

Computer Graphics (CS 543)

Lecture 10 (Part 1): 3D Clipping

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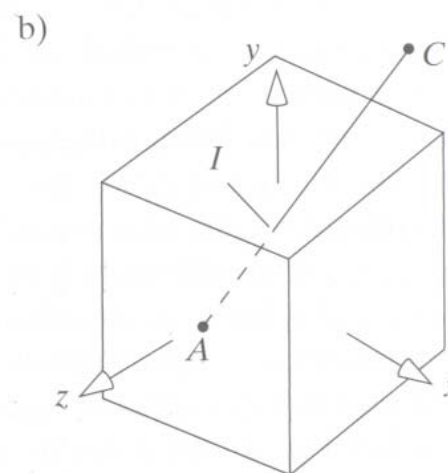
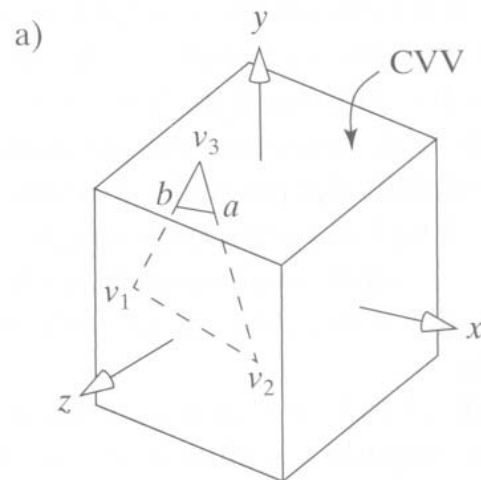
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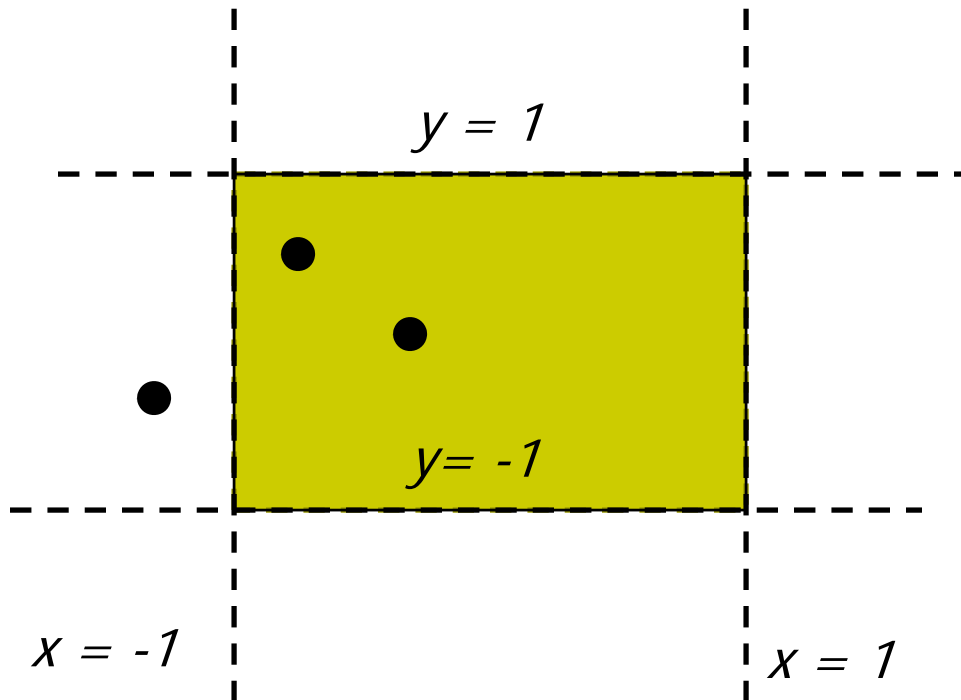
Liang-Barsky 3D Clipping

- **Goal:** Clip object edge-by-edge against Canonical View volume (CVV)
- **Problem:**
 - 2 end-points of edge: $A = (A_x, A_y, A_z, A_w)$ and $C = (C_x, C_y, C_z, C_w)$
 - If edge intersects with CVV, compute intersection point $I = (I_x, I_y, I_z, I_w)$





Determining if point is inside CVV



- **Problem:** Determine if point (x,y,z) is inside or outside CVV?

Point (x,y,z) is **inside CVV** if

$$(-1 \leq x \leq 1)$$

and $(-1 \leq y \leq 1)$

and $(-1 \leq z \leq 1)$

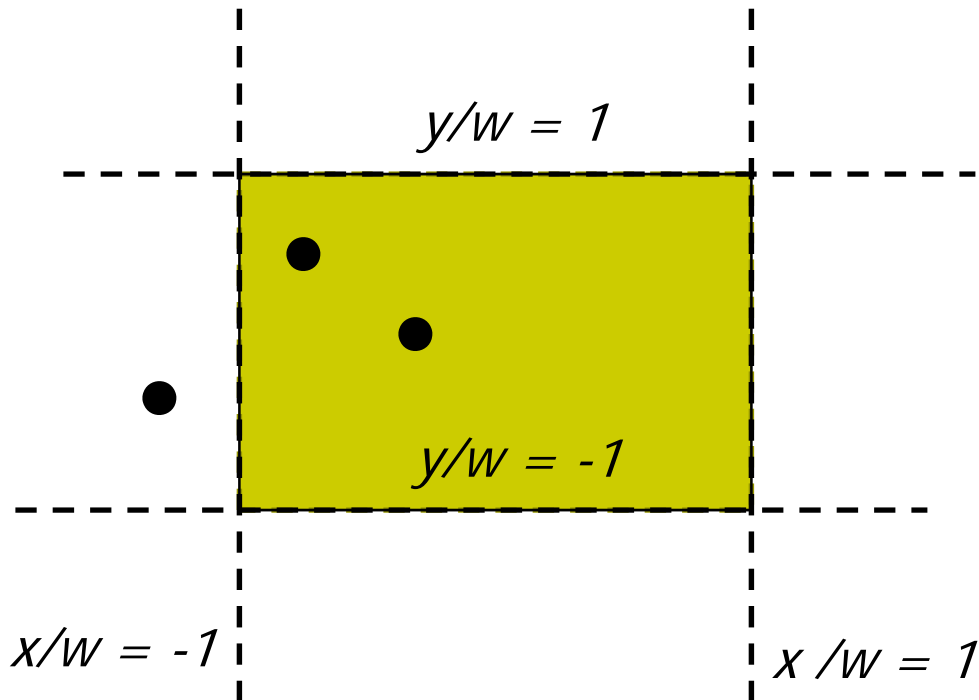
else point **is outside CVV**

- CVV == 6 infinite planes $(x=-1,1; y=-1,1; z=-1,1)$



Determining if point is inside CVV

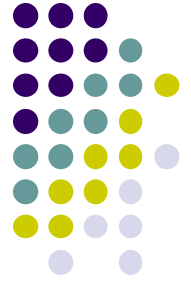
- If point specified as (x,y,z,w)
 - **Test $(x/w, y/w, z/w)$!**



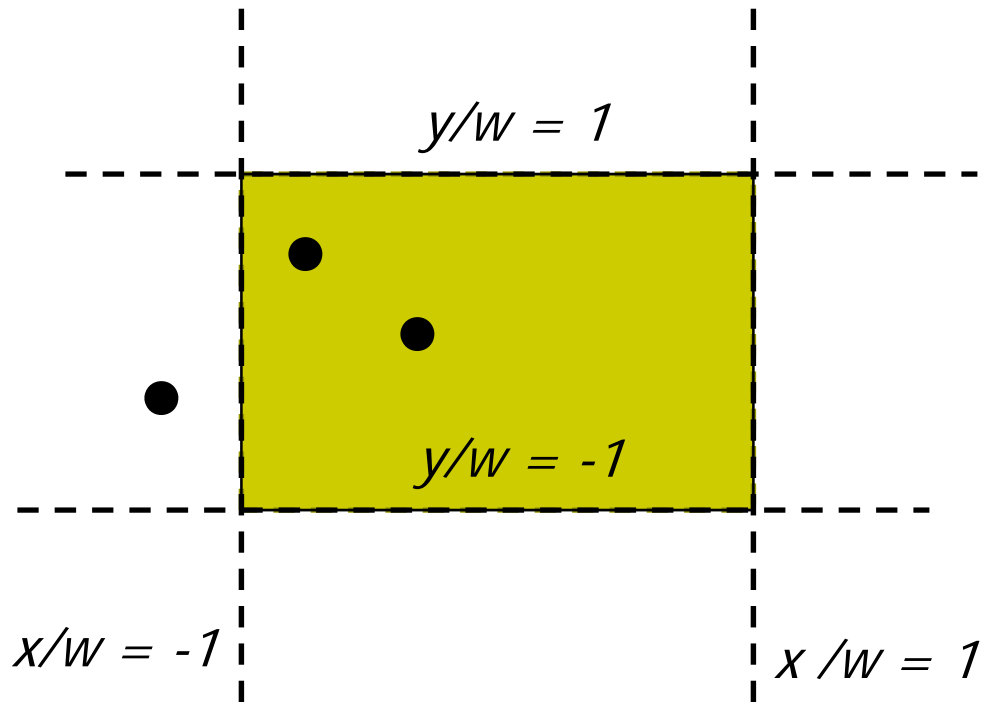
Point $(x/w, y/w, z/w)$ is inside CVV

if $(-1 \leq x/w \leq 1)$
and $(-1 \leq y/w \leq 1)$
and $(-1 \leq z/w \leq 1)$

else point is outside CVV



Modify Inside/Outside Tests Slightly



Our test: $(-1 < \mathbf{x/w} < 1)$

Point (x,y,z,w) inside plane $x = 1$ if

$$\begin{aligned} & x/w < 1 \\ \Rightarrow & \mathbf{w - x > 0} \end{aligned}$$

Point (x,y,z,w) inside plane $x = -1$ if

$$\begin{aligned} & -1 < x/w \\ \Rightarrow & \mathbf{w + x > 0} \end{aligned}$$



Numerical Example: Inside/Outside CVV Test

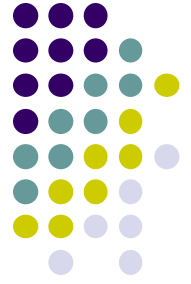
- Point (x,y,z,w) is
 - inside plane $x=-1$ **if $w+x > 0$**
 - inside plane $x=1$ **if $w - x > 0$**



- Example Point $(0.5, 0.2, 0.7)$ inside planes $(x = -1, 1)$ because $- 1 \leq 0.5 \leq 1$
- If $w = 10$, $(0.5, 0.2, 0.7) = (5, 2, 7, 10)$
- Can either **divide by w** then test: $- 1 \leq 5/10 \leq 1$ **OR**

To test if inside $x = - 1$, **$w + x = 10 + 5 = 15 > 0$**

To test if inside $x = 1$, **$w - x = 10 - 5 = 5 > 0$**

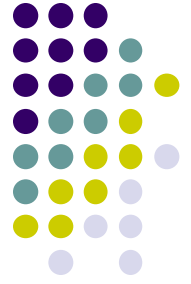


3D Clipping

- Do same for y, z to form boundary coordinates for 6 planes as:

Boundary coordinate (BC)	Homogenous coordinate	Clip plane	Example (5,2,7,10)
BC0	$w+x$	$x=-1$	15
BC1	$w-x$	$x=1$	5
BC2	$w+y$	$y=-1$	12
BC3	$w-y$	$y=1$	8
BC4	$w+z$	$z=-1$	17
BC5	$w-z$	$z=1$	3

- **Consider line that goes from point A to C**
 - **Trivial accept:** 12 BCs (6 for pt. A, 6 for pt. C) > 0
 - **Trivial reject:** Both endpoints outside (-ve) for same plane



Edges as Parametric Equations

- Implicit form $F(x, y) = 0$
- Parametric forms:
 - points specified based on single parameter value
 - Typical parameter: time t

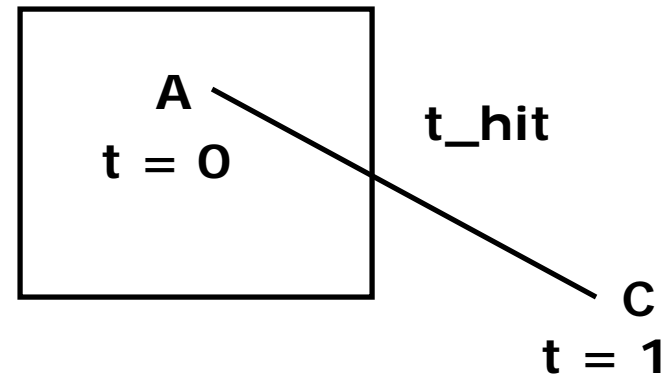
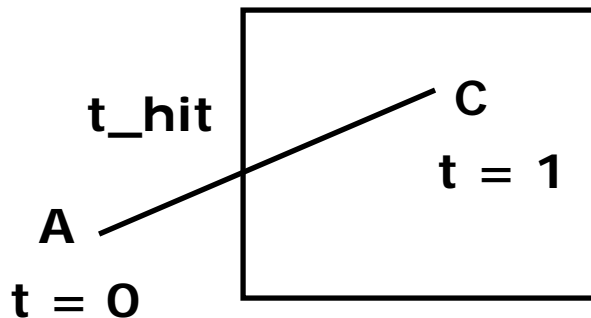
$$P(t) = P_0 + (P_1 - P_0) * t \quad 0 \leq t \leq 1$$

- Some algorithms work in parametric form
 - Clipping: exclude line segment ranges
 - Animation: Interpolate between endpoints by varying t
- Represent each edge parametrically as $A + (C - A)t$
 - at time $t=0$, point at A
 - at time $t=1$, point at C



Inside/outside?

- Test A, C against 6 walls ($x=-1,1$; $y=-1,1$; $z=-1,1$)
- There is an intersection if BCs have opposite signs. i.e. if either
 - A is outside (< 0), C is inside (> 0) or
 - A inside (> 0), C outside (< 0)
- Edge intersects with plane at some t_{hit} between $[0,1]$





Calculating hit time (t_{hit})

- How to calculate t_{hit} ?
- Represent an edge t as:

$$Edge(t) = ((Ax + (Cx - Ax)t, (Ay + (Cy - Ay)t, (Az + (Cz - Az)t, (Aw + (Cw - Aw)t)$$

- E.g. If $x = 1$,

$$\frac{Ax + (Cx - Ax)t}{Aw + (Cw - Aw)t} = 1$$

- Solving for t above,

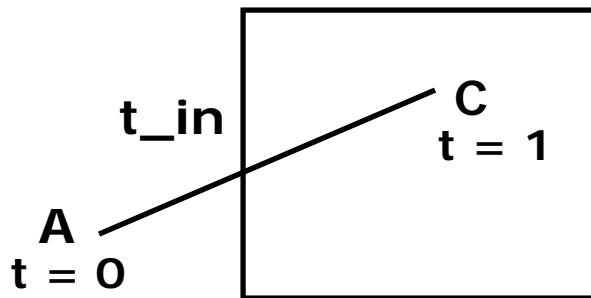
$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$



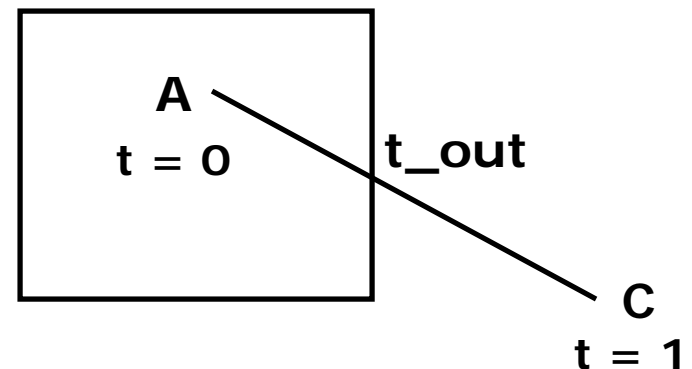
Inside/outside?

- t_{hit} can be “entering (t_{in})” or “leaving (t_{out})”
- Define: “entering” if A outside, C inside
 - Why? As t goes $[0-1]$, edge goes from outside (at A) to inside (at C)
- Define “leaving” if A inside, C outside
 - Why? As t goes $[0-1]$, edge goes from inside (at A) to outside (at C)

Entering



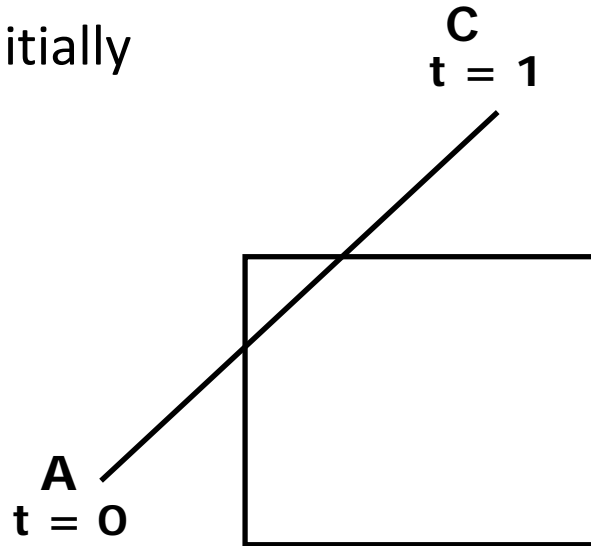
Leaving





Chop step by Step against 6 planes

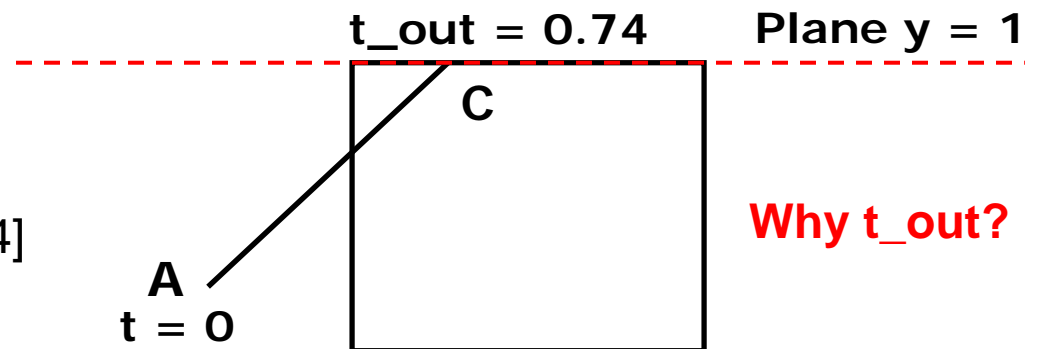
- Initially



$t_{in} = 0, \quad t_{out} = 1$
Candidate Interval (CI) = [0 to 1]

- Chop against each of 6 planes

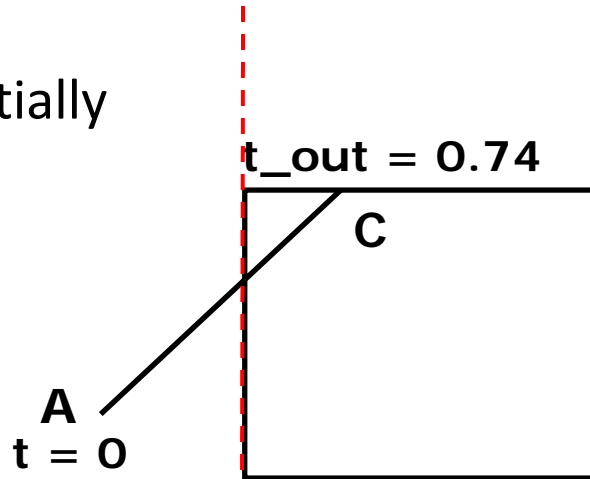
$t_{in} = 0, \quad t_{out} = 0.74$
Candidate Interval (CI) = [0 to 0.74]





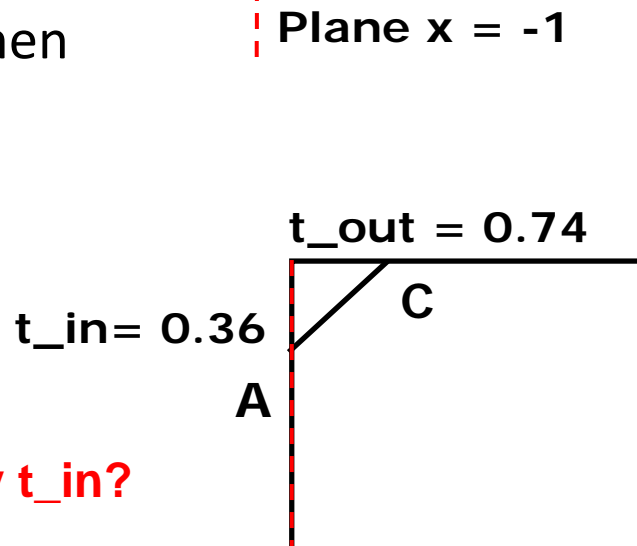
Chop step by Step against 6 planes

- Initially

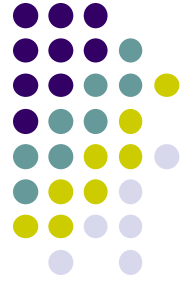


$t_{in} = 0, \quad t_{out} = 0.74$
Candidate Interval (CI) = [0 to 0.74]

- Then

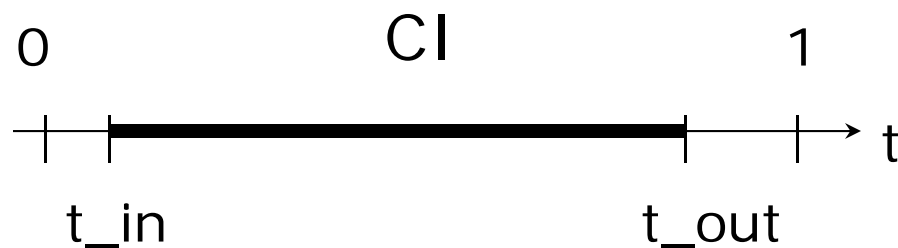


$t_{in} = 0.36, \quad t_{out} = 0.74$
Candidate Interval (CI) CI = [0.36 to 0.74]

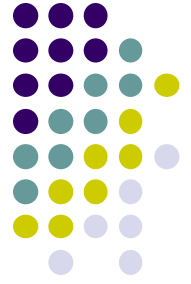


Candidate Interval

- Candidate Interval (CI): time interval during which edge might still be inside CVV. i.e. $CI = t_{in}$ to t_{out}
- Initialize CI to $[0,1]$
- For each of 6 planes, calculate t_{in} or t_{out} , shrink CI

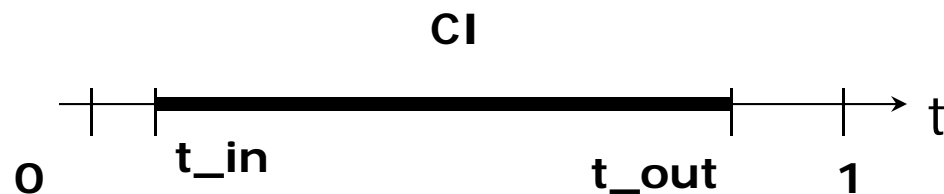


- Conversely: values of t outside CI = edge is outside CVV

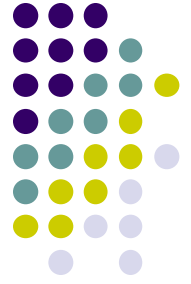


Shortening Candidate Interval

- **Algorithm:**
 - Test for trivial accept/reject (stop if either occurs)
 - Set CI to $[0,1]$
 - For each of 6 planes:
 - Find hit time t_{hit}
 - If t_{in} , new $t_{in} = \max(t_{in}, t_{hit})$
 - If t_{out} , new $t_{out} = \min(t_{out}, t_{hit})$
 - If $t_{in} > t_{out} \Rightarrow$ exit (no valid intersections)

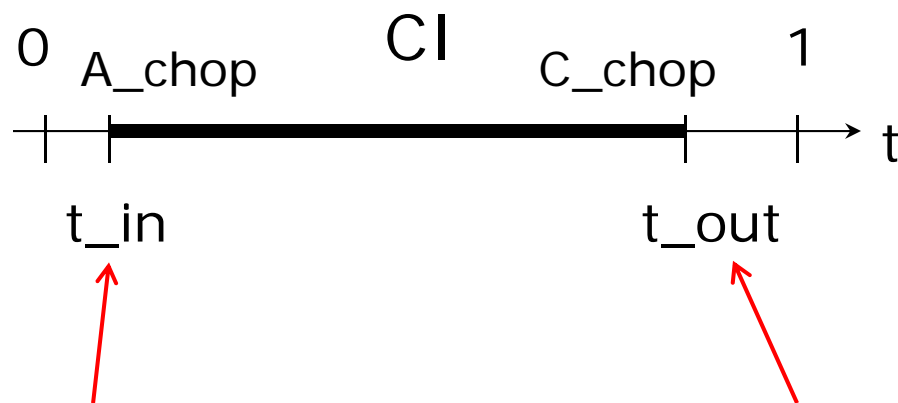


Note: seeking smallest valid CI without t_{in} crossing t_{out}



Calculate chopped A and C

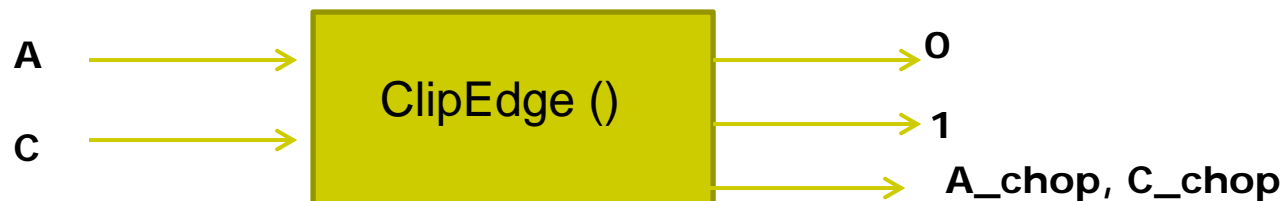
- If valid t_{in} , t_{out} , calculate adjusted edge endpoints A, C as
- $A_{chop} = A + t_{in} (C - A)$ (calculate for A_x, A_y, A_z)
- $C_{chop} = A + t_{out} (C - A)$ (calculate for C_x, C_y, C_z)





3D Clipping Implementation

- Function clipEdge()
- Input: two points A and C (in homogenous coordinates)
- Output:
 - 0, if AC lies **complete outside** CVV
 - 1, **complete inside** CVV
 - Returns clipped A and C otherwise
- Calculate 6 BCs for A, 6 for C





Store BCs as Outcodes

- Use outcodes to track in/out
 - Number walls $x = +1, -1$; $y = +1, -1$, and $z = +1, -1$ as 0 to 5
 - Bit i of A's **outcode** = 1 if A is outside i th wall
 - 1 otherwise
- **Example:** outcode for point outside walls 1, 2, 5

Wall no.	0	1	2	3	4	5
OutCode	0	1	1	0	0	1

↑ ↑ ↑



Trivial Accept/Reject using Outcodes

- **Trivial accept:** inside (not outside) any walls

Wall no.	0	1	2	3	4	5
A Outcode	0	0	0	0	0	0
C OutCode	0	0	0	0	0	0

Logical bitwise test: $A | C == 0$

- **Trivial reject:** point outside **same** wall. Example Both A and C outside wall 1

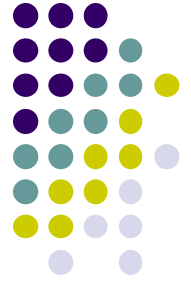
Wall no.	0	1	2	3	4	5
A Outcode	0	1	0	0	1	0
C OutCode	0	1	1	0	0	0

Logical bitwise test: $A \& C \neq 0$



3D Clipping Implementation

- Compute BCs for A,C store as outcodes
- Test A, C outcodes for trivial accept, trivial reject
- If not trivial accept/reject, for each wall:
 - Compute tHit
 - Update t_in, t_out
 - If $t_{in} > t_{out}$, early exit



3D Clipping Pseudocode

```
int clipEdge(Point4& A, Point4& C)
{
    double tIn = 0.0, tOut = 1.0, tHit;
    double aBC[6], cBC[6];
    int aOutcode = 0, cOutcode = 0;

    .....find BCs for A and C
    .....form outcodes for A and C

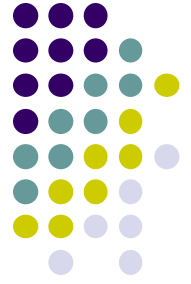
    if((aOutCode & cOutcode) != 0) // trivial reject
        return 0;
    if((aOutCode | cOutcode) == 0) // trivial accept
        return 1;
```

3D Clipping Pseudocode



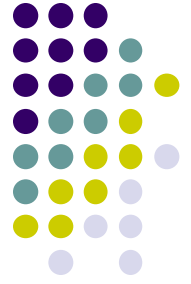
```
for(i=0;i<6;i++) // clip against each plane
{
    if(cBC[i] < 0) // C is outside wall i (exit so tOut)
    {
        tHit = aBC[i]/(aBC[i] - cBC[i]); // calculate tHit
        tOut = MIN(tOut, tHit);
    }
    else if(aBC[i] < 0) // A is outside wall i (enters so tIn)
    {
        tHit = aBC[i]/(aBC[i] - cBC[i]); // calculate tHit
        tIn = MAX(tIn, tHit);
    }
    if(tIn > tOut) return 0; // CI is empty: early out
}
```

$$t = \frac{A_w - A_x}{(A_w - A_x) - (C_w - C_x)}$$



3D Clipping Pseudocode

```
Point4 tmp; // stores homogeneous coordinates
If(aOutcode != 0) // A is outside: tIn has changed. Calculate A_chop
{
    tmp.x = A.x + tIn * (C.x - A.x);
    // do same for y, z, and w components
}
If(cOutcode != 0) // C is outside: tOut has changed. Calculate C_chop
{
    C.x = A.x + tOut * (C.x - A.x);
    // do same for y, z and w components
}
A = tmp;
Return 1; // some of the edges lie inside CVV
}
```



Polygon Clipping

- Not as simple as line segment clipping
 - Clipping a line segment yields at most one line segment
 - Clipping a polygon can yield multiple polygons



- Clipping a convex polygon can yield at most one other polygon



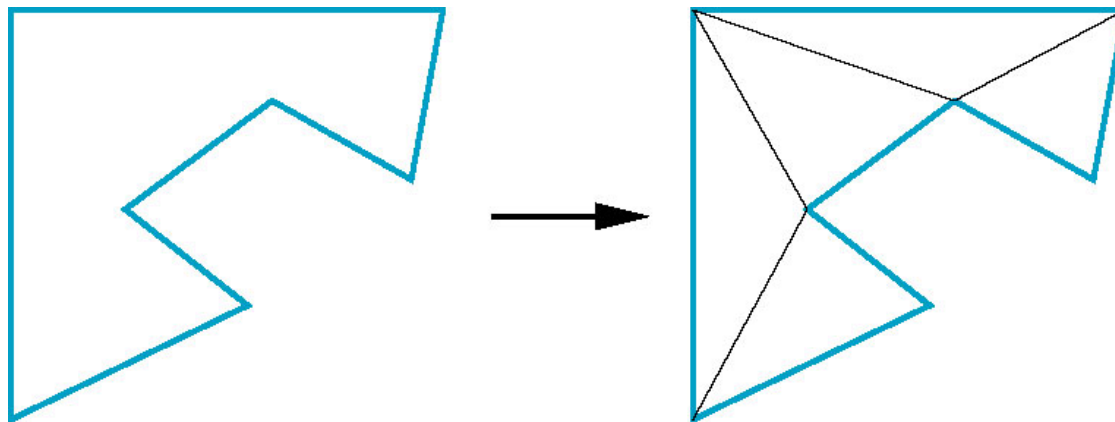
Clipping Polygons

- Need more sophisticated algorithms to handle polygons:
 - *Sutherland-Hodgman*: any a given polygon against a **convex** clip polygon (or window)
 - *Weiler-Atherton*: Both subject polygon and clip polygon can be **concave**



Tessellation and Convexity

- One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)
- Also makes fill easier





References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition