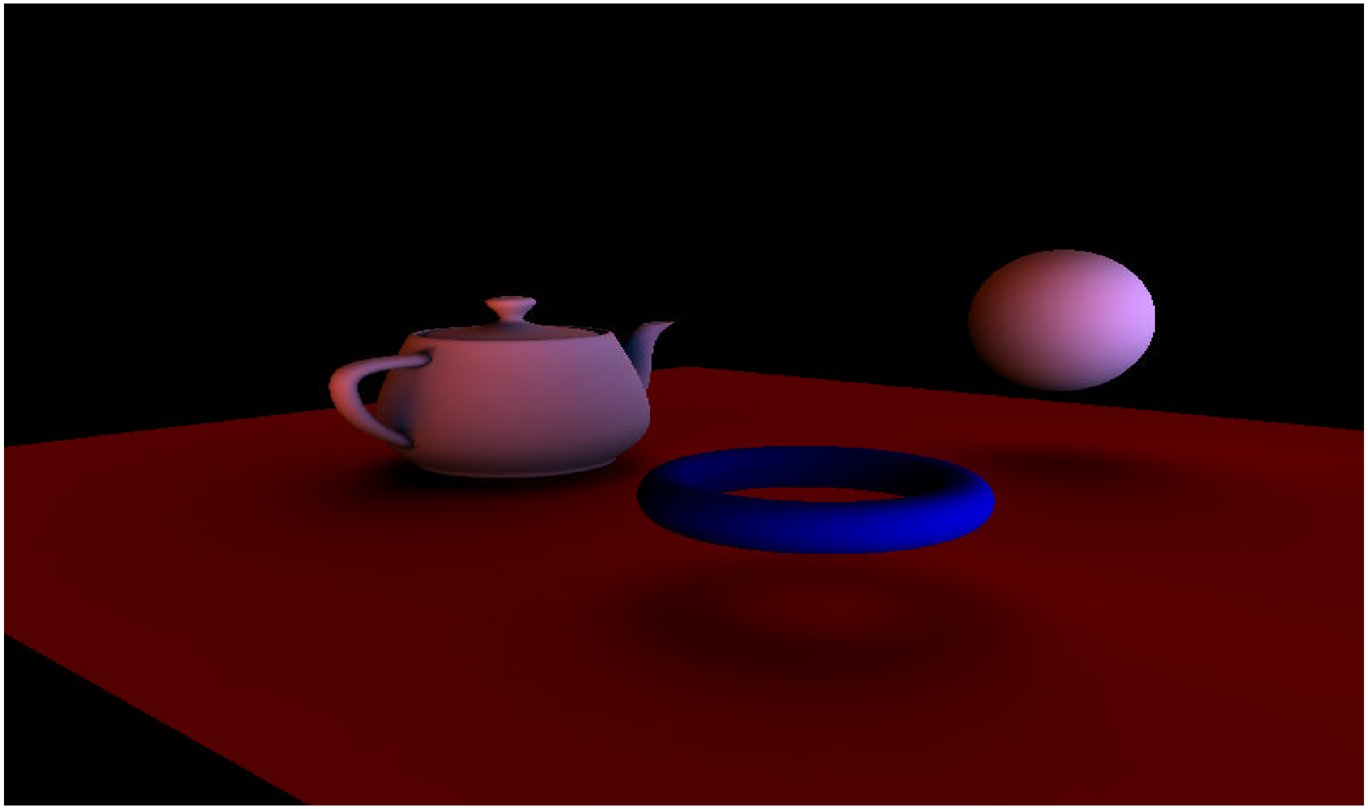


CS 563 Advanced Topics in Computer Graphics Spherical Harmonic Lighting

by Mark Vessella

Courtesy of http://www.yasrt.org/shlighting/

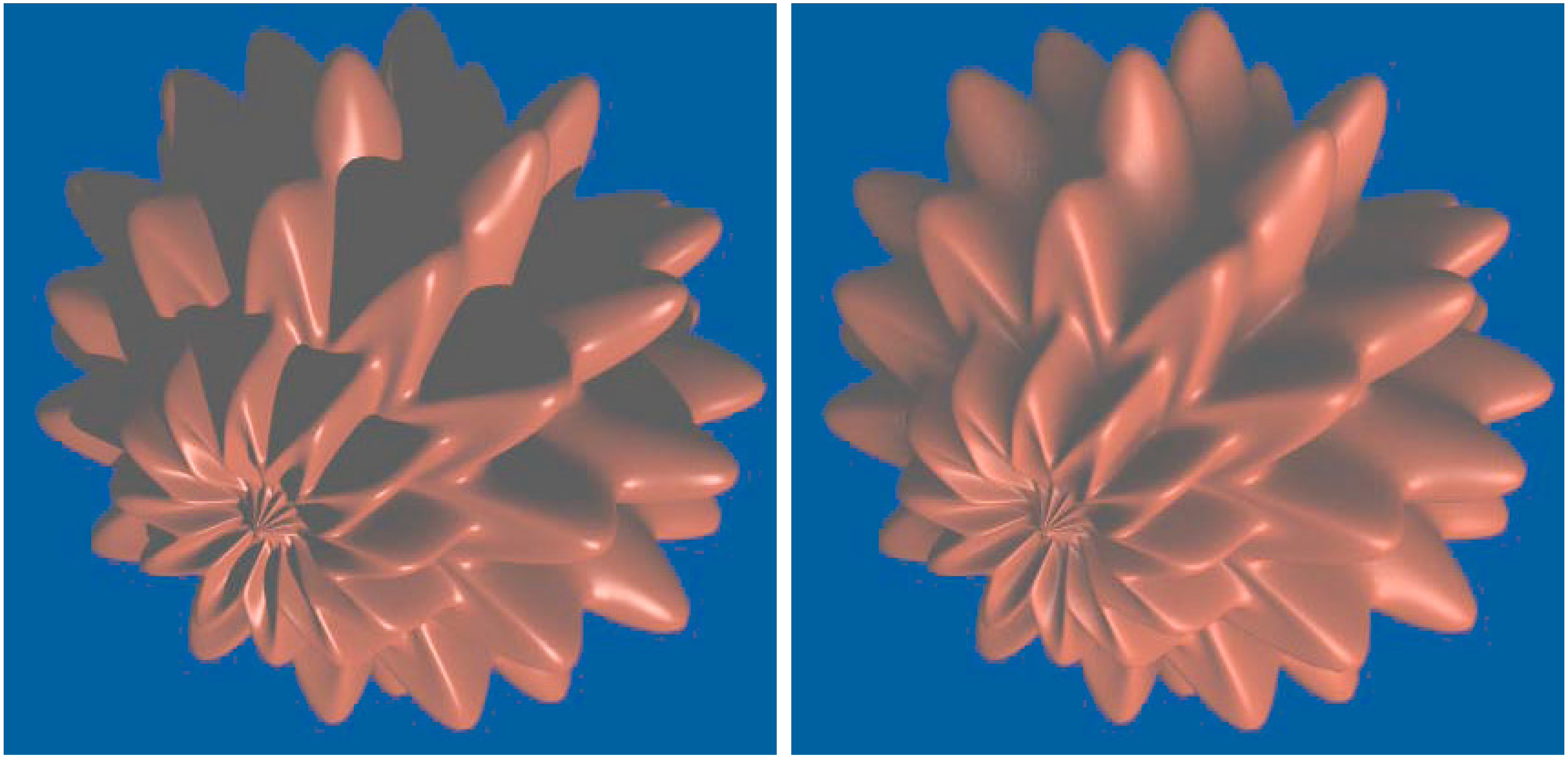
Outline for the Night

- Introduction to Spherical Harmonic Lighting
- Description of how to implement it
- Demo’s
- Questions

Introduction

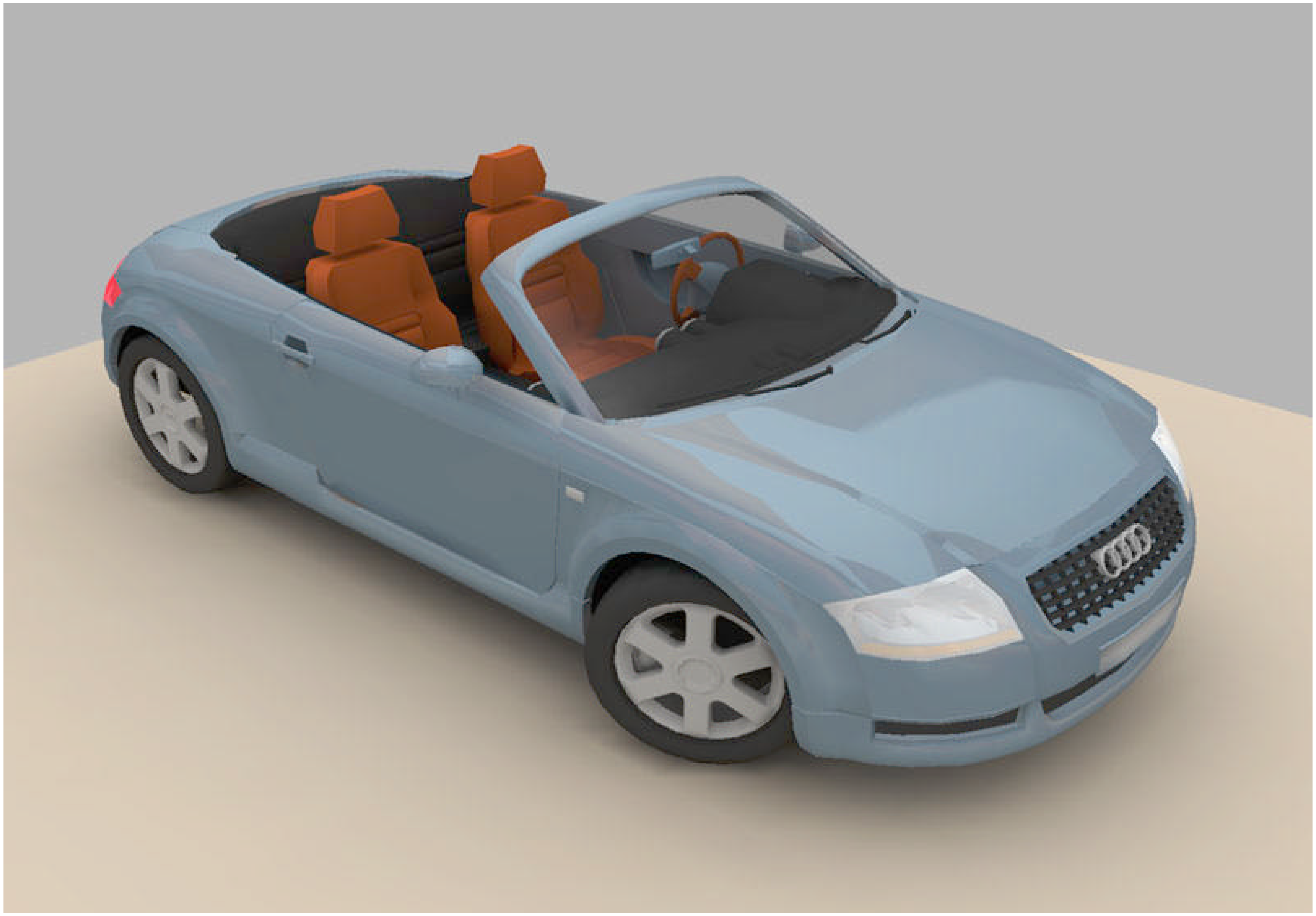
- What is Spherical Harmonic Lighting?
  - Technique used to light objects in real-time
- Why do we want to use it?
  - Supposedly can render complex objects in real-time
  - Comparable to global illumination
  - Point light sources don’t give realistic images
  - Makes self-shadowing and interrflections possible

Point Light Source vs. Spherical Harmonics



Courtesy of Precomputed Radiance Transfer, Teemu Mäki-Patola

Another Sample Image



Courtesy of Spherical Harmonic Lighting – The Gritty Details, Robin Green

Background Needed

- Computer graphics – lighting models, ray-tracers,...
- Strong mathematical ability – Calculus, probability, numerical methods, spherical coordinates, ...
- Computer programming skill

Overview of the Algorithm

1. Calculate coefficients for SH function from the given lighting function
   - Monte Carlo integration and Associated Legendre Polynomials are needed
   - This is pre-computed – Very time consuming
2. Reconstruct an approximation of the original function using these coefficients
3. Use this approximation to render your image

Common Lighting Models in CG

1. Diffuse illumination
2. Ambient illumination
3. Specular highlights

➢ An important property of these are they add together linearly

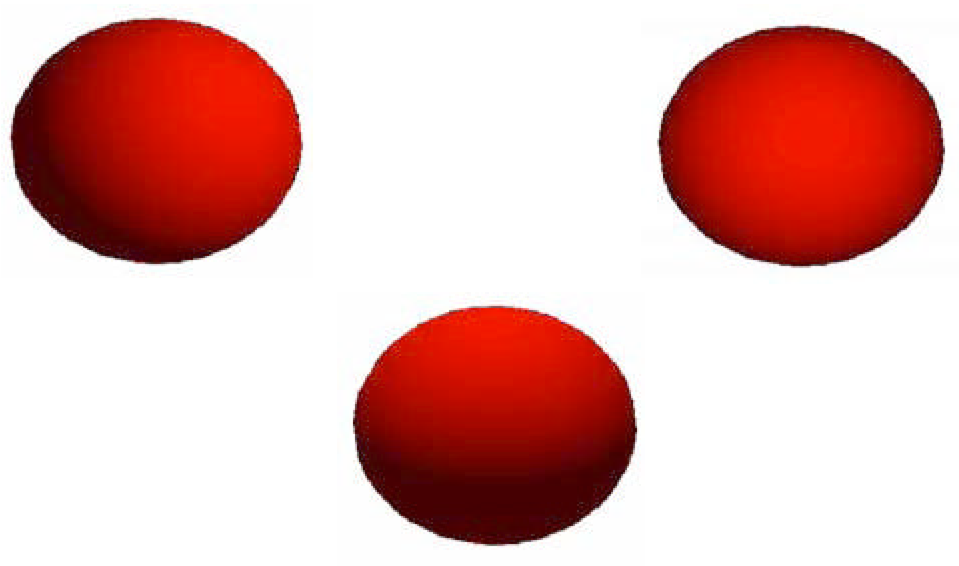
Diffuse Illumination

- Brightness is proportional to the cosine of the angle between the light source and the normal of the surface
- The surface appears equally bright from all directions

Courtesy of Illumination Models and Shading, Foley and Van Dam

Diffuse Illumination

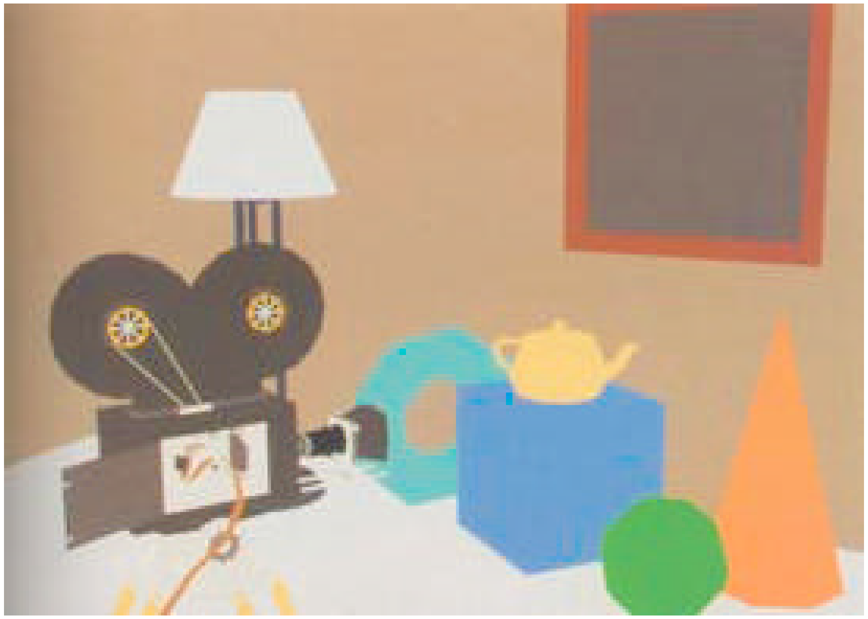
- Different lighting directions



Courtesy of Illumination Models and Shading, Foley and Van Dam

Ambient Illumination

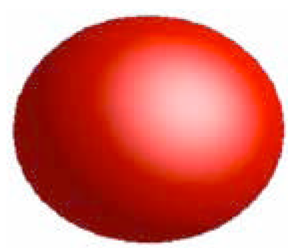
- Sort of like a background “glow” that takes into account reflections.



Courtesy of Illumination Models and Shading, Foley and Van Dam

Specular highlights

- When a light source hits an object it does not reflect light in all directions evenly. This makes the object appear shiny by adding highlights.



Courtesy of Illumination Models and Shading, Foley and Van Dam

Combining All 3 Light Models

Example:

Ambient and Diffuse

Specular highlights

Courtesy of Illumination Models and Shading, Foley and Van Dam

- SH Lighting can be used for the ambient and diffuse parts while specular highlights will be added on top

Rendering Equation

- Formula for lighting in computer graphics based solely on physics:

$$L(\mathbf{x},\vec{w}_0) = L_e(\mathbf{x},\vec{w}_0) + \int_S f_r(\mathbf{x},\vec{w}_i \rightarrow \vec{w}_0) L(\mathbf{x}',\vec{w}_i) G(\mathbf{x},\mathbf{x}') V(\mathbf{x},\mathbf{x}') dw_i$$

Courtesy of Spherical Harmonic Lighting – The Gritty Details, Robin Green

- Calculating an integral is <u>NOT</u> Real-Time friendly

Solution!?

- Monte Carlo Integration
  - Basic idea
    1. Choose random number in the range of the integral
    2. If the random number is in the integral area than count it
    3. Repeat steps 1 and 2 N times
        1. The higher the value of N the more accurate the result will be
    4. Divide the number of random numbers in the area of the integral by the total number, N, of random variables

Monte Carlo Estimator

- This method eventually turns into...

$$\int f(x) \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i)w(x_i)$$

- Integration turns into a series of multiplies and adds

Afterthoughts on “ MC Integration”

There are probably better methods

- Quasi-Monte Carlo Integration???
- Improved sampling techniques

Orthogonal Basis Functions

- “Small pieces of signal that can be scaled and combined to produce an approximation to an original function”[2]

Orthogonal Basis Functions

- Procedure:
  - Calculate the integral of the original function times the basis functions over the full range of f to get a series of coefficients

$$c_l^m = \int_S f(s) P_l^m(s)\, ds$$

  - To reconstruct the signal multiply each of these coefficients by the basis function and sum the result

$$\tilde{f}(s) = \sum_{i=0}^{n^2} c_i P_i(s)$$

- They come in families of functions, i.e. Chebyshev, Jacobi, ...
- The integral of the product of two in the same family is a constant if they are the same member of the family and is 0 if they are different members of the family

Associated Legendre Polynomials

- Spherical Harmonics uses Associated Legendre Polynomials
- They are also orthonomal:
  - The integral of the product of two is 1 if they are the same member of the family and 0 if they are different members

Calculating Associated Legendre Polynomials

- Calculate the polynomials recursively:

$$1.\ (l-m)P_l^m = x(2l-1)P_{l-1}^m - (l+m-1)P_{l-2}^m$$

$$2.\ P_m^m = (-1)^m (2m-1)!!(1-x^2)^{m/2}$$

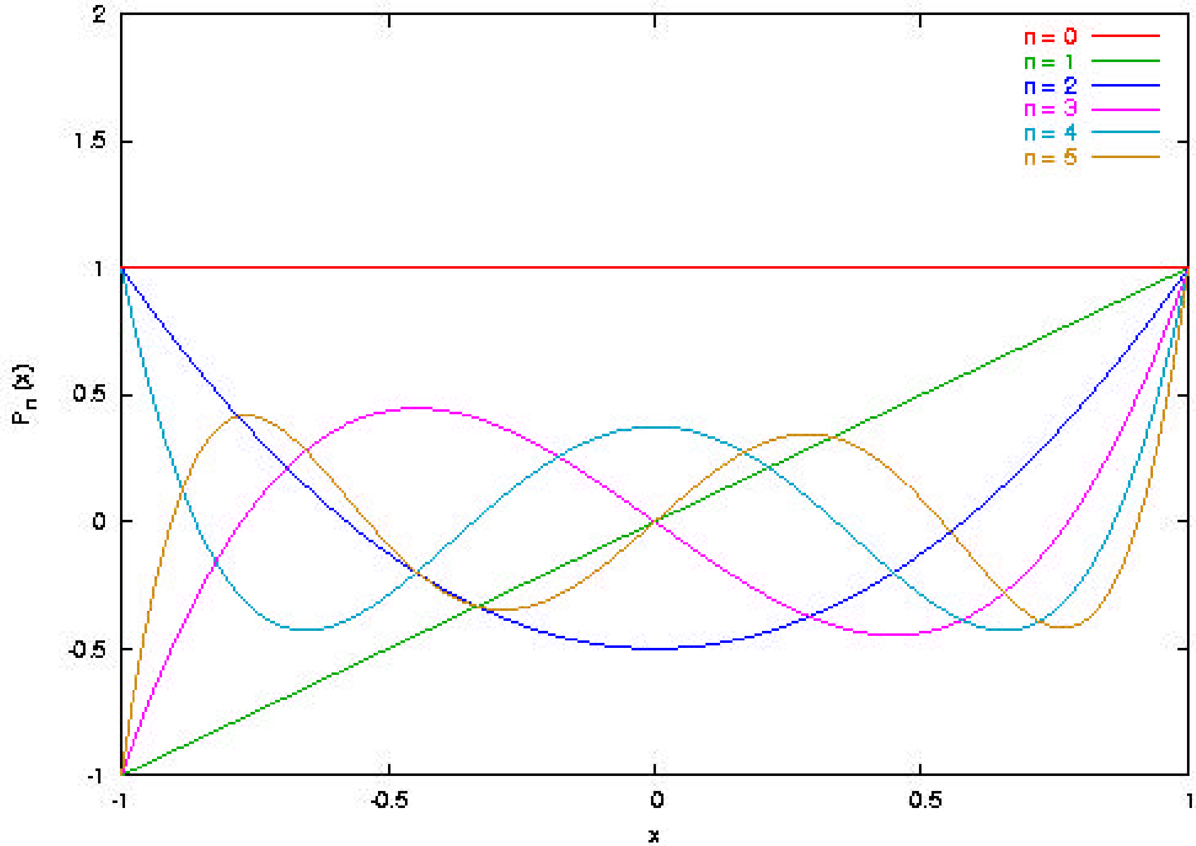
$$3.\ P_{m+1}^m = x(2m+1)P_m^m$$

Courtesy of Spherical Harmonic Lighting – The Gritty Details, Robin Green

  - l is a “band index” with a range of [0, any positive integer]
    - Orthogonal with respect to a constant term
  - m is has a range of [0, l]
    - Orthogonal with respect to a different term
- It can also be done with a Taylor series expansion

Graphs of Associated Legendre Polynomials

- (m = 0)



Courtesy of en.wikipedia.org/wiki/Image:Lpoly.png

Spherical Harmonic Functions

- Convert the Associated Legendre Polynomials into spherical coordinates

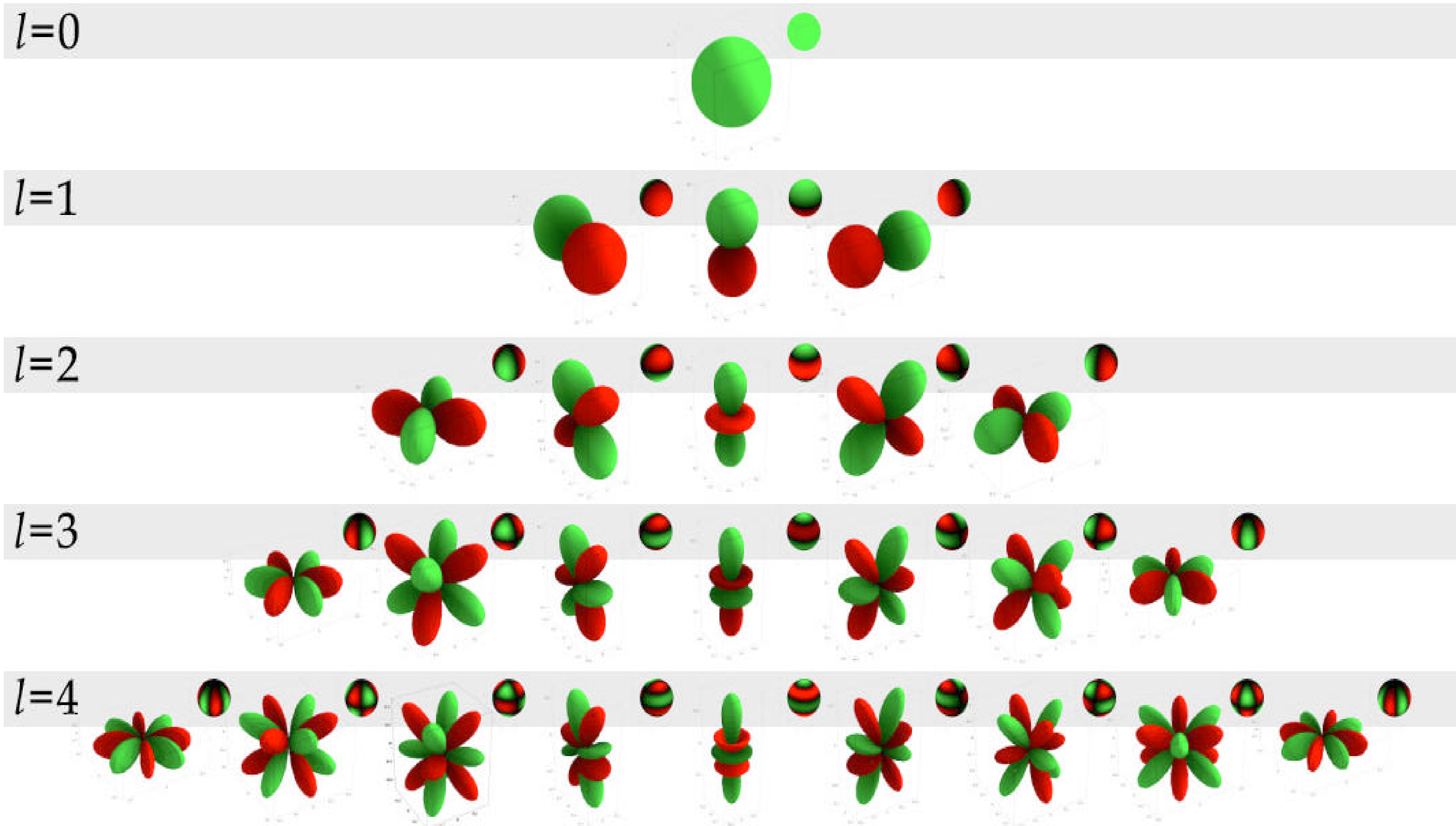
$$
y_l^m(q, j) = \begin{cases} \sqrt{2} K_l^m \cos(m j) P_l^m(\cos q), m > 0 \\ \sqrt{2} K_l^m \sin(-m j) P_l^{-m}(\cos q), m < 0 \\ K_l^0 P_l^0(\cos q), m = 0 \end{cases}
$$

- P is the Associated Legendre Polynomial and
- K is a scaling factor:

$$
K_l^m = \sqrt{\frac{(2l+1)}{4p} \frac{(l-|m|)!}{(l+|m|)!}}
$$

Courtesy of Spherical Harmonic Lighting – The Gritty Details, Robin Green

Spherical Harmonic Functions



Courtesy of Spherical Harmonic Lighting – The Gritty Details, Robin Green

Spherical Harmonic Properites

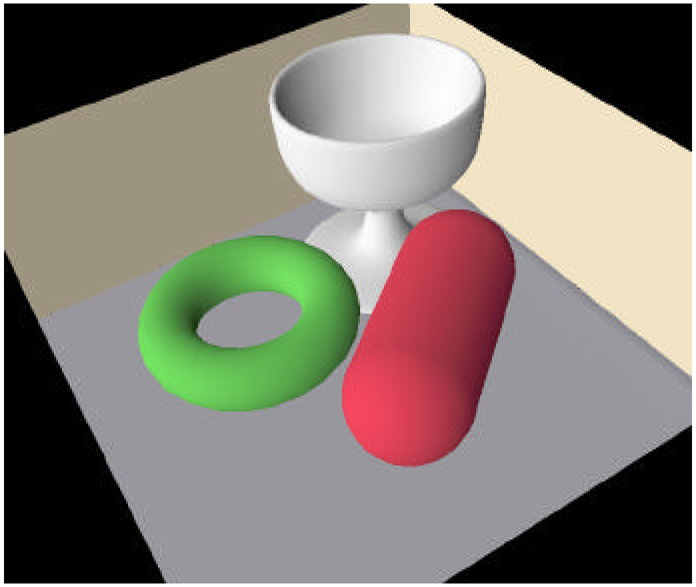
- Orthogonal and orthonormal
- Rotationally invariant
  - As a scene changes, i.e. a light changes position or a model rotates, the lighting of the model will not fluctuate
- Can convert integration to a series of multiples and adds:

$$\int_S \tilde{L}(s)\tilde{t}(s)ds = \sum_{i=0}^{n^2} L_i t_i$$

Spherical Harmonic Lighting Models

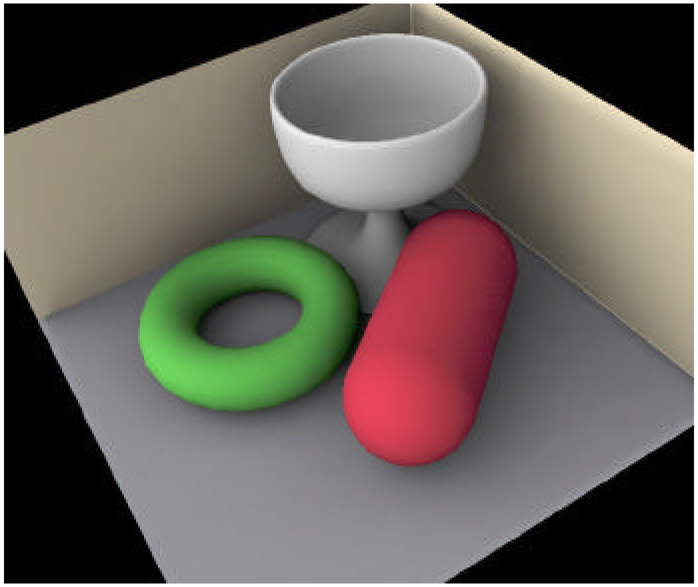
- Diffuse Unshadowed Transfer
- Shadowed Diffuse Transfer
- Diffuse Interreflected Transfer

Diffuse Unshadowed Transfer



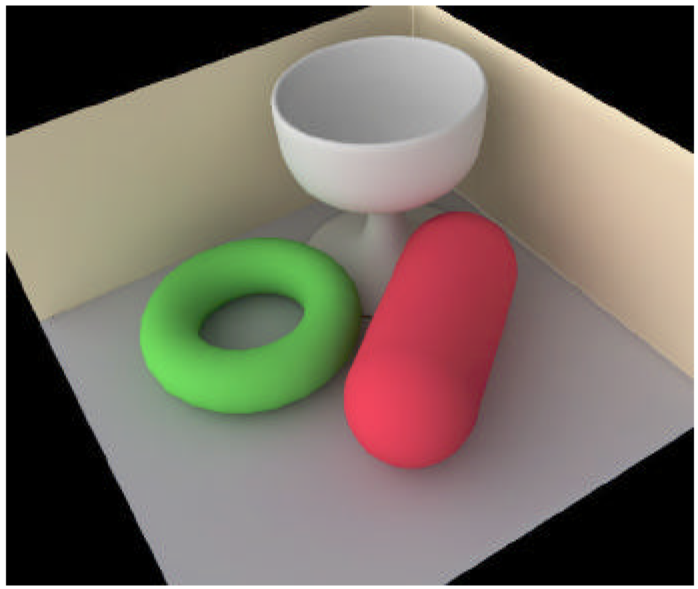
Courtesy of Illumination Models and Shading, Foley and Van Dam

Shadowed Diffuse Transfer



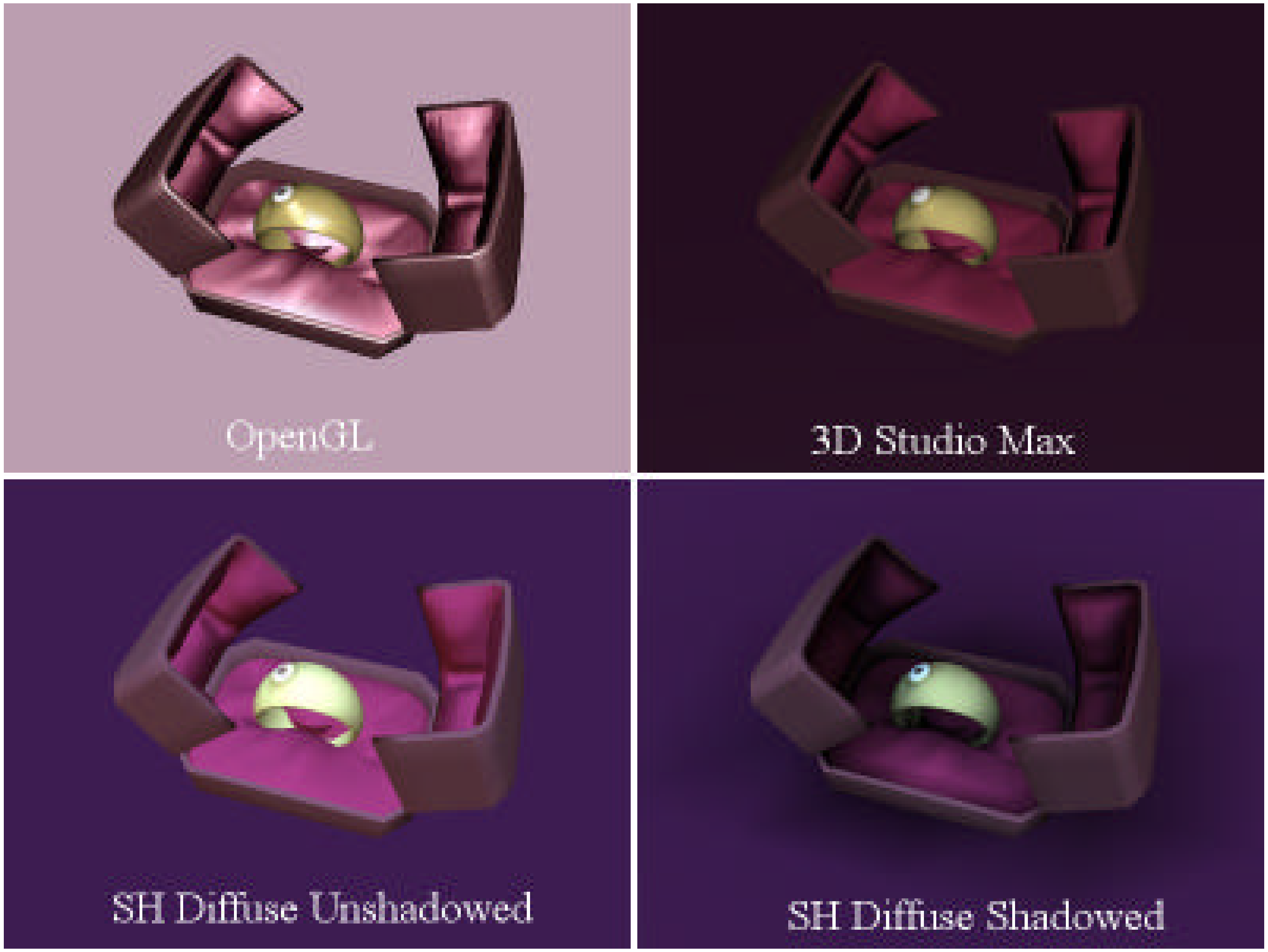
Courtesy of Illumination Models and Shading, Foley and Van Dam

Diffuse Interreflected Transfer



Courtesy of Illumination Models and Shading, Foley and Van Dam

Example Lighting Techniques



Courtesy of http://www.yasrt.org/shlighting/

Rendering

- Spherical harmonic lighting for ambient and diffuse surfaces
- Add specular highlights on top of it

Advanced SH techniques

- Exploiting hardware for SH lighting
- Using the preprocessor for various things, i.e. models with mixed materials, reflections,...
- Real time translucency

Shortcomings

- “Objects do not very generally shadow and affect each other’s lighting.”[4]
- You can’t deform objects

Advice - IMHO

- So you want to REALLY learn it!?
  - Learn(or relearn, re-familiarize, ...):
    1. Spherical coordinates
    2. Ignore the implementations given on the web – Figure it out for yourself!
    3. Associated Legendre polynomials
  - Don't waste time with
    - Monte Carlo Integration – just blindly use someone else's code

Demos

- First demo can be found at:
  - http://www.paulsprojects.net/opengl/sh/sh.html
  - Complete with all the source code
  - If you are going to run it I recommend downloading the coefficients
    - I was unable to generate them on my own – It could have been because my screensaver was coming on in the middle generating them
- Second demo can be found at:
  - http://www.yasrt.org/shlighting

Unrelated Interesting Observations

- Everyone knows what k! means
  - k! = factorial of k = k * k-1 *k-2...*1
  - What does k!! Mean?
    - k!! = double factorial of k = k * k-2 *k-4...*1
      - (k is always odd)
- I put text in front of an image and forgot the image
  - It didn’t show up when I viewed the slides in PowerPoint
  - It did show up merged with the text when I viewed the slides in PowerPoint Viewer

Thank You

- Cliff Lindsay for reviewing my preliminary slides and giving me a few pointers
  - Note that Cliff Lindsay and Prof. Agu come up as number 23 (sometimes 20) when searching “spherical harmonic lighting” in Yahoo!

References

- Papers
  - [1] Illumination Models and Shading, Foley and Van Dam
  - [2] Spherical Harmonic Lighting – The Gritty Details, Robin Green
  - [3]Accuracy of Spherical Harmonic Approximations for Images of Lambertian Objects Under Far and Near Lighting, Darya Frolova, Denis Simakov, and Ronen Basri
  - [4]Precomputed Radiance Transfer, Teemu Mäki-Patola
  - [5]A Quick Rendering Method Using Basis Functions for Interactive Lighting Design, Yoshinori Dobashi, Kazufumi Kaneda, Hideki Nakatani, Hideo Yamashita

References

- Web pages
  - en.wikipedia.org/wiki/Image:Lpoly.png
  - http://www.paulsprojects.net/opengl/sh/sh.html
  - http://www.yasrt.org/shlighting
  - http://www.itu.dk/edu/documentation/mathworks/math/math/l/l175.htm