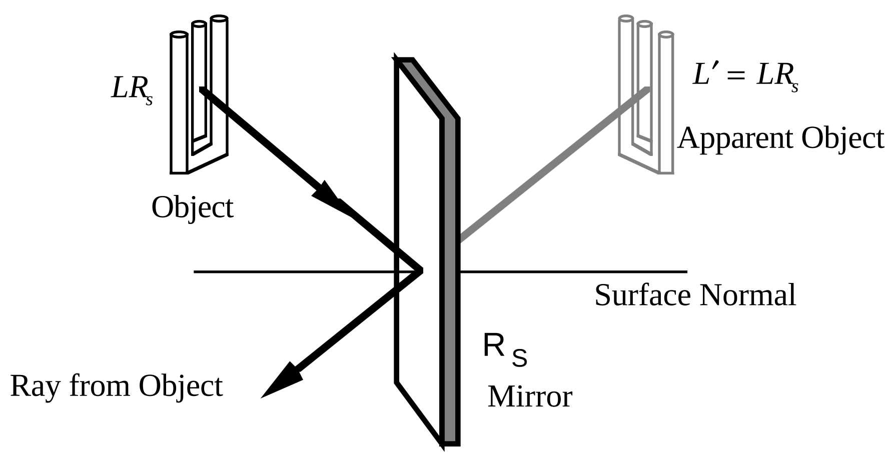

Figure 11. Specular Reflection**.**

Specular reflection is characterized by a specular reflectivity R_s, which is the ratio of reflected flux to incident flux. The reflecting surface, is not seen. All rays that appear to come from the surface originate at some other surface whose location can be determined by using the equality of the angles of incidence and reflection, Figure 11. If the object has luminance L, then

[14] N Wittels and SH Zisk, "Lighting Design for Industrial Machine Vision", **Proc. SPIE**, **728**, 47-56, 1987.

[15] Two standard references are the papers are by BT Phong, "Illumination for Computer Generated Pictures", **Comm. ACM**, **18**, 311-317, 1975 and H Gouraud, "Continuous Shading of Curved Surfaces", **IEEE Trans. Computers**, **C-20**, 623-629, 1971.

it appears as if the object has luminance LR_s. The important distinction about specular reflection is that incident rays retain their directional information during reflection.

Diffuse reflection is characterized by a *diffuse reflectivity* R_d, which is the ratio of reflected flux to incident flux. With diffuse reflection there is a total loss of information about the direction of incidence. The two cases shown in Figure 12 are totally equivalent - no measurement made on the reflected light can distinguish between them. Whether the illumination comes from a point source, an extended source, or any other type of source, the amount of light diffusely reflected depends only on the total illumination of the reflector. If the incident illuminance is E ($E = I\cos\theta / r^2$ for the cases shown), then the reflected illuminance is ER_d. Since the surface is diffuse the axial luminance of the reflected light is $L_0 = ER_d / \pi$; the luminance in other directions is described by equation (6).

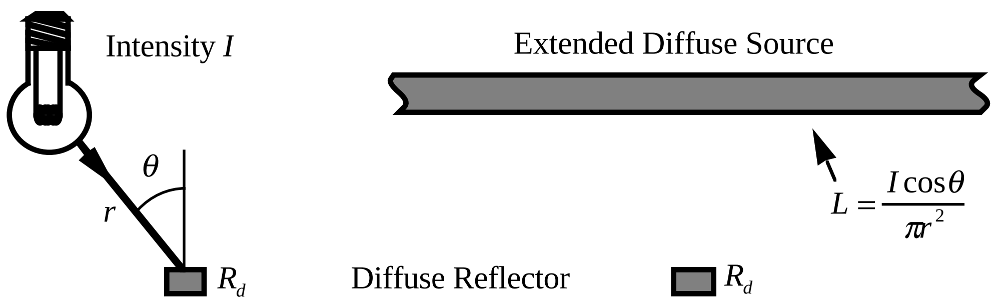

Figure 12. Diffuse Reflection.

Specular and diffuse reflection can be combined to describe the total reflection[16] which is characterized by its total reflectivity R_t, the reflected flux divided by the incident flux:

$$R_t = R_s + R_d \tag{16}$$

The total reflectivity R_t must always be less than one. Its components are measured in two experiments. First, diffuse reflectivity is measured by illuminating with a known flux along a known direction and measuring the total reflected flux in all directions except the specular one, see the left side of Figure 13. The hole in the detector is important: it excludes specularly reflected light. Hemispherical detectors are difficult to make so the arrangement shown on the right side of Figure 13 can be used. It is fundamental that all systems which transfer light must

[16] This is a simplification that ignores polarization effects. They can be important because specular reflection generally preserves or enhances linear polarization while diffuse reflection generally reduces or eliminated linear polarization.

be reversible: if source and detector are exchanged, the same measurements will result.[17] Next the specular reflectivity is measured using equal angles of incidence and measurement, Figure 14. The contribution from diffuse reflection to the measured flux in this experiment must be subtracted to get the correct specular reflectivity. Note that Figures 13 and 14 are schematic diagrams only. Reflectometers are more complex than these diagrams, but photometric instrument design is beyond the scope of these notes.

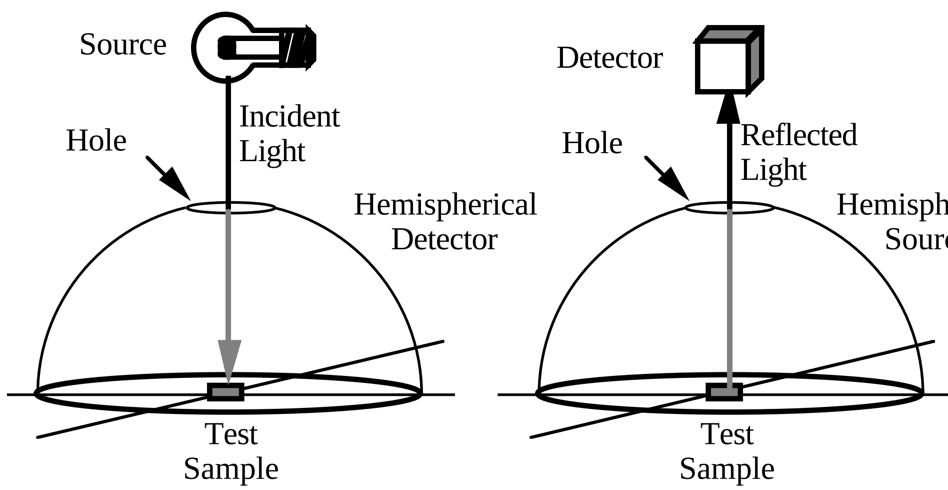

Figure 13. Measuring Diffuse Reflectivity.

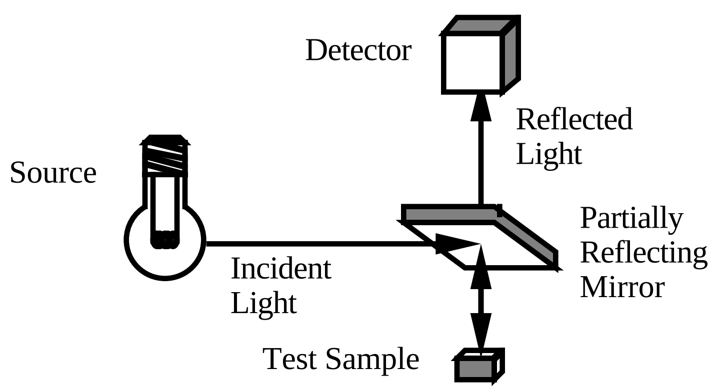

Figure 14. Measuring Specular Reflectivity.

8. Spectral Content

Photometry is based on the notion that the sensors match the spectral sensitivity of the human eye. The situation is usually more complicated. First, human spectral sensitivity is different under low and high flux conditions. Also individual humans differ in their sensitivities

[17] This is a result of the second law of thermodynamics. An optical system transfers radiant energy. If the energy flowing from source to detector were different after the exchange, it would form the basis for a perpetual motion machine. Of course systems which violate reversibility can be built, but they require energy input (Maxwell's demon must be fed).

- which is one reason that the spectral luminous efficiency function is now standardized in terms of SI units instead of being standardized by measurements on people. Finally, no camera perfectly matches the human spectral response. The solution is to expand all photometric units to include the notion of derivatives with respect to wavelength. They are denoted by adding the words "spectral" in front of the terms. For example, *spectral luminance* is the derivative with respect to area, solid angle, and wavelength:

$$L_\lambda = \frac{d^3\Phi}{dA\,d\Omega\,d\lambda} \tag{17}$$

It has units cd/m^3, since wavelength has units of length. All of the calculations done in these notes are valid for the corresponding spectral units. Any of the photometric units can be recovered by integrating the respective spectral units over all wavelengths between 380 and 780nm:

$$E = \int_{380\text{nm}}^{780\text{nm}} E_\lambda d\lambda; \quad L = \int_{380\text{nm}}^{780\text{nm}} R_{s\lambda} L_\lambda d\lambda, \ \text{etc.}$$

One final word of caution is in order regarding spectral photometric units. They describe spectral content of light, which is not the same as color. *Color* is a combined physiological and psychological response to light. The spectral content of light determines in principle the color of the light but the determination is not unique. A color description does not imply a particular spectral distribution. That is fortunate because it allow us to produce color pictures and photographs using mixtures of only three or four inks or dyes. Matching spectral content is much more difficult than matching color: at least an order of magnitude increase in the number of inks or dies would be required. An experimentally derived set of primary colors (red, green, blue) and a corresponding set of three spectral luminous efficiency functions has been found to give acceptable results when matching colors. This model is called the tri-chromatic system. Thus, color calculations can be performed with sets of three spectrally weighted analogs for each of the photometric functions mentioned above. Further discussion of color is beyond the scope of this introduction but it can be found elsewhere.[18]

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[18] The book by TN Cornsweet, **Visual Perception** (1970, Academic Press) has a lengthy discussion of the physiological bases for color response in people. A description of how the system is used is contained in the book by DB Judd and G Wyszecki, **Color in Business, Science, and Industry** (1963, J Wiley, NY). A review article on color has been written by G Wyszecki, “The Measurement of Brightness and Color”, **Metrologia**, **2**, 111-125, 1966.