Computer Graphics (CS 543)
Lecture 10 (Part 3): Rasterization: Line Drawing

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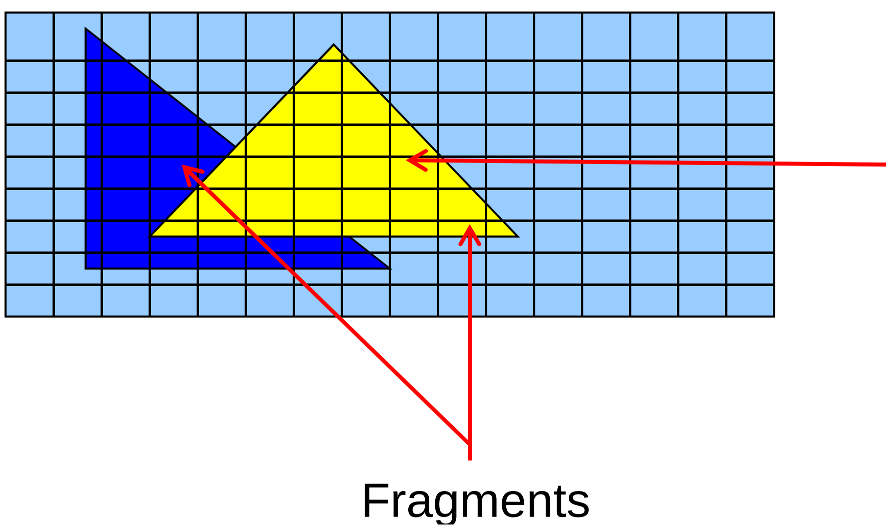
Rasterization

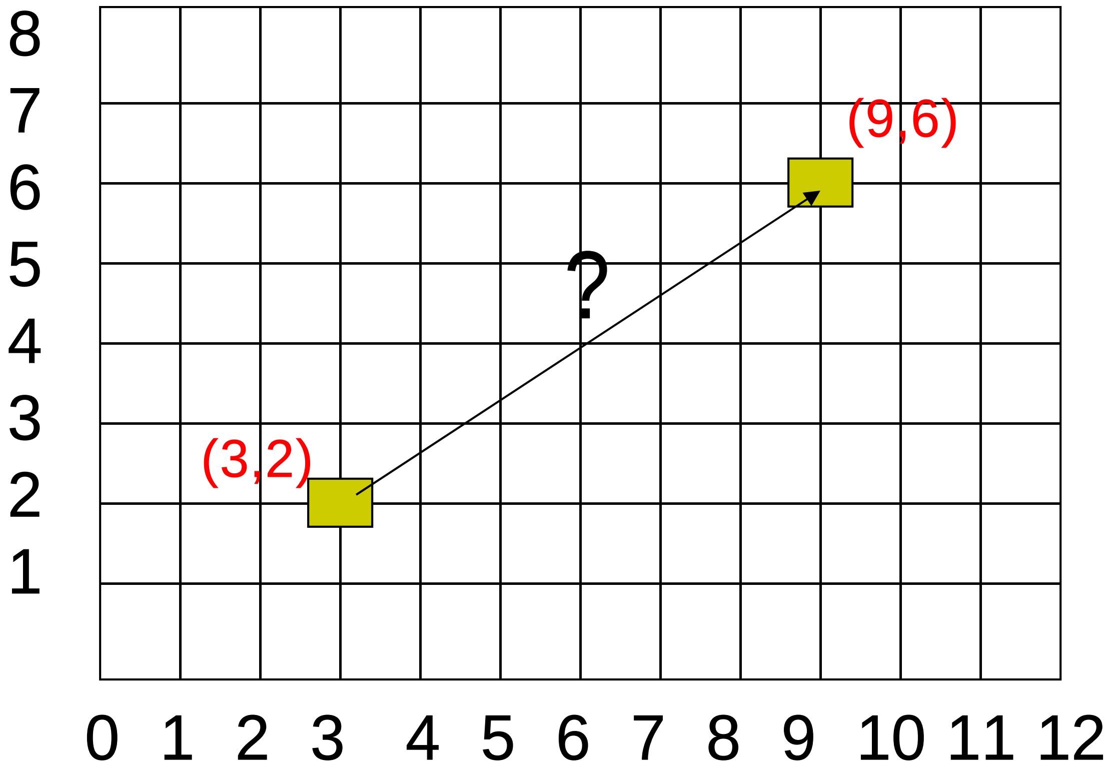
- Rasterization produces set of **fragments**
- Implemented by graphics hardware
- Rasterization algorithms for primitives (e.g lines, circles, triangles, polygons)



Rasterization: Determine Pixels (fragments) each primitive covers

Line drawing algorithm

- Programmer specifies (x,y) of end pixels
- Need algorithm to determine pixels on line path

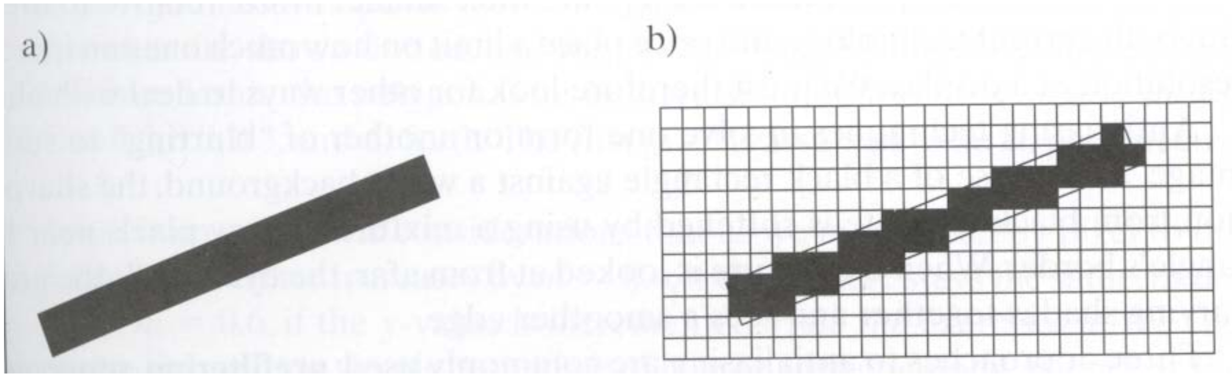


Line: (3,2) -> (9,6)

Which intermediate pixels to turn on?

Line drawing algorithm

- Pixel (x,y) values constrained to integer values
- Computed intermediate values may be floats
- Rounding may be required. E.g. (10.48, 20.51) rounded to (10, 21)
- Rounded pixel value is off actual line path (jaggy!!)
- Sloped lines end up having jaggies
- Vertical, horizontal lines, no jaggies

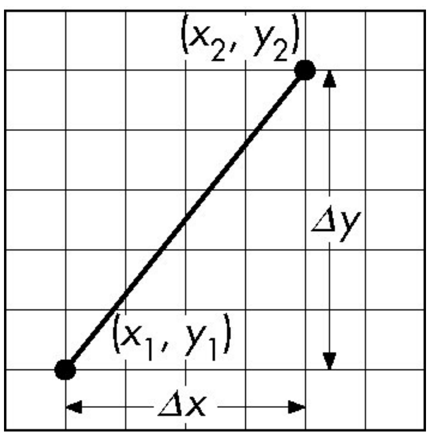


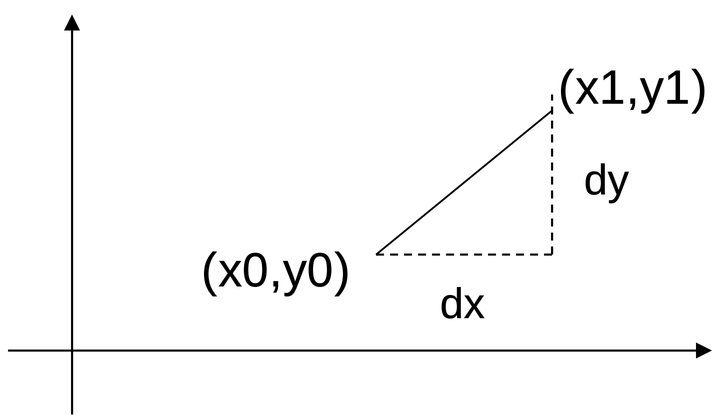
Line Drawing Algorithm

- Slope-intercept line equation
  - y = mx + b
  - Given 2 end points (x0,y0), (x1, y1), how to compute m and b?

$$m = \frac{dy}{dx} = \frac{y1 - y0}{x1 - x0}$$

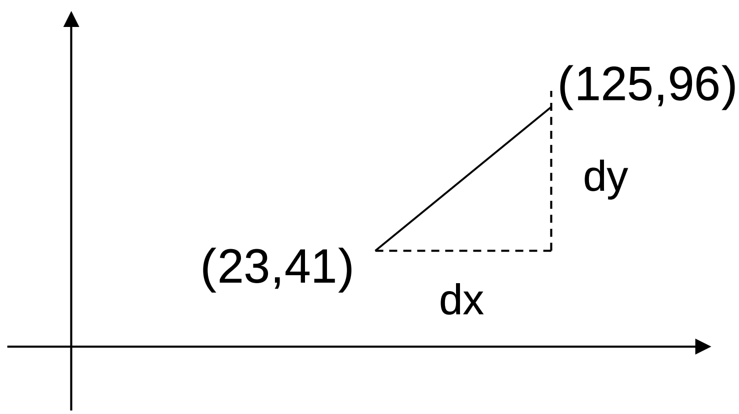
$$y0 = m * x0 + b$$
$$\Rightarrow b = y0 - m * x0$$





Line Drawing Algorithm

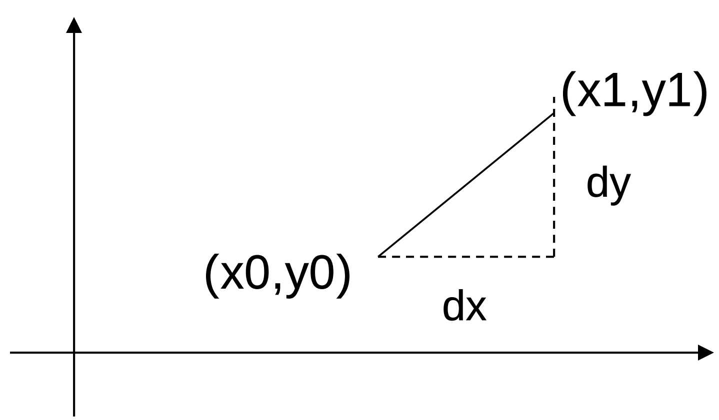
- Numerical example of finding slope m:
  - (Ax, Ay) = (23, 41), (Bx, By) = (125, 96)

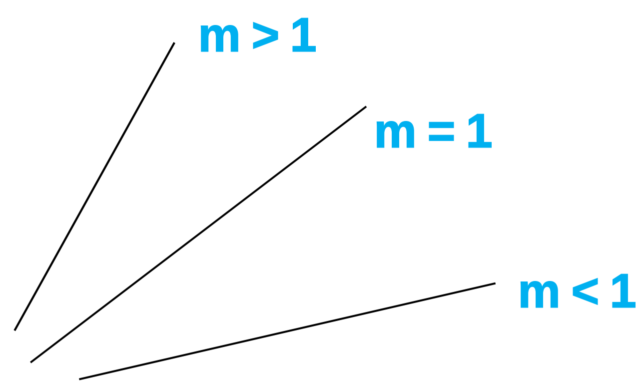


$$m = \frac{By - Ay}{Bx - Ax} = \frac{96 - 41}{125 - 23} = \frac{55}{102} = 0.5392$$

Digital Differential Analyzer (DDA): Line Drawing Algorithm

Consider slope of line, m:



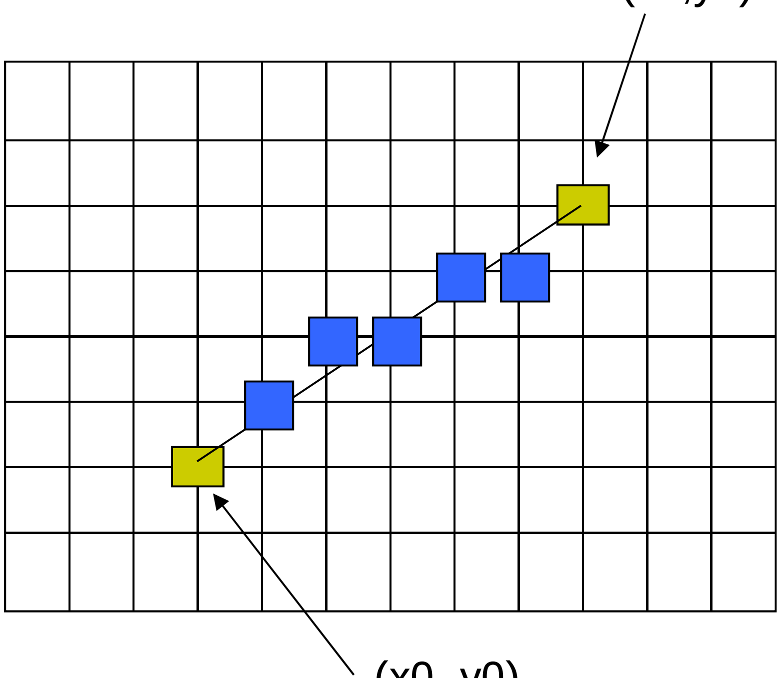


- Step through line, starting at (x0,y0)
- **Case a: (m < 1)** x incrementing faster
  - Step in x=1 increments, compute y (a fraction) and round
- **Case b: (m > 1)** y incrementing faster
  - Step in y=1 increments, compute x (a fraction) and round

DDA Line Drawing Algorithm (Case a: m < 1)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} = \frac{y_{k+1} - y_k}{1}$$

$$\Rightarrow y_{k+1} = y_k + m$$



x = x0 y = y0

Illuminate pixel (x, round(y))

x = x + 1 y = y + m

Illuminate pixel (x, round(y))

x = x + 1 y = y + m

Illuminate pixel (x, round(y))

...

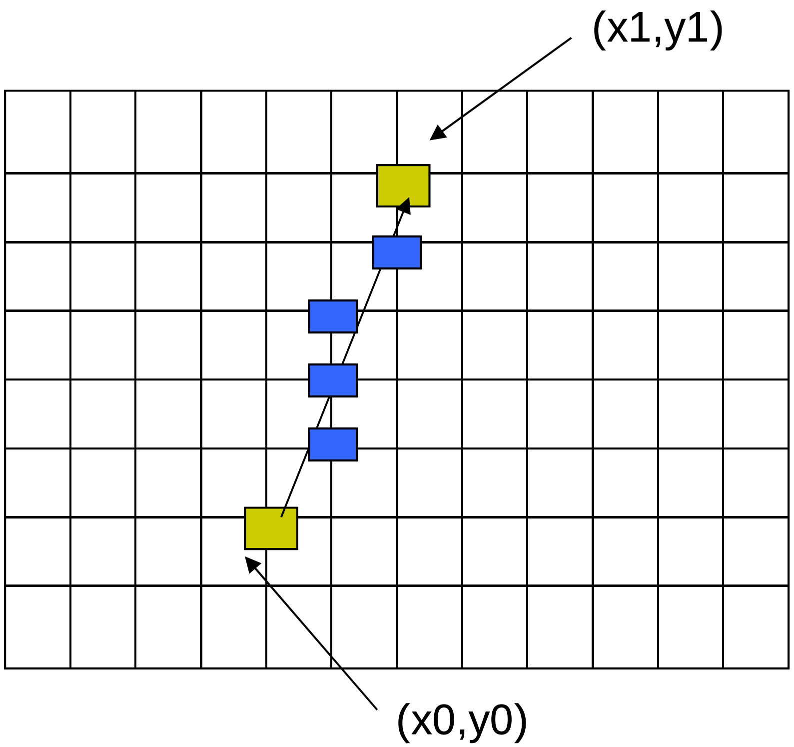
Until x == x1

Example, if first end point is (0,0)
Example, if m = 0.2
Step 1: x = 1, y = 0.2 => shade (1,0)
Step 2: x = 2, y = 0.4 => shade (2, 0)
Step 3: x= 3, y = 0.6 => shade (3, 1)
... etc

DDA Line Drawing Algorithm (Case b: m > 1)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} = \frac{1}{x_{k+1} - x_k}$$

$$\Rightarrow x_{k+1} = x_k + \frac{1}{m}$$



x = x0 y = y0

Illuminate pixel (round(x), y)

y = y + 1 x = x + 1/m

Illuminate pixel (round(x), y)

y = y + 1 x = x + 1 /m

Illuminate pixel (round(x), y)

...

Until y == y1

Example, if first end point is (0,0)
if 1/m = 0.2
Step 1: y = 1, x = 0.2 => shade (0,1)
Step 2: y = 2, x = 0.4 => shade (0, 2)
Step 3: y= 3, x = 0.6 => shade (1, 3)
... etc

DDA Line Drawing Algorithm Pseudocode

```
compute m;
if m < 1:
{
   float y = y0;        // initial value
   for(int x = x0;  x <= x1;  x++, y += m)
               setPixel(x, round(y));
}
else   // m > 1
{
   float x = x0;        // initial value
   for(int y = y0;  y <= y1;  y++, x += 1/m)
              setPixel(round(x), y);
}
```

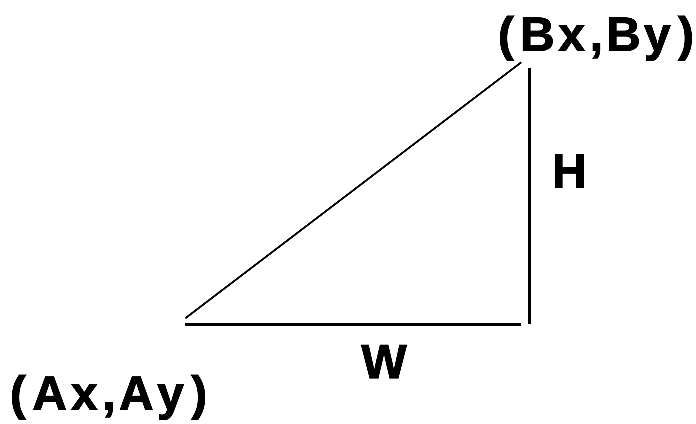
- **Note:** `setPixel(x, y)` writes current color into pixel in column x and row y in frame buffer

Line Drawing Algorithm Drawbacks

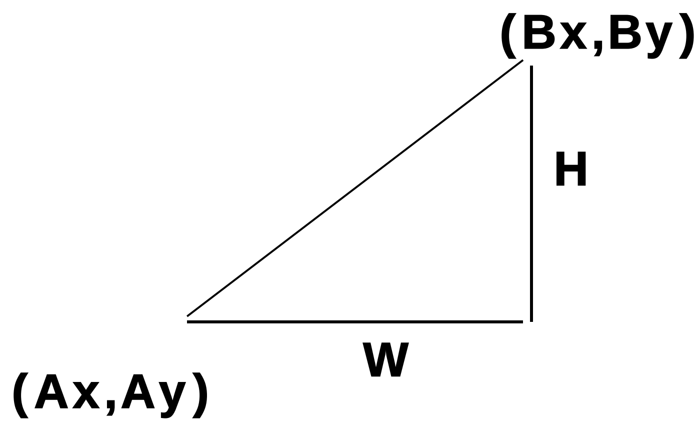
- DDA is the simplest line drawing algorithm
  - Not very efficient
  - Round operation is expensive
- Optimized algorithms typically used.
  - Integer DDA
  - E.g.Bresenham algorithm
- Bresenham algorithm
  - Incremental algorithm: current value uses previous value
  - Integers only: avoid floating point arithmetic
  - Several versions of algorithm: we'll describe midpoint version of algorithm

Bresenham's Line-Drawing Algorithm

- **Problem:** Given endpoints (Ax, Ay) and (Bx, By) of line, determine intervening pixels
- First make two simplifying assumptions (remove later):
  - (Ax < Bx) and
  - (0 < m < 1)
- Define
  - Width W = Bx – Ax
  - Height H = By - Ay



Bresenham's Line-Drawing Algorithm



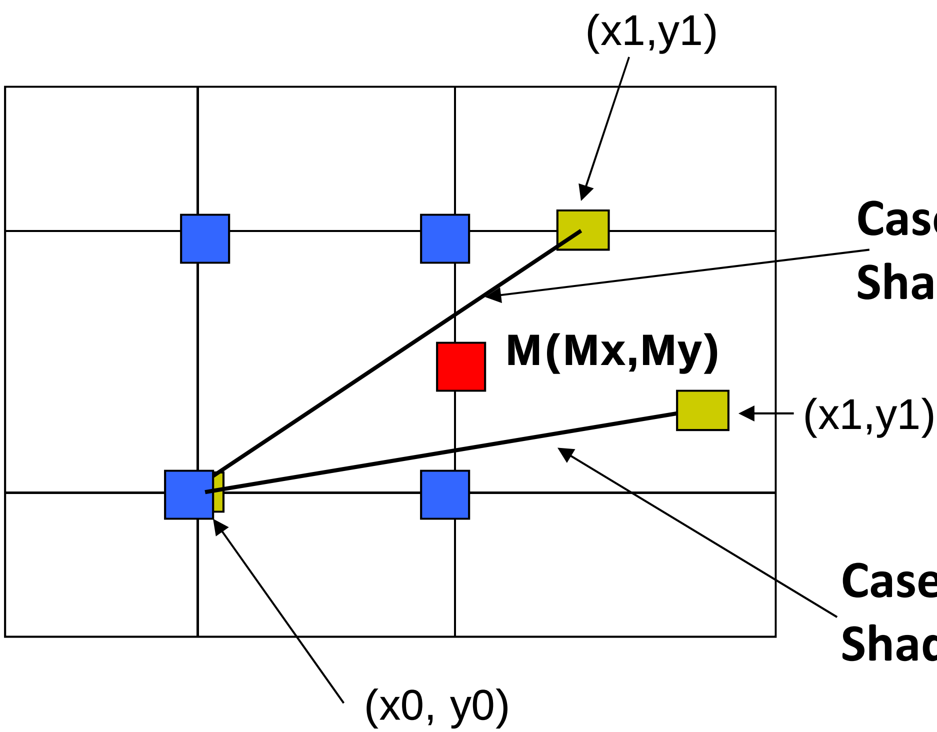
- Based on assumptions (Ax < Bx) and (0 < m < 1)
  - W, H are +ve
  - H < W
- Increment x by +1, y incr by +1 or stays same
- Midpoint algorithm determines which happens

Bresenham's Line-Drawing Algorithm

What Pixels to turn on or off?

Consider pixel midpoint M(Mx, My) = (x + 1, y + ½)

Build equation of actual line, compare to midpoint



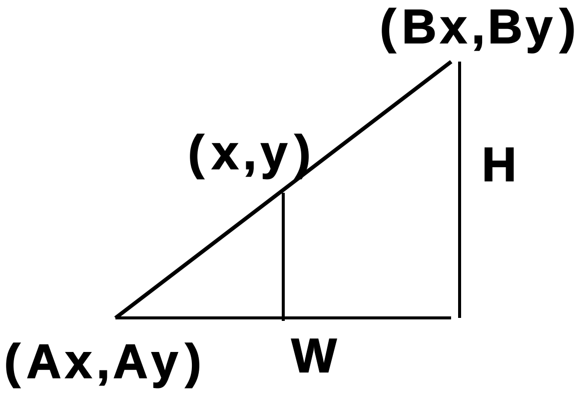
Case a: If midpoint (red dot) is below line,
Shade upper pixel, (x + 1, y + 1)

Case b: If midpoint (red dot) is above line,
Shade lower pixel, (x + 1, y)

Build Equation of the Line

- Using similar triangles:

$$\frac{y - Ay}{x - Ax} = \frac{H}{W}$$



$$H(x - Ax) = W(y - Ay)$$
$$-W(y - Ay) + H(x - Ax) = 0$$

- Above is equation of line from (Ax, Ay) to (Bx, By)
- Thus, any point (x,y) that lies on ideal line makes eqn = 0
- Double expression (to avoid floats later), and call it F(x,y)

$$F(x,y) = -2W(y - Ay) + 2H(x - Ax)$$

Bresenham's Line-Drawing Algorithm

- So, $F(x,y) = -2W(y - Ay) + 2H(x - Ax)$

- Algorithm, If:
  - $F(x, y) < 0$, (x, y) above line
  - $F(x, y) > 0$, (x, y) below line

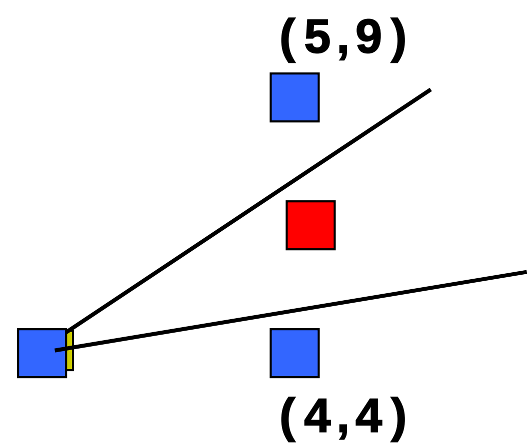
- **Hint:** $F(x, y) = 0$ is on line
- Increase y keeping x constant, $F(x, y)$ becomes more negative

Bresenham's Line-Drawing Algorithm

- **Example:** to find line segment between (3, 7) and (9, 11)

$$F(x,y) = -2W(y - Ay) + 2H(x - Ax)$$
$$= (-12)(y - 7) + (8)(x - 3)$$

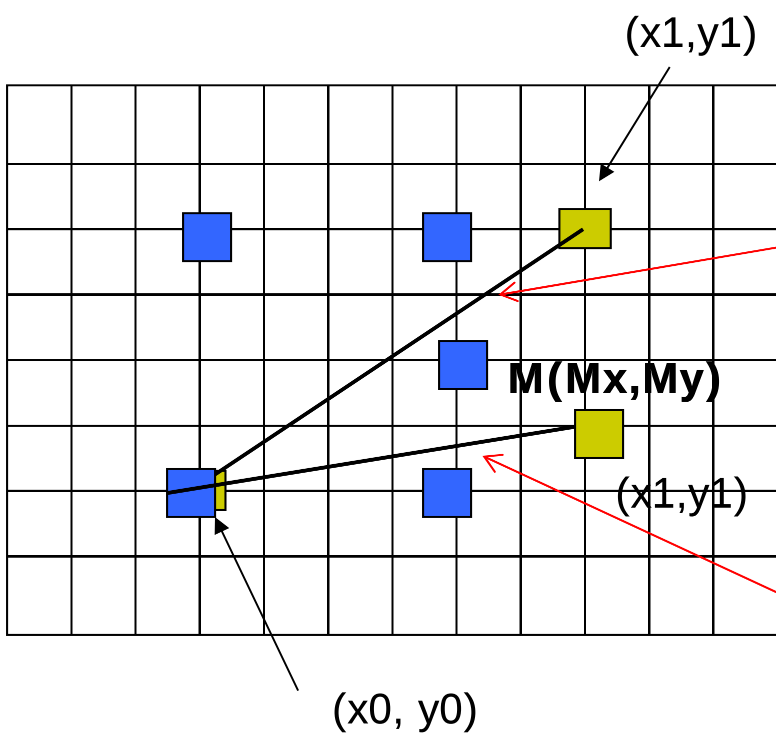
- For points on line. E.g. (7, 29/3), F(x, y) = 0
- A = (4, 4) lies below line since F = 44
- B = (5, 9) lies above line since F = -8



Bresenham's Line-Drawing Algorithm

What Pixels to turn on or off?

Consider pixel midpoint M(Mx, My) = (x0 + 1, Y0 + ½)



Case a: If M below actual line
F(Mx, My) < 0
shade upper pixel (x + 1, y + 1)

Case b: If M above actual line
F(Mx,My) > 0
shade lower pixel (x + 1, y)

Can compute F(x,y) incrementally

Initially, midpoint M = (Ax + 1, Ay + ½)

$$F(Mx, My) = -2W(y - Ay) + 2H(x - Ax)$$

i.e. F(Ax + 1, Ay + ½) = 2H – W

Can compute F(x,y) for next midpoint incrementally

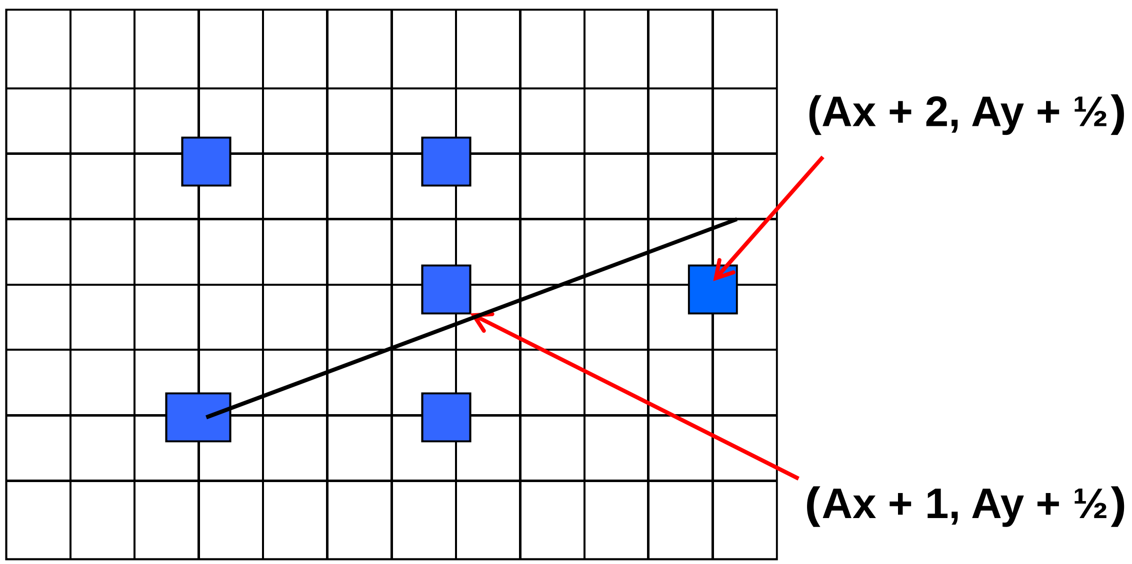
If we increment to (x + 1, y), compute new F(Mx,My)

F(Mx, My) += 2H

i.e. F(Ax + 2, Ay + ½)

- F(Ax + 1, Ay + ½)

= 2H

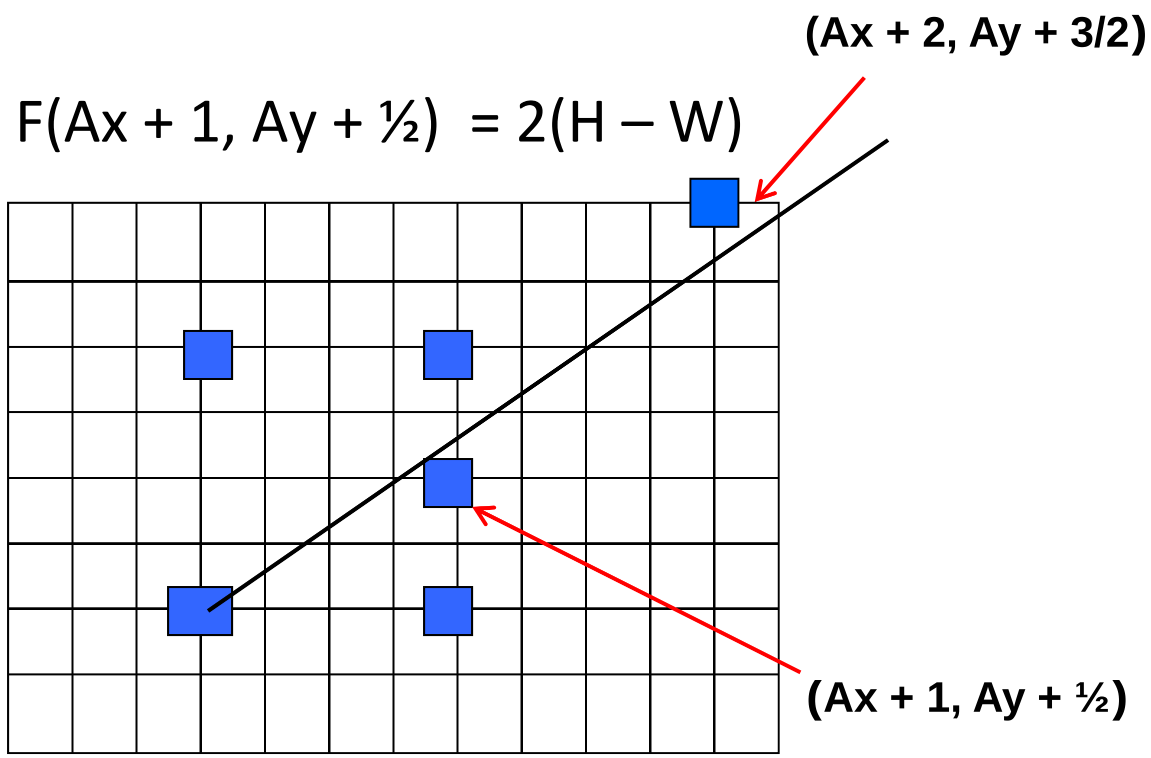


Can compute F(x,y) incrementally

If we increment to (x +1, y + 1)

$F(Mx, My) \mathrel{+}= 2(H - W)$

i.e. $F(Ax + 2, Ay + 3/2) - F(Ax + 1, Ay + ½) = 2(H - W)$



Bresenham's Line-Drawing Algorithm

```
Bresenham(IntPoint a, InPoint b)
{ // restriction: a.x < b.x and 0 < H/W < 1
      int y = a.y, W = b.x – a.x, H = b.y – a.y;
      int F = 2 * H – W;   // current error term
      for(int x = a.x;   x <= b.x;   x++)
      {
        setpixel at (x, y);  // to desired color value
             if F < 0            // y stays same
                   F = F + 2H;
             else{
                   Y++,  F = F  + 2(H – W)     // increment y
             }
      }
}
```

- Recall: F is equation of line

Bresenham's Line-Drawing Algorithm

- Final words: we developed algorithm with restrictions

  $0 < m < 1$ and $Ax < Bx$

- Can add code to remove restrictions
  - When $Ax > Bx$ (swap and draw)
  - Lines having $m > 1$ (interchange x with y)
  - Lines with $m < 0$ (step x++, decrement y not incr)
  - Horizontal and vertical lines (pretest a.x = b.x and skip tests)

References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Chapter 9