Computer Graphics (CS 543)
Lecture 12a: 3D Clipping

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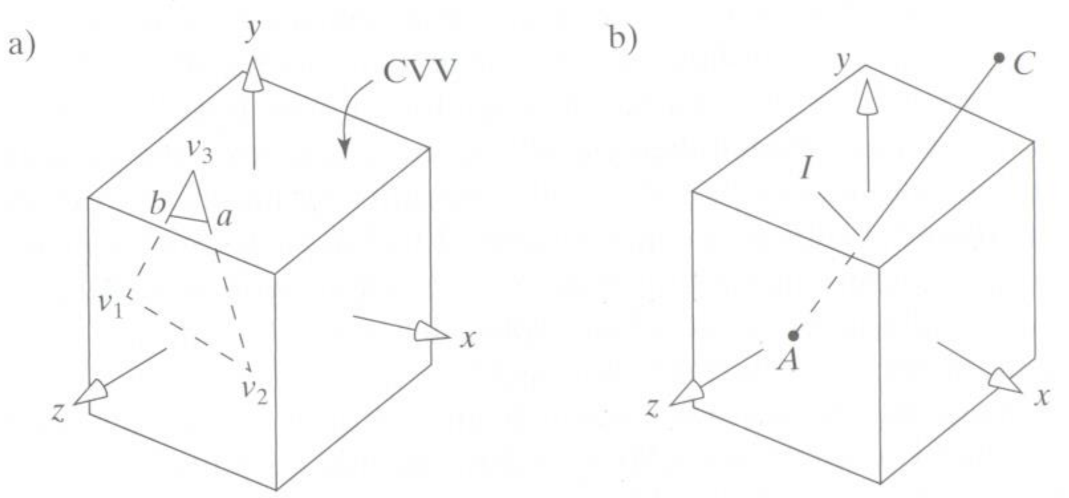
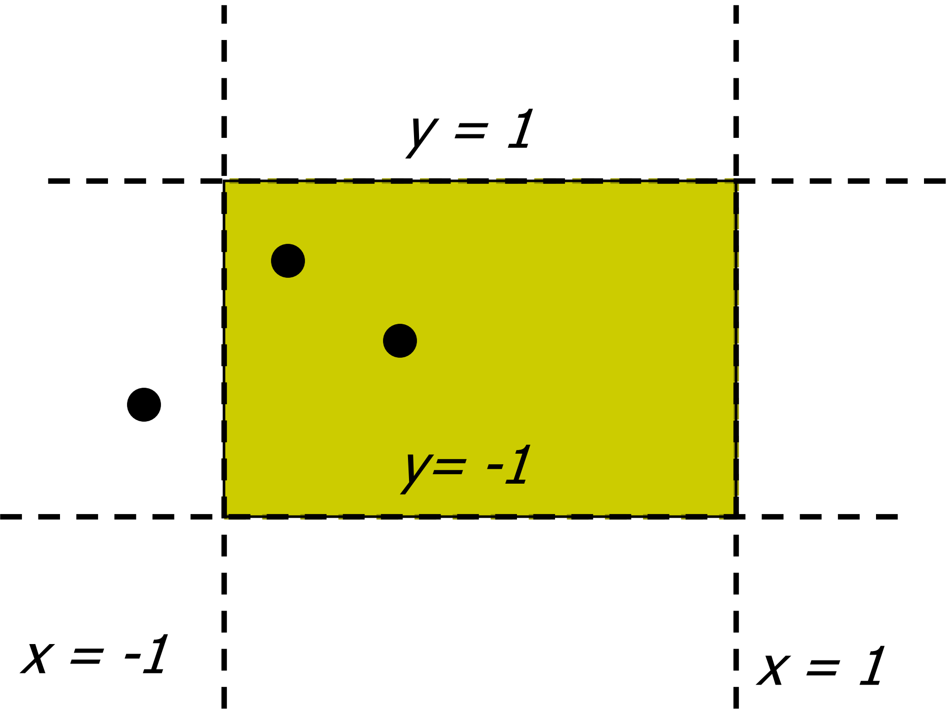
Liang-Barsky 3D Clipping

Ref: Computer Graphics using OpenGL, Hill and Kelley, 3rd edition, pages 356-360

- **Goal:** Clip object edge-by-edge against Canonical View volume (CVV)
- **Problem:**
  - 2 end-points of edge: **A = (Ax, Ay, Az, Aw)** and **C = (Cx, Cy, Cz, Cw)**
  - If edge intersects with CVV, compute intersection point **I =(Ix,Iy,Iz,Iw)**



Determining if point is inside CVV



- **Problem:** Determine if point (x,y,z) is inside or outside CVV?

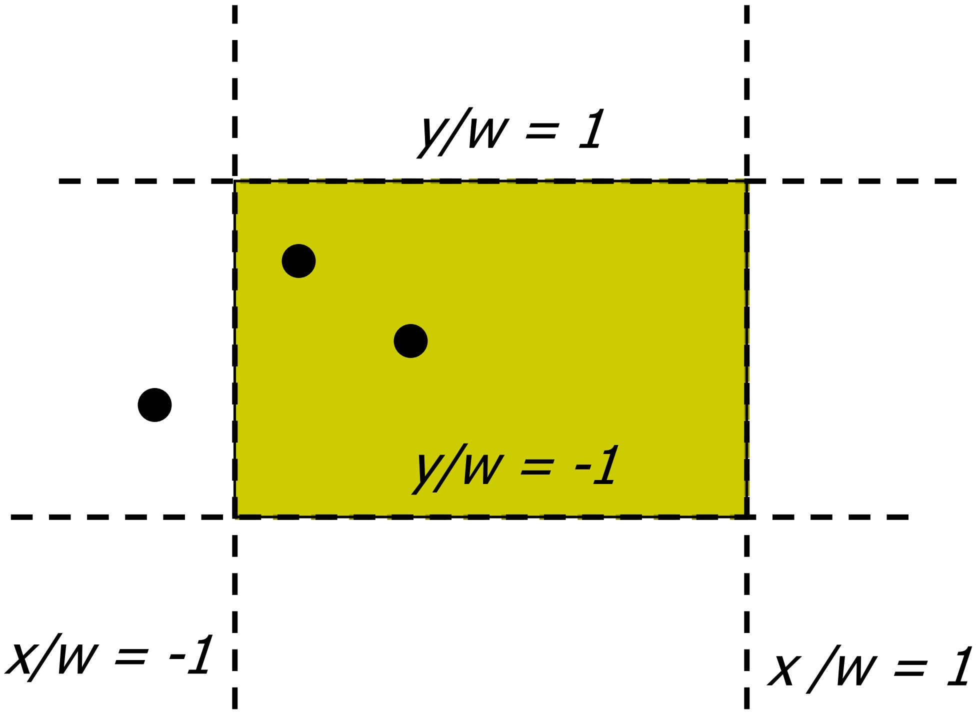
Point (x,y,z) is **inside CVV if**

(-1 <= x <= 1)
and (-1 <= y <= 1)
and (-1 <= z <= 1)

else point **is outside CVV**

- **CVV == 6 infinite planes (x=-1,1; y=-1,1; z=-1,1)**

Determining if point is inside CVV



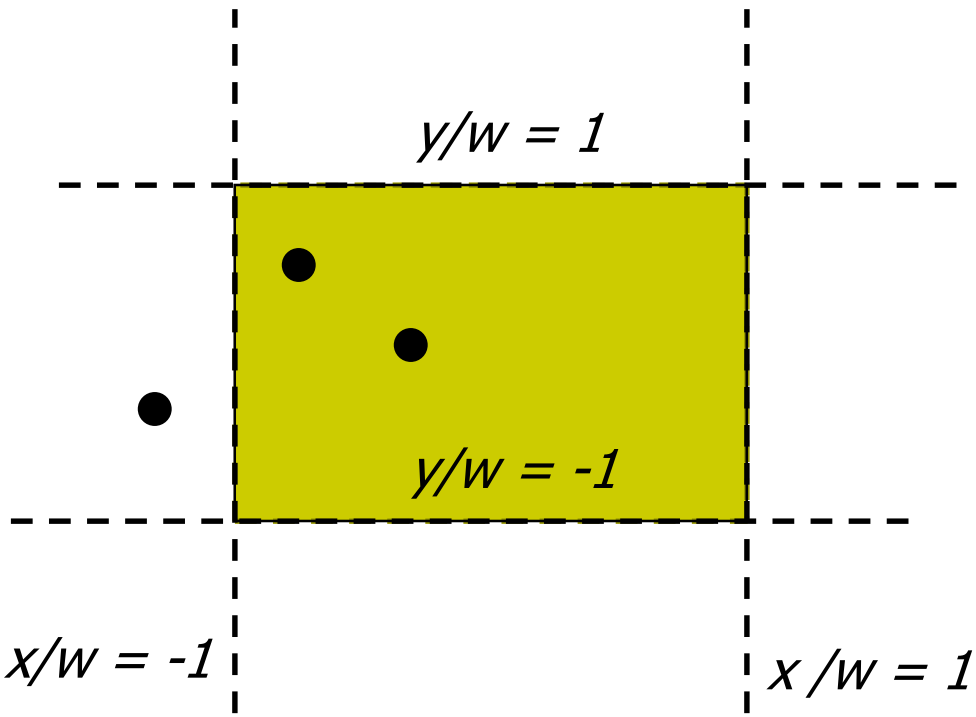
- If point specified as (x,y,z,w)
- **Test (x/w, y/w , z/w)!**

Point (x/w, y/w, z/w) is inside CVV

if (-1 <= **x/w** <= 1)
and (-1 <= **y/w** <= 1)
and (-1 <= **z/w** <= 1)

else point is outside CVV

Modify Inside/Outside Tests Slightly



Our test: (-1 < **x/w** < 1)

Point (x,y,z,w) inside plane x = 1 if

x/w < 1

=> **w – x > 0**

Point (x,y,z,w) inside plane x = -1 if

-1 < x/w

=> **w + x > 0**

Numerical Example: Inside/Outside CVV Test

- Point (x,y,z,w) is
  - inside plane x=-1 **if w+x > 0**
  - inside plane x=1 **if w – x > 0**

- Example Point (0.5, 0.2, 0.7) inside planes (x = -1,1) because - 1 <= 0.5 <= 1

- If w = 10, (0.5, 0.2, 0.7) = (5, 2, 7, 10)
- Can either **divide by w** then test: – 1 <= 5/10 <= 1 **OR**

To test if inside x = - 1, **w + x** = 10 + 5 = 15 > 0

To test if inside x = 1, **w - x** = 10 - 5 = 5 > 0

3D Clipping

- Do same for y, z to form boundary coordinates for 6 planes as:

Boundary coordinate (BC)	**Homogenous coordinate**	**Clip plane**	**Example (5,2,7,10)**
BC0	w+x	x=-1	15
BC1	w-x	x=1	5
BC2	w+y	y=-1	12
BC3	w-y	y=1	8
BC4	w+z	z=-1	17
BC5	w-z	z=1	3

- **Consider line that goes from point A to C**
  - **Trivial accept:** 12 BCs (6 for pt. A, 6 for pt. C) > 0
  - **Trivial reject:** Both endpoints outside (-ve) for same plane

Edges as Parametric Equations

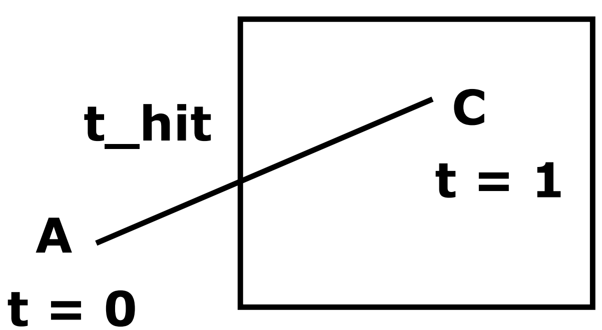
- Implicit form $F(x, y) = 0$
- Parametric forms:
  - points specified based on single parameter value
  - Typical parameter: time t

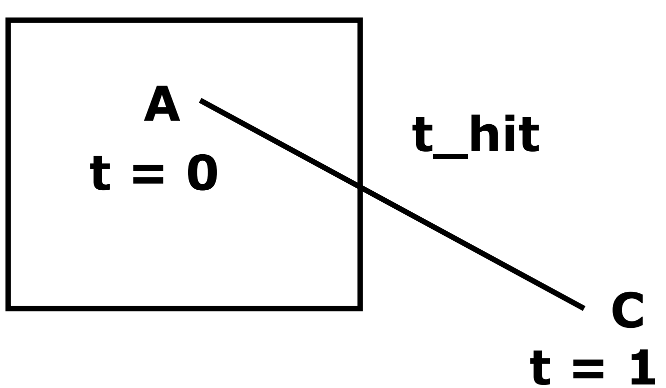
$$P(t) = P_0 + (P_1 - P_0) * t \qquad 0 \le t \le 1$$

- Some algorithms work in parametric form
  - Clipping: exclude line segment ranges
  - Animation: Interpolate between endpoints by varying t
- Represent each edge parametrically as A + (C – A)t
  - at time t=0, point at A
  - at time t=1, point at C

Inside/outside?

- Test A, C against 6 walls **(x=-1,1;   y=-1,1;   z=-1,1)**
- There is an intersection if BCs have opposite signs. i.e. if either
  - A is outside (< 0), C is inside (> 0)   or
  - A inside (> 0) , C outside (< 0)
- Edge intersects with plane at some t_hit between [0,1]





Calculating hit time (t_hit)

- How to calculate t_hit?
- Represent an edge t as:

$$Edge(t) = ((Ax + (Cx - Ax)t, (Ay + (Cy - Ay)t, (Az + (Cz - Az)t, (Aw + (Cw - Aw)t)$$

- E.g. If x = 1,

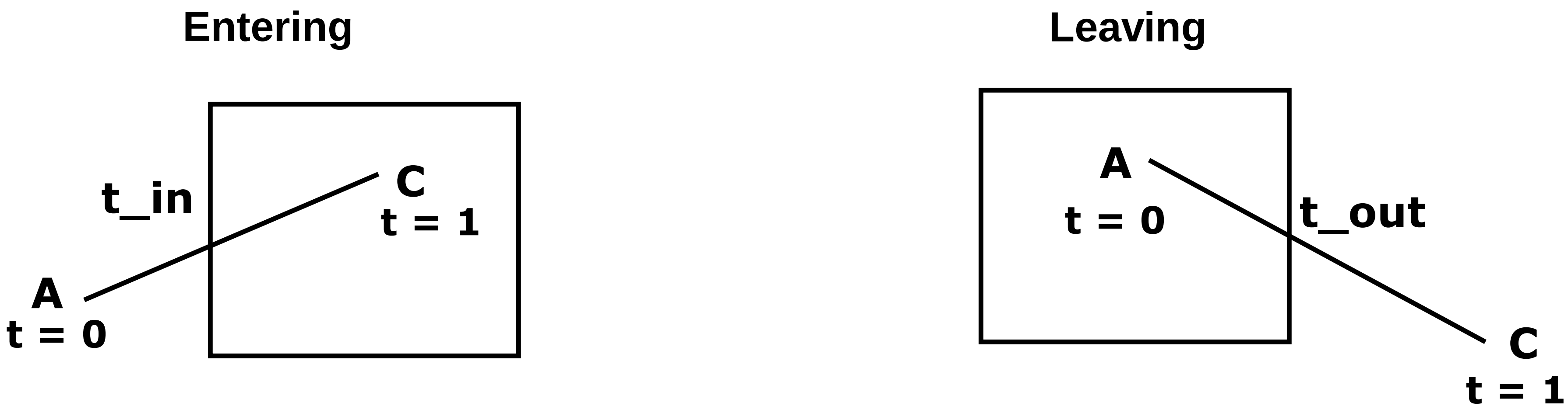
$$\frac{Ax + (Cx - Ax)t}{Aw + (Cw - Aw)t} = 1$$

- Solving for t above,

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

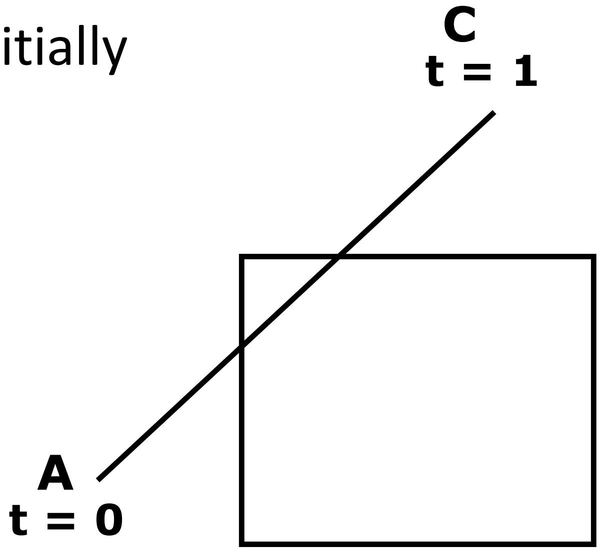
Inside/outside?

- t_hit can be **"entering (t_in) "** or **"leaving (t_out)"**
- Define: **"entering"** if A outside, C inside
  - Why? As t goes [0-1], edge goes from outside (at A) to inside (at C)
- Define **"leaving"** if A inside, C outside
  - Why? As t goes [0-1], edge goes from inside (at A) to inside (at C)



Chop step by Step against 6 planes

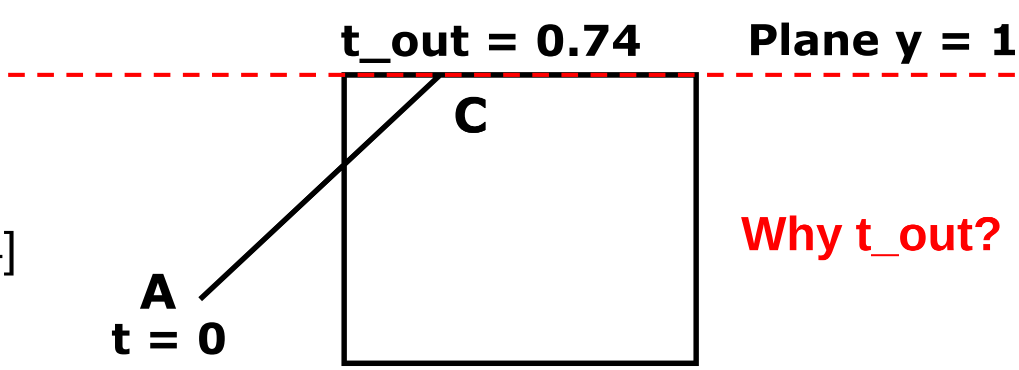
- Initially



t_in = 0, t_out = 1
Candidate Interval (CI) = [0 to 1]

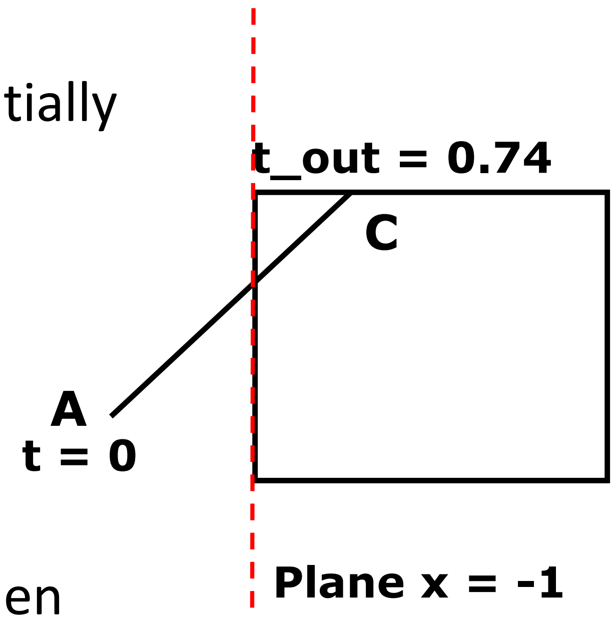
- Chop against each of 6 planes

t_in = 0, t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]



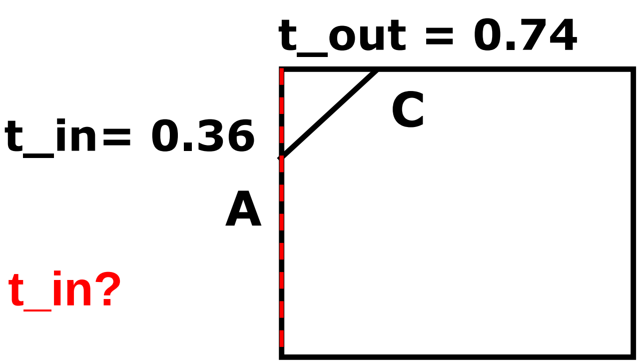
Chop step by Step against 6 planes

- Initially



t_in = 0,   t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]

- Then

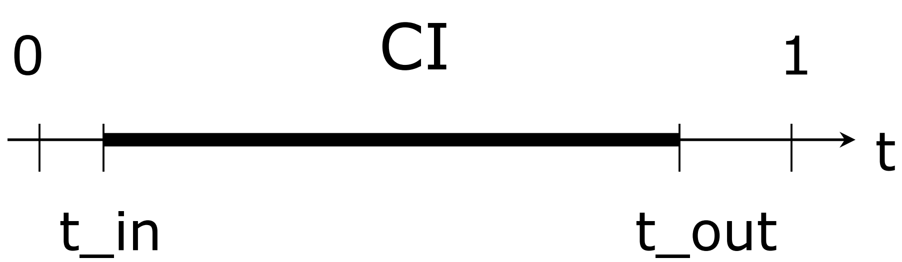


Why t_in?

t_in = 0.36,   t_out = 0.74
Candidate Interval (CI) CI = [0.36 to 0.74]

Candidate Interval

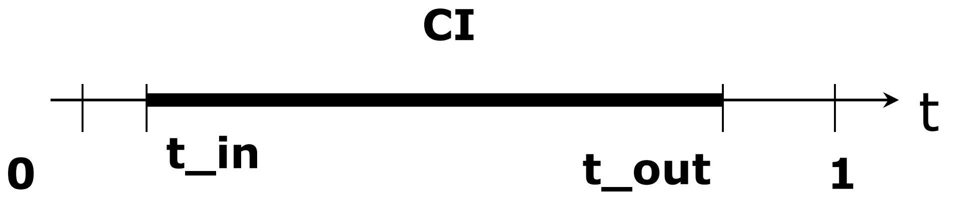
- Candidate Interval (CI): time interval during which edge might still be inside CVV. i.e. CI = t_in to t_out
- Initialize CI to [0,1]
- For each of 6 planes, calculate **t_in** or **t_out**, shrink CI



- Conversely: values of t outside CI = edge is outside CVV

Shortening Candidate Interval

- **Algorithm:**
  - Test for trivial accept/reject (stop if either occurs)
  - Set CI to [0,1]
  - For each of 6 planes:
    - Find hit time t_hit
    - If t_in,   new t_in = max(t_in,t_hit)
    - If t_out, new t_out = min(t_out, t_hit)
    - If t_in > t_out => exit (no valid intersections)

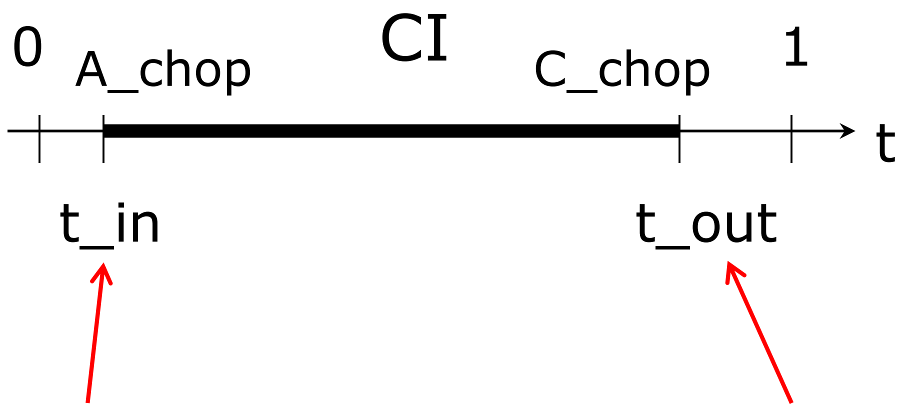


Note: seeking smallest valid CI without t_in crossing t_out

Calculate choppped A and C

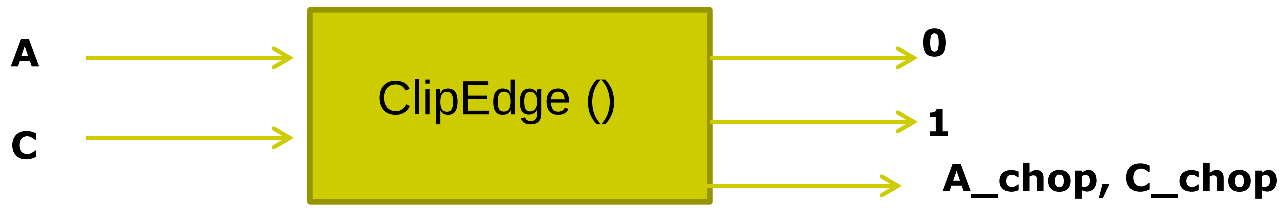
- If valid t_in, t_out, calculate adjusted edge endpoints A, C as

- A_chop = A + t_in (C – A) (calculate for Ax,Ay, Az)
- C_chop = A + t_out (C – A) (calculate for Cx,Cy,Cz)



3D Clipping Implementation

- Function clipEdge( )
- Input: two points A and C (in homogenous coordinates)
- Output:
  - 0, if AC lies **complete outside** CVV
  - 1, **complete inside** CVV
  - Returns clipped A and C otherwise
- Calculate 6 BCs for A, 6 for C



Store BCs as Outcodes

- Use outcodes to track in/out
  - Number walls x = +1, -1; y = +1, -1, and z = +1, -1 as 0 to 5
  - Bit *i* of A's **outcode = 1** if A is outside ith wall
  - 1 otherwise
- **Example:** outcode for point outside walls 1, 2, 5

Wall no.	0	1	2	3	4	5
OutCode	0	1	1	0	0	1

Trivial Accept/Reject using Outcodes

- **Trivial accept:** inside (not outside) any walls

Wall no.	0	1	2	3	4	5
A Outcode	0	0	0	0	0	0
C OutCode	0	0	0	0	0	0

Logical bitwise test: A | C == 0

- **Trivial reject:** point outside **same** wall. Example Both A and C outside wall 1

Wall no.	0	1	2	3	4	5
A Outcode	0	1	0	0	1	0
C OutCode	0	1	1	0	0	0

Logical bitwise test: A & C != 0

3D Clipping Implementation

- Compute BCs for A,C store as outcodes
- Test A, C outcodes for trivial accept, trivial reject
- If not trivial accept/reject, for each wall:
  - Compute tHit
  - Update t_in, t_out
  - If t_in > t_out, early exit

3D Clipping Pseudocode

```
int clipEdge(Point4& A, Point4& C)
{
    double tIn = 0.0, tOut = 1.0, tHit;
    double aBC[6], cBC[6];
    int aOutcode = 0, cOutcode = 0;

    .....find BCs for A and C
    .....form outcodes for A and C

    if((aOutCode & cOutcode) != 0) // trivial reject
        return 0;
    if((aOutCode | cOutcode) == 0) // trivial accept
        return 1;
```

3D Clipping Pseudocode

```
for(i=0;i<6;i++)  // clip against each plane
{
    if(cBC[i] < 0)  // C is outside wall i (exit so tOut)
    {
        tHit = aBC[i]/(aBC[i] – cBC[I]);     // calculate tHit
        tOut = MIN(tOut, tHit);
    }
    else if(aBC[i] < 0)  // A is outside wall I (enters so tIn)
    {
        tHit = aBC[i]/(aBC[i] – cBC[i]);     // calculate tHit
        tIn = MAX(tIn, tHit);
    }
    if(tIn > tOut) return 0; // CI is empty: early out
}
```

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

3D Clipping Pseudocode

```
Point4 tmp;  // stores homogeneous coordinates
If(aOutcode != 0) // A is outside: tIn has changed. Calculate A_chop
{
    tmp.x = A.x + tIn * (C.x – A.x);
    // do same for y, z, and w components
}
If(cOutcode != 0) // C is outside: tOut has changed. Calculate C_chop
{
    C.x = A.x + tOut * (C.x – A.x);
    // do same for y, z and w components
}
A = tmp;
Return 1; // some of the edges lie inside CVV
}
```

Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a concave polygon can yield multiple polygons

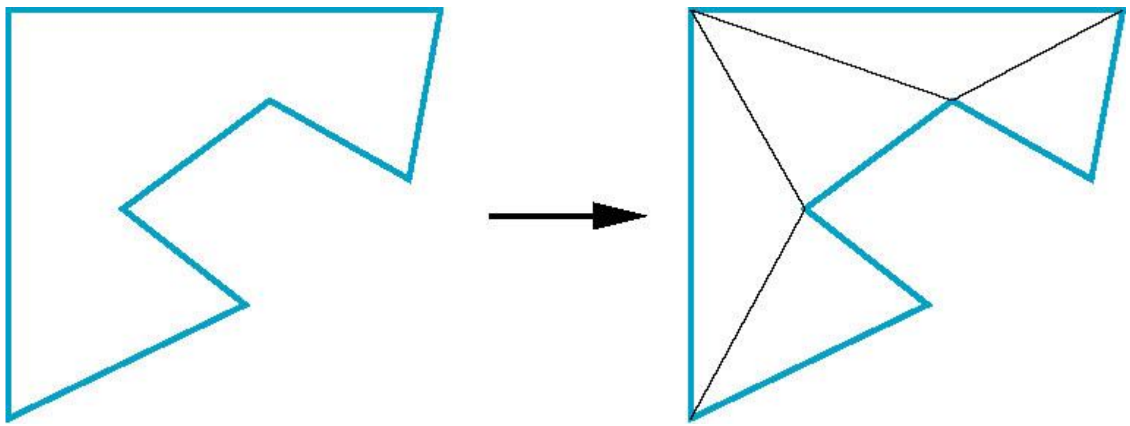
- Clipping a convex polygon can yield at most one other polygon

Clipping Polygons

- Need more sophisticated algorithms to handle polygons:
  - ***Sutherland-Hodgman:*** clip any given polygon against a convex clip polygon (or window)
  - ***Weiler-Atherton:*** Both clipped polygon and clip polygon (or window) can be concave

Tessellation and Convexity

- One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)
- Also makes fill easier



References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Chapter 9