

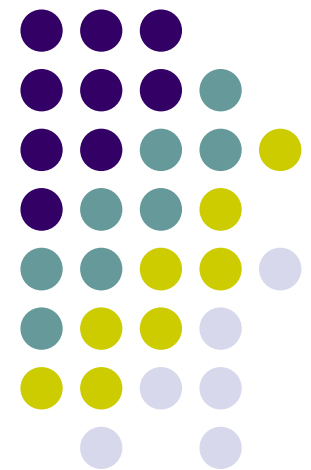
# Computer Graphics (CS 543)

## Lecture 5 (Part 3): Hierarchical 3D Models

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Prof Emmanuel Agu

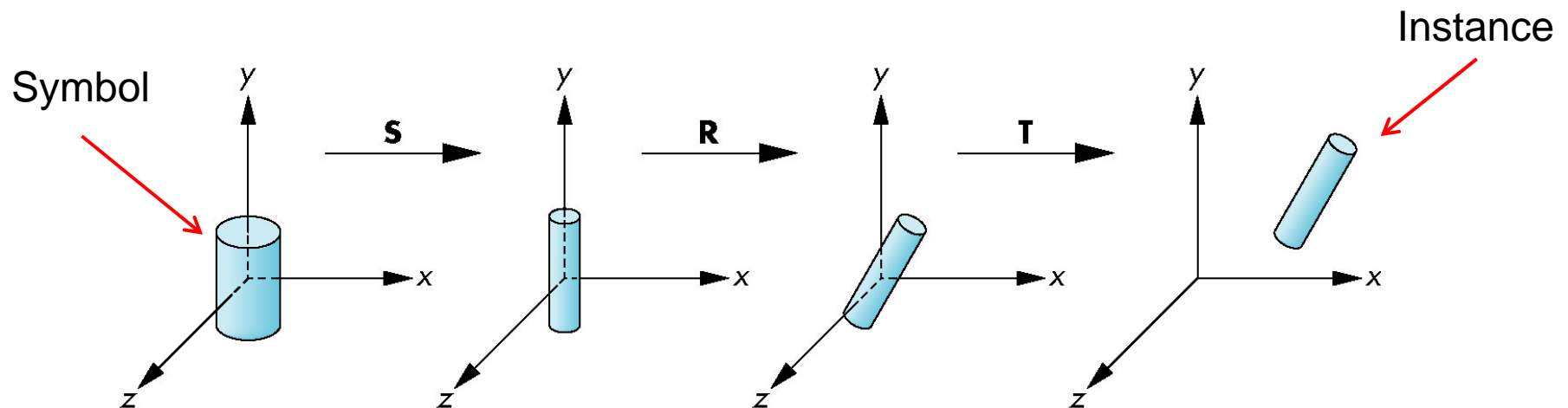
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# Instance Transformation

- Start with unique object (a *symbol*)
- Each appearance of object in model is an *instance*
  - Must scale, orient, position
  - Defines instance transformation

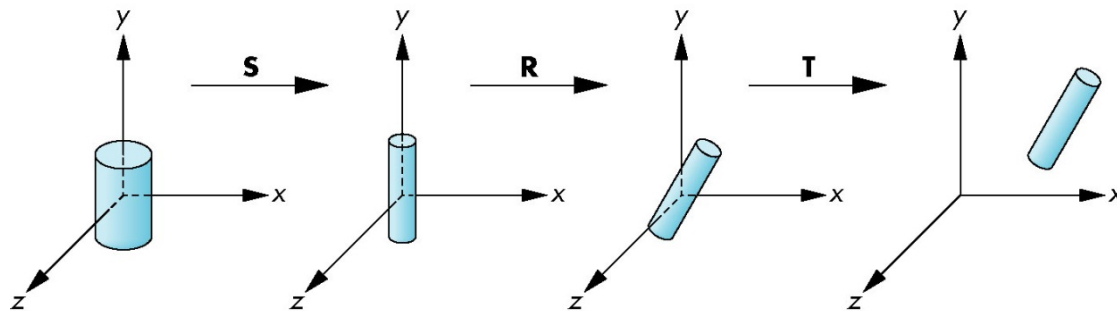




# Symbol-Instance Table

Can store **instances + instance transformations**

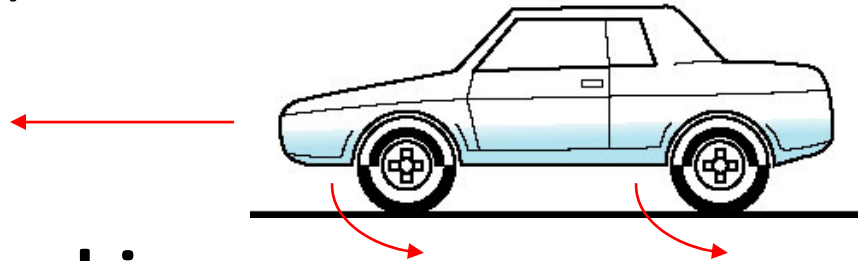
Symbol	Scale	Rotate	Translate
1	$s_{x'}, s_{y'}, s_{z'}$	$\theta_{x'}, \theta_{y'}, \theta_{z'}$	$d_{x'}, d_{y'}, d_{z'}$
2			
3			
1			
1			
.			
.			





# Problems with Symbol-Instance Table

- Symbol-instance table does not show relationships between parts of model
- Consider model of car
  - Chassis (body) + 4 identical wheels
  - Two symbols

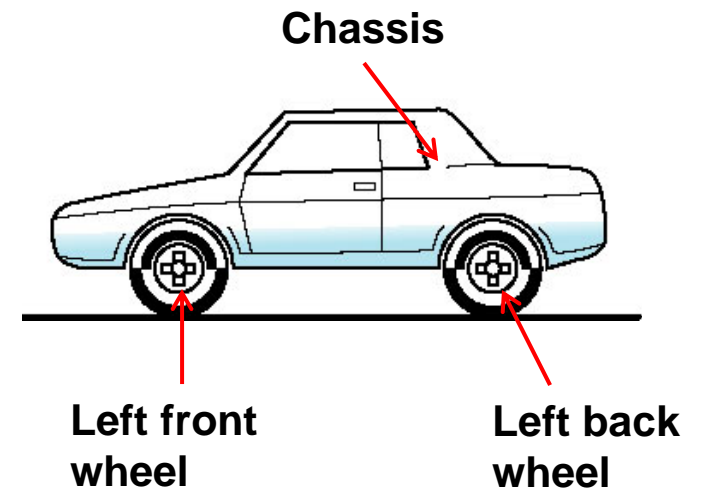


- **Relationships:**
  - Wheels connected to chassis
  - Chassis motion determined by rotational speed of wheels



# Structure Program Using Function Calls?

```
car(speed)
{
    chassis()
    wheel(right_front);
    wheel(left_front);
    wheel(right_rear);
    wheel(left_rear);
}
```

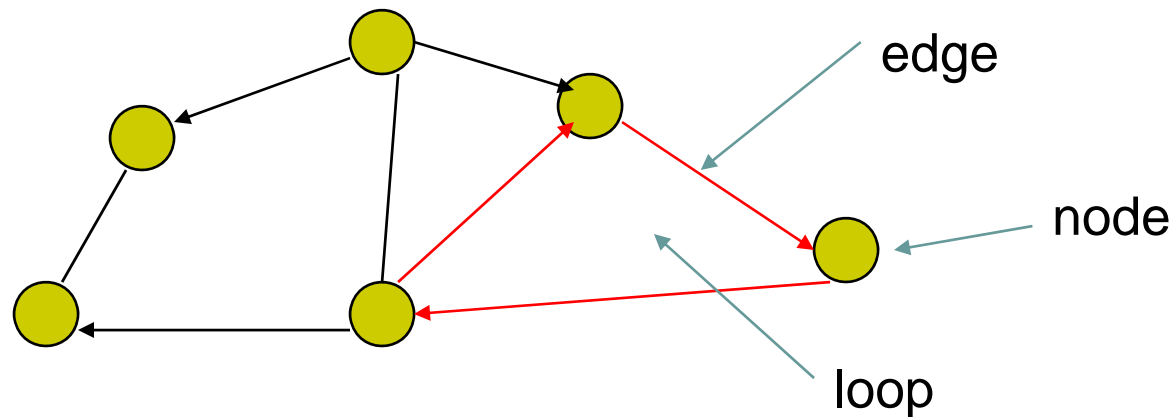


- Fails to show relationships between parts
- Look into graph representation



# Graphs

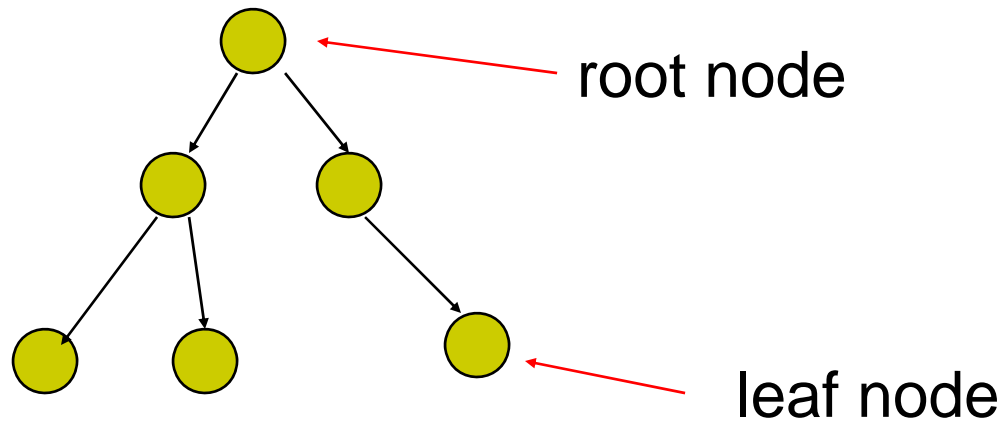
- Set of *nodes* + *edges (links)*
- **Edge** connects a pair of nodes
  - Directed or undirected
- **Cycle**: directed path that is a loop





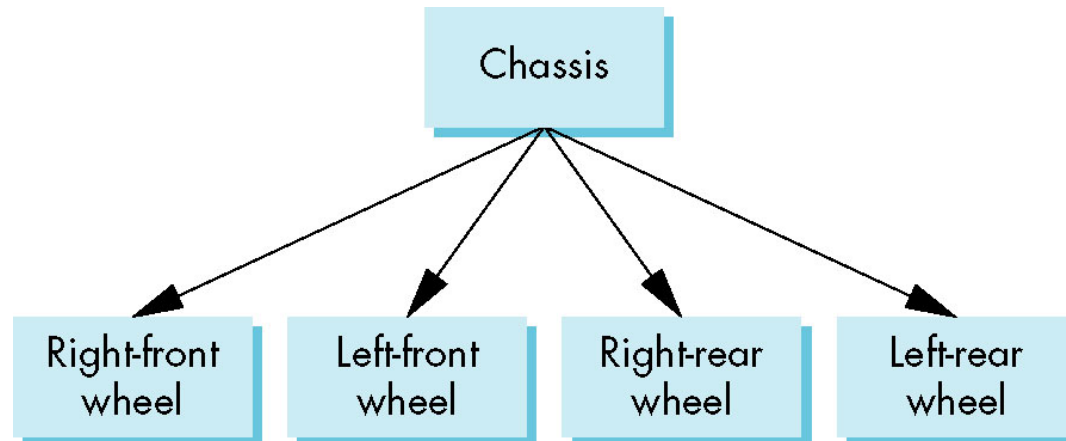
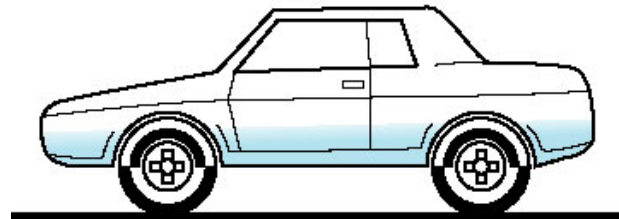
# Tree

- Graph in which each node (except root) has exactly one parent node
  - A parent may have multiple children
  - Leaf node: no children





# Tree Model of Car



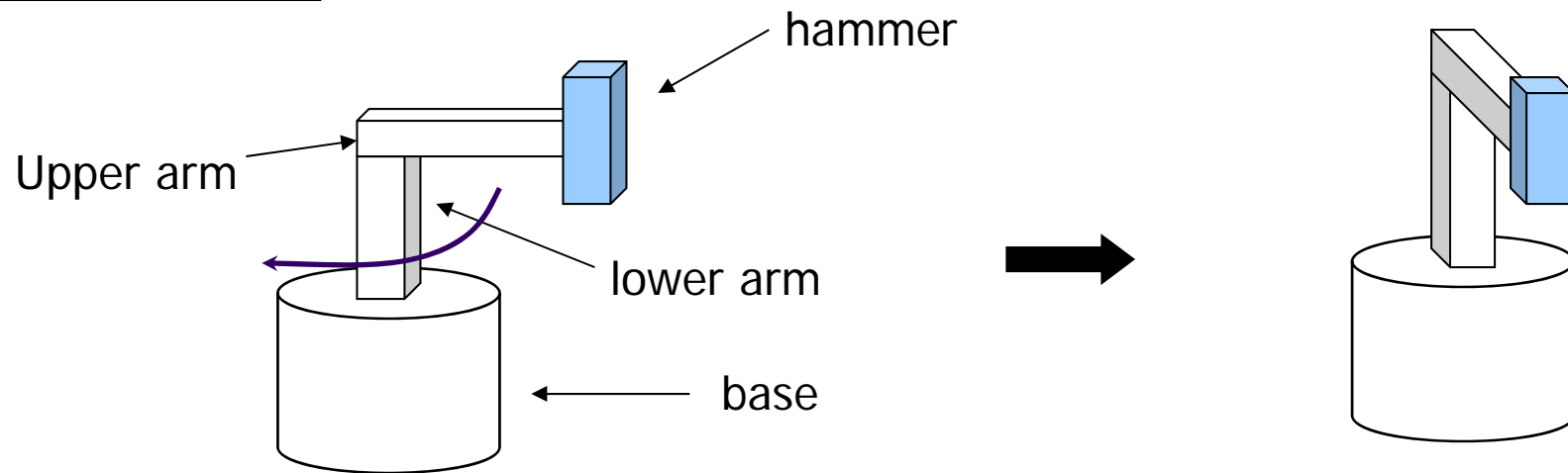




# Hierarchical Transforms

- **Robot arm:** Many small **connected** parts
- Attributes (position, orientation, etc) depend on each other

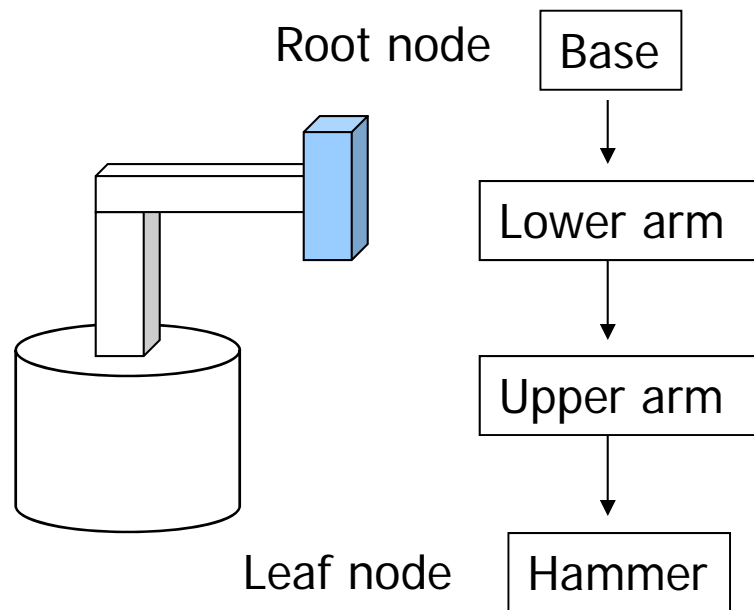
**A ROBOT HAMMER!**





# Hierarchical Transforms

- Object dependency description using tree structure



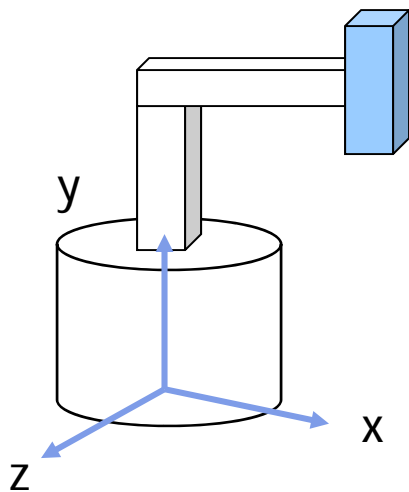
Object position and orientation can be affected by its parent, grand-parent, grand-grand-parent ... nodes

Hierarchical representation is known as **Scene Graph**

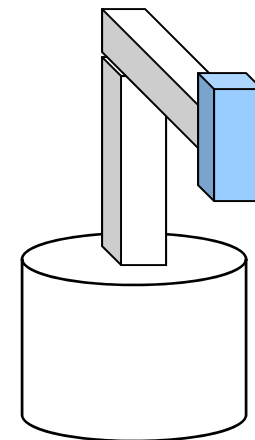
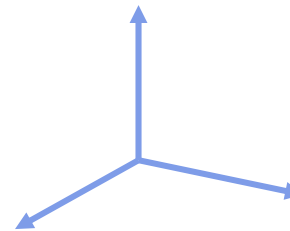


# Transformations

- Two ways to specify transformations:
  - **(1) Absolute transformation:** each part transformed independently (relative to origin)



Translate the base by  $(5,0,0)$ ;  
Translate the lower arm by  $(5,0,0)$ ;  
Translate the upper arm by  $(5,0,0)$ ;  
...

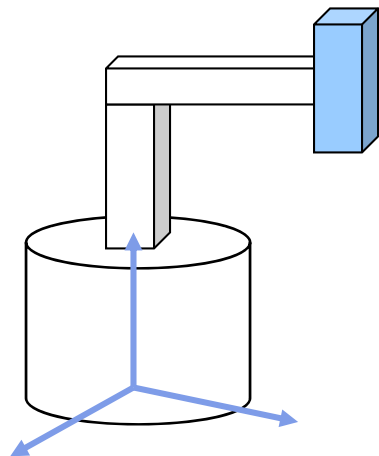




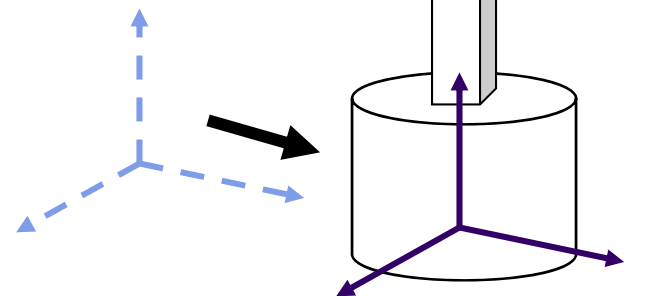
# Relative Transformation

A better (and easier) way:

(2) **Relative transformation:** Specify transformation for each object relative to its parent



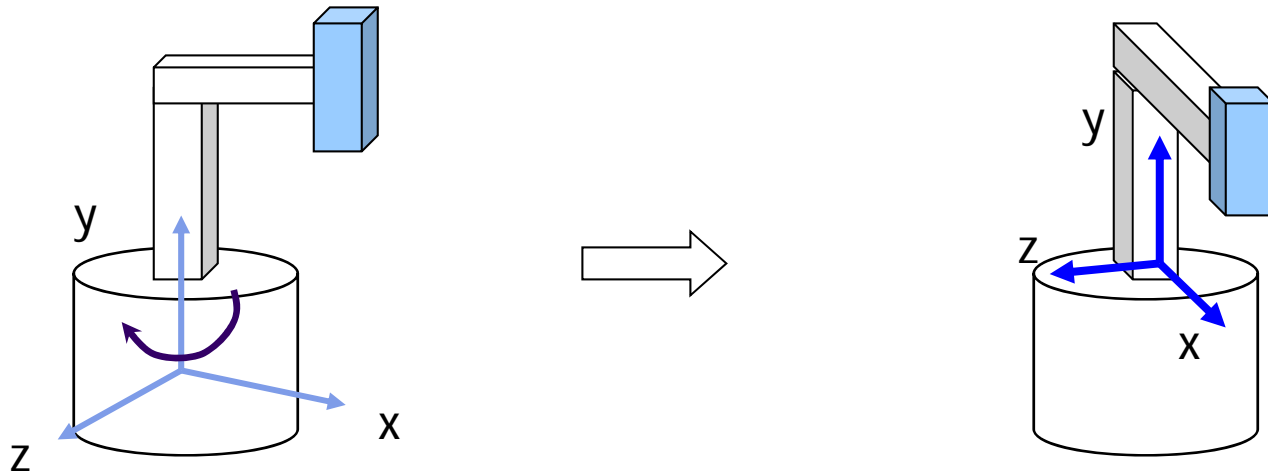
Step 1: Translate base and its descendants by  $(5,0,0)$ ;



# Relative Transformation



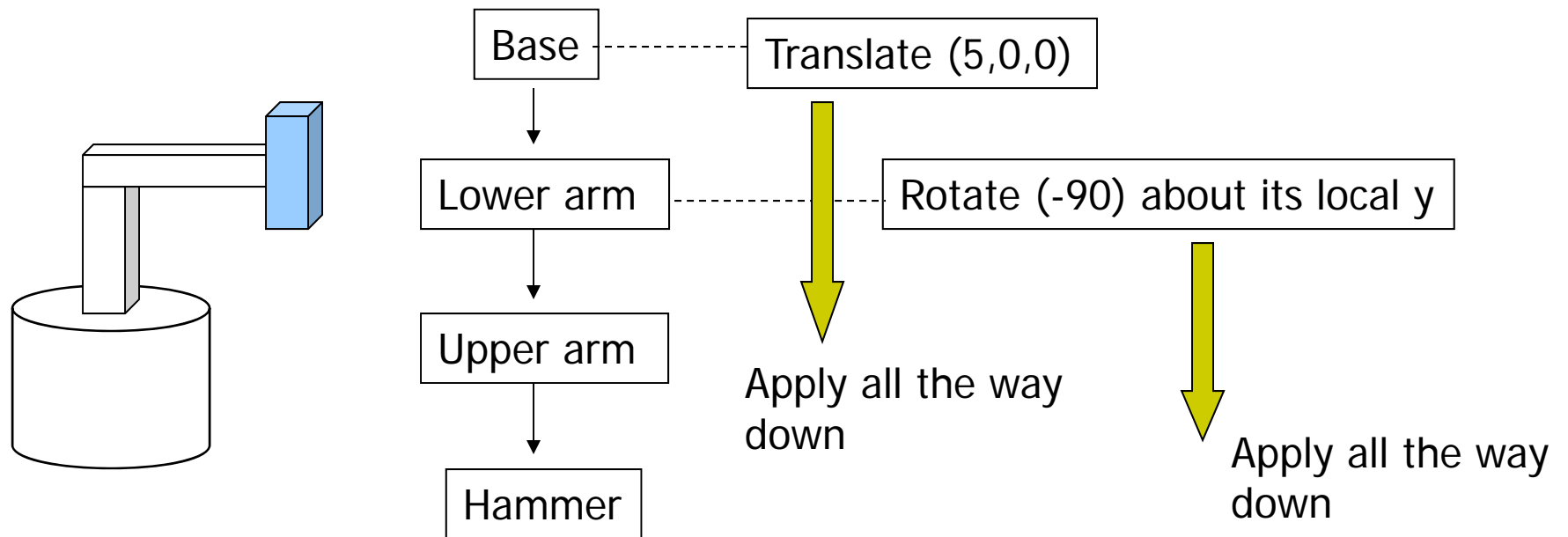
**Step 2: Rotate the lower arm and all its descendants relative to the base's local y axis by -90 degree**





# Relative Transformation

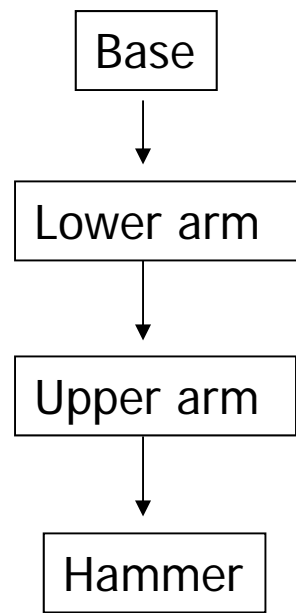
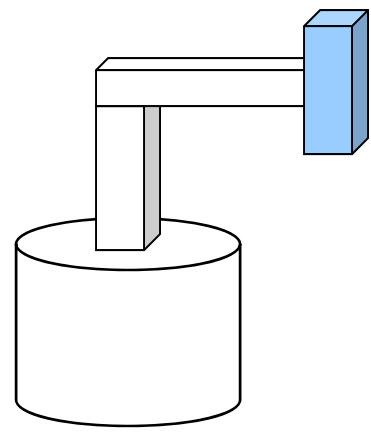
- Relative transformation using scene graph





# Hierarchical Transforms Using OpenGL

- Translate base and all its descendants by (5,0,0)
- Rotate lower arm and its descendants by -90 degree about local y



```
ctm = LoadIdentity();  
  
... // setup your camera  
  
ctm = ctm * Translatef(5,0,0);  
  
Draw_base();  
  
ctm = ctm * Rotatef(-90, 0, 1, 0);  
  
Draw_lower_arm();  
Draw_upper_arm();  
Draw_hammer();
```



# Hierarchical Modeling

- Previous CTM had 1 level
- **Hierarchical modeling:** extend CTM to stack with multiple levels using linked list

Current top  
Of CTM stack  $\longrightarrow$  
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$





# PushMatrix

- **PushMatrix( )**: Save current modelview matrix in stack
- Positions 1 & 2 in linked list are same after PushMatrix

*Before PushMatrix*

Current top  
Of CTM stack → 
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



*After PushMatrix*

← Current top  
Of CTM stack 
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



# PushMatrix

- Further Rotate, Scale, Translate affect only top matrix
- E.g.  $ctm = ctm * \text{Translate}(3, 8, 6)$

*After PushMatrix*

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

✘

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

← Translate(3,8,6) applied  
only to current top  
Of CTM stack

↓

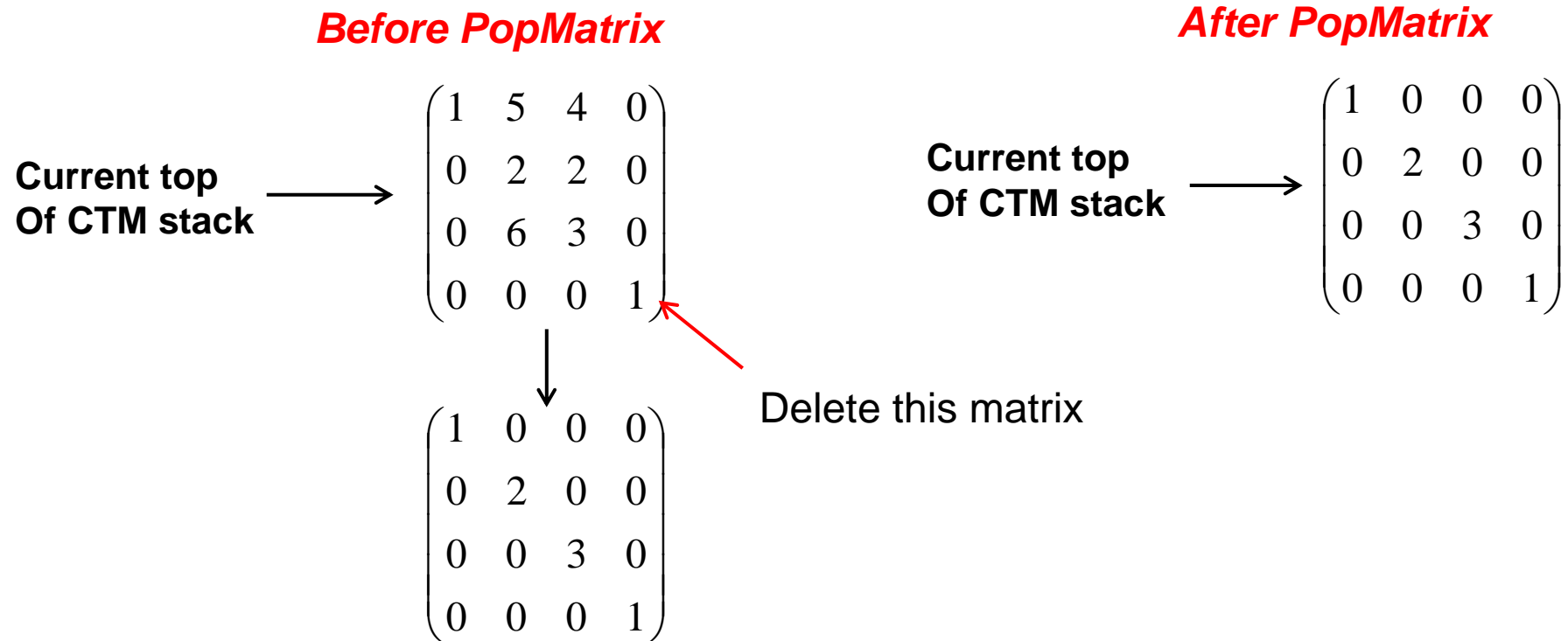
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

← Matrix in second position saved.  
Unaffected by Translate(3,8,6)



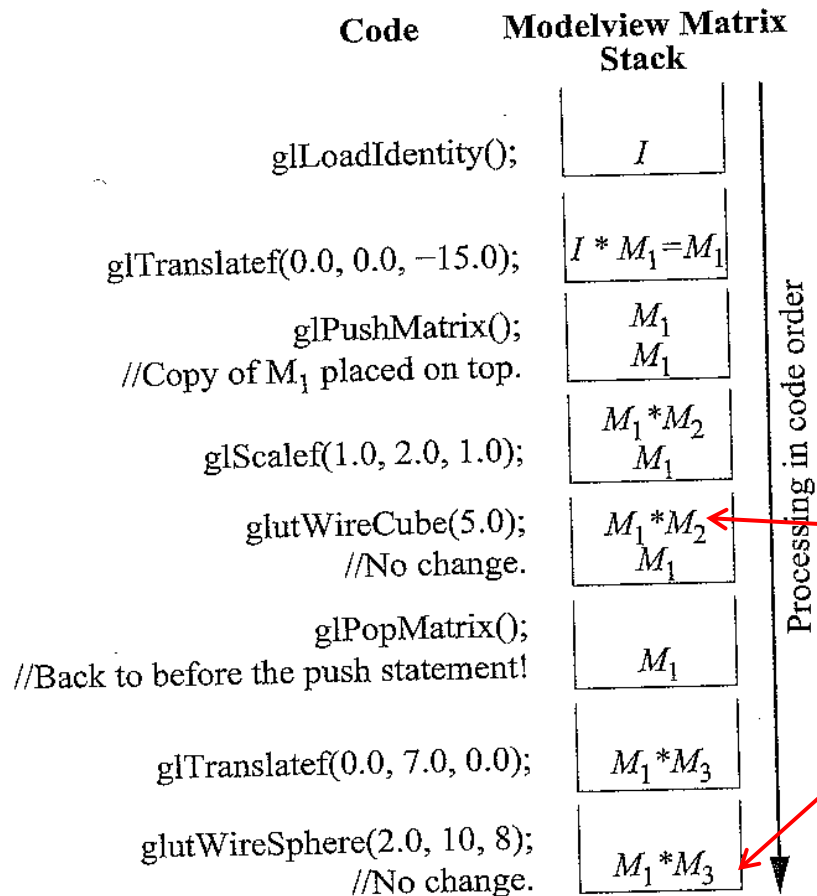
# PopMatrix

- **PopMatrix( )**: Delete position 1 matrix, position 2 matrix becomes top





# PopMatrix and PushMatrix Illustration



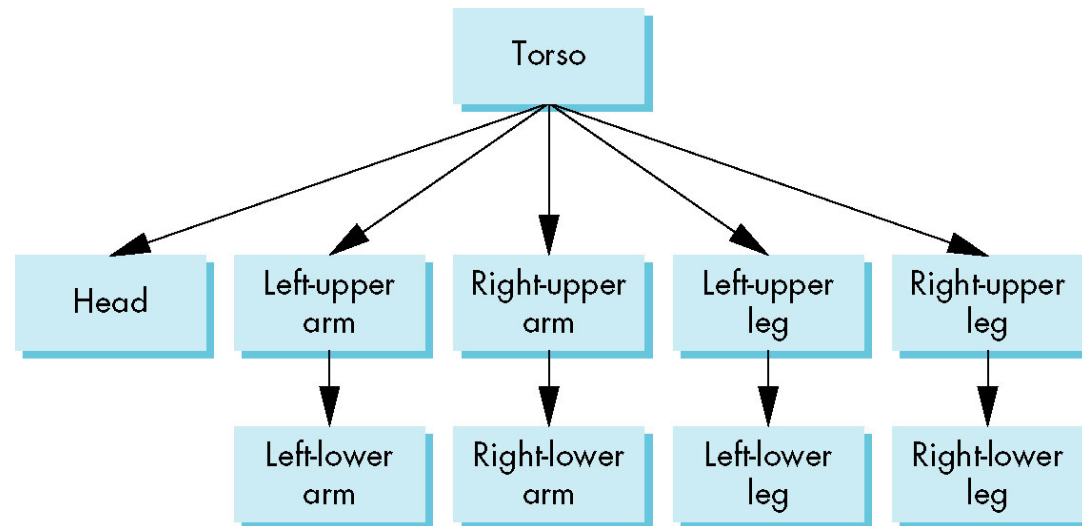
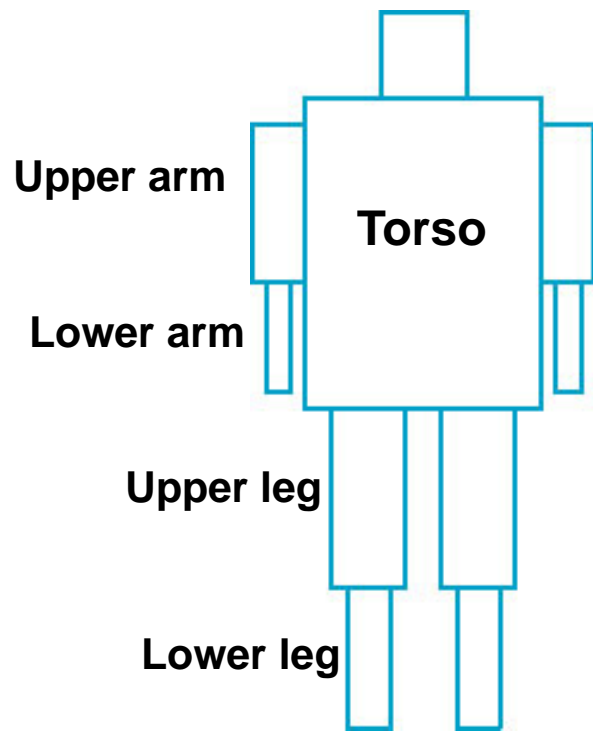
- Note: Diagram uses old `glTranslate`, `glScale`, etc commands
- We want same behavior though

Apply matrix at top of CTM to vertices of object created

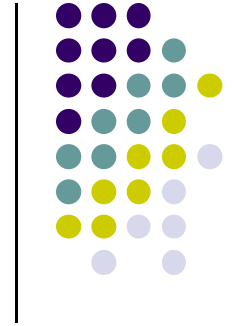
**Ref:** Computer Graphics Through OpenGL by Guha

Figure 4.19: Transitions of the modelview matrix stack.

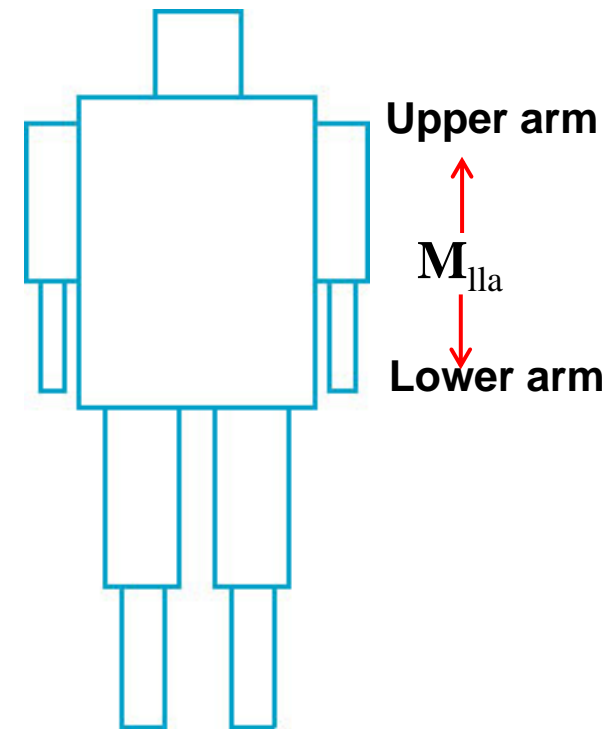
# Humanoid Figure



# Building the Model



- Draw each part as a function
  - `torso()`
  - `left_upper_arm()`, etc
- **Transform Matrices:** transform of node wrt its parent
  - $M_{lla}$  positions left lower arm with respect to left upper arm
- Stack based traversal (push, pop)





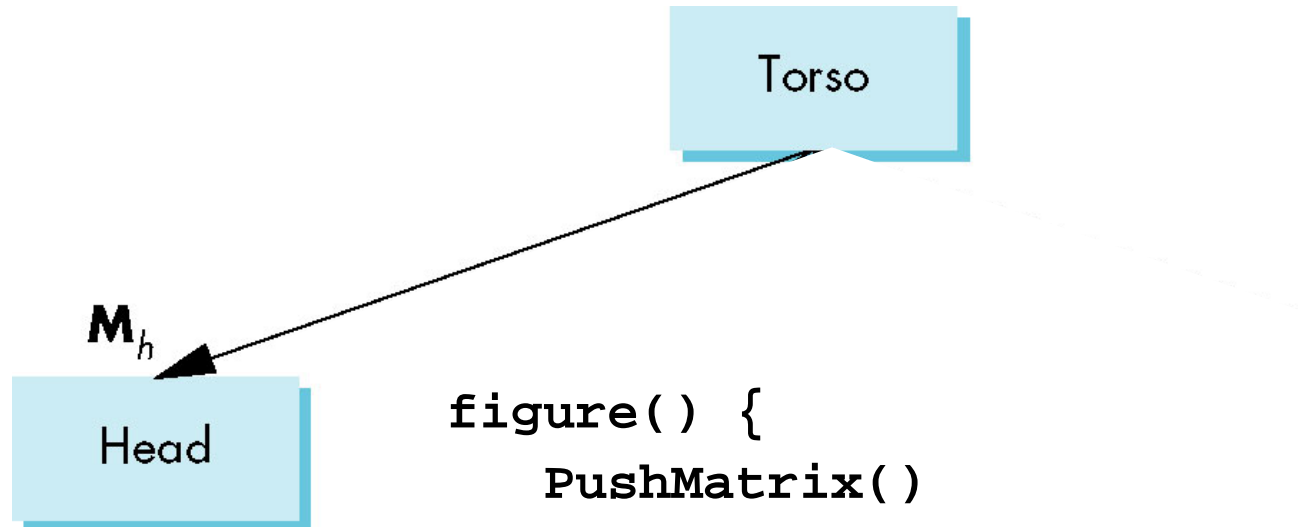
# Draw Humanoid using Stack

Torso

```
figure() {  
  PushMatrix() ← save present model-view matrix  
  torso(); ← draw torso  
}
```



# Draw Humanoid using Stack



```
figure() {  
  PushMatrix()  
  torso();  
  Rotate (...);  
  head();  
}
```

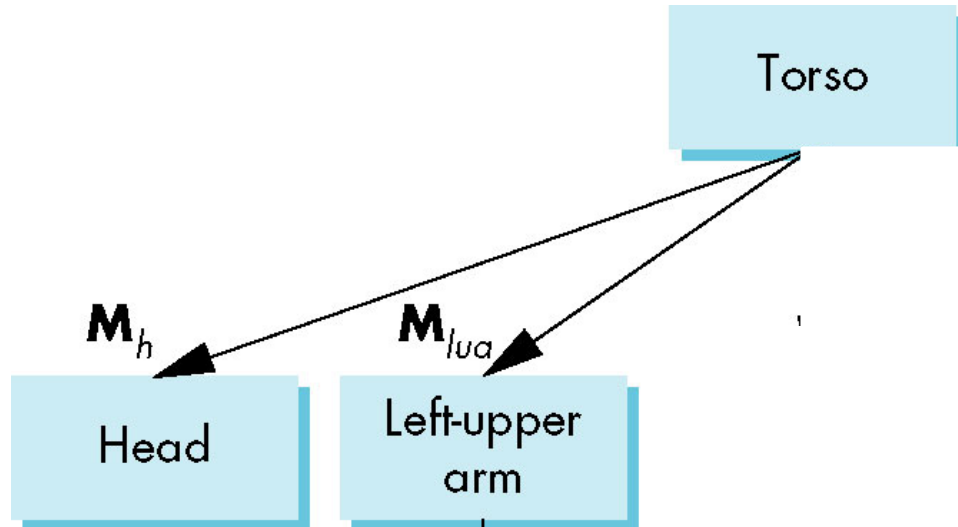
$(M_h)$  Transformation of head  
Relative to torso

draw head





# Draw Humanoid using Stack



Go back to torso matrix, and save it again

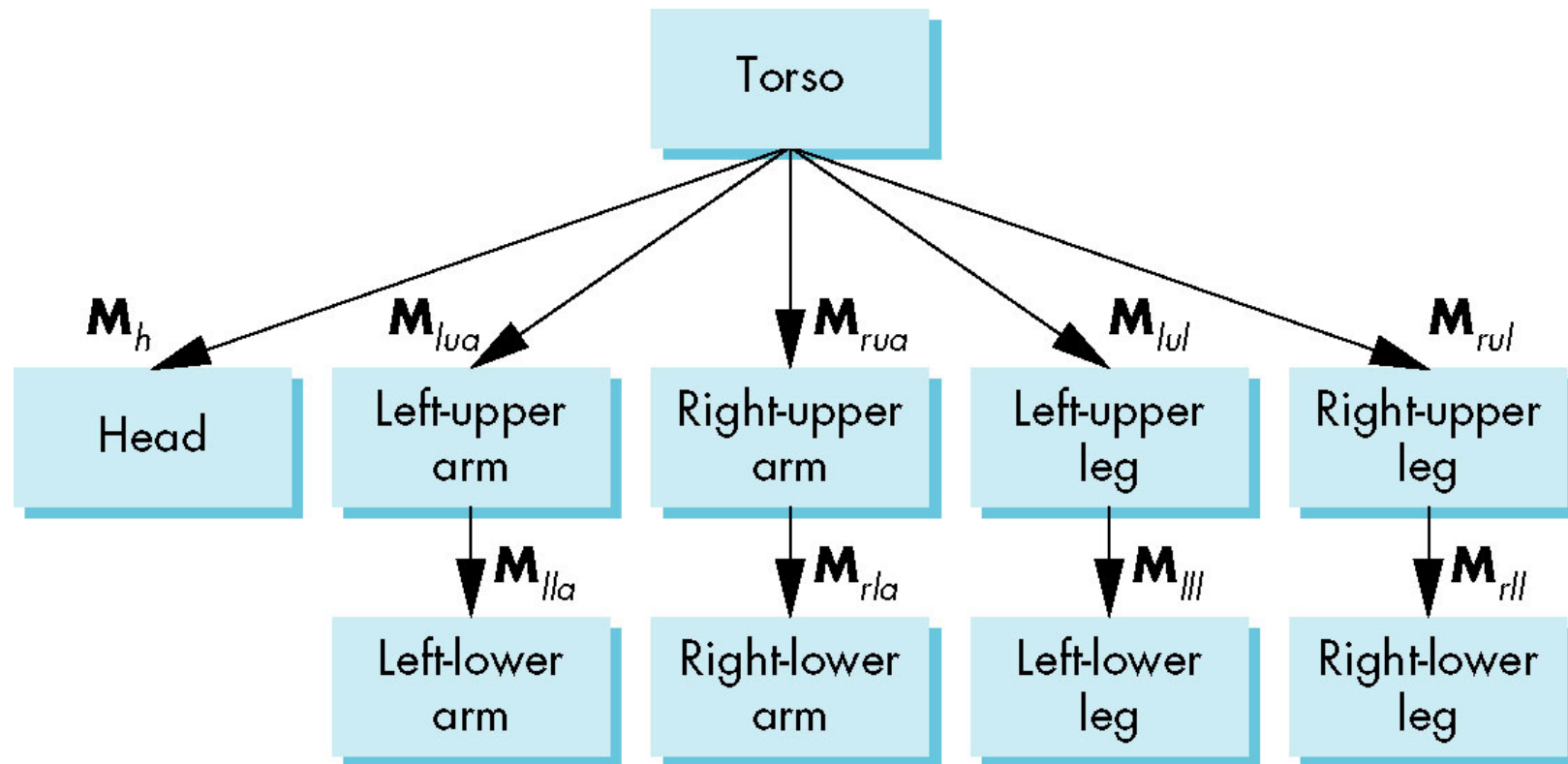
( $M_{lua}$ ) Transformation(s) of left upper arm relative to torso

draw left-upper arm

```
PushMatrix();
torso();
Rotate (...);
head();
PopMatrix();
PushMatrix();
Translate(...);
Rotate(...);
left_upper_arm();
.....
// rest of code()
```



# Complete Humanoid Tree with Matrices

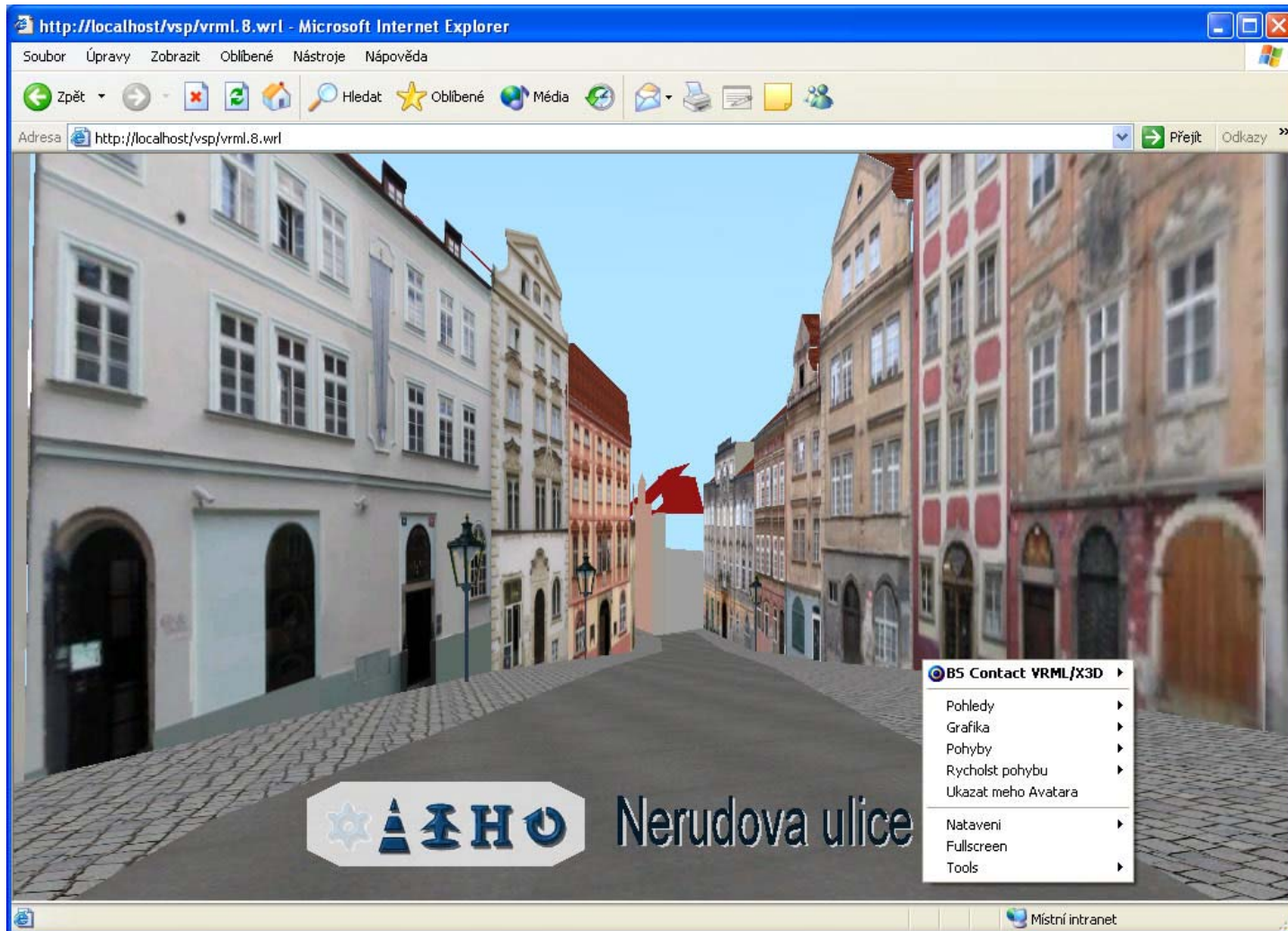


# VRML



- Scene graph introduced by SGI Open Inventor
- Used in many graphics applications (Maya, etc)
- Want scene graph for World Wide Web
- Need links scene parts in distributed data bases
- Virtual Reality Markup Language
  - Based on Inventor data base
  - Implemented with OpenGL

# VRML World Example





## References

- Angel and Shreiner, Interactive Computer Graphics (6<sup>th</sup> edition), Chapter 8