CS 563 Advanced Topics in Computer Graphics
Theoretical Foundations

by Scott Ingram

Question : Why ??

Answer:

A fundamental understanding of the theory behind how light interacts with matter is needed in order to understand how others are implementing and modeling light interaction.

Agenda Outline

- Basic terminology
- Angular Dependencies
- Notation and Direction
- Radiance and Irradiance
- Spectral Representation
- BRDFs
- Reflectance
- Lambertian BRDF
- The Rendering Equation
- Monte Carlo Integration

- <u>Radiometric Quantities</u>

- *Radiant Energy* ***Q*** : The basic unit of electromagnetic energy. Measured in joules (units: J)

- Radiant Flux ***Φ*** : The amount of radiant energy passing through a surface or region of space per second. ***dQ/dt***. (units: $J * s^{-1}$)

Background Terminology Cont.

- *Radiant flux density* : The radiant flux per differential area. $\boldsymbol{d\Phi/dA}$ (units : W m^{-2})
- *Irradiance* ***E:*** The flux density that arrives at a surface. Same units as Radiant flux density... W m^{-2}
- *Radiant exitance* ***M:*** The flux density that leaves a surface. Also, same units as Radiant flux density... W m^{-2}
- *Radiant Intensity* **I:** Flux density per unit solid angle : $\boldsymbol{d\Phi/d\omega}$. Radiant energy in time and direction. (units : W m^{-2} s^{-1}).

Background Terminology Cont.

- *Radiance* **L**: The flux per unit projected per unit solid angle. Units are the same as Radiant intensity (units : W m^{-2} s^{-1}), ***BUT*** coming from a *specific direction* …

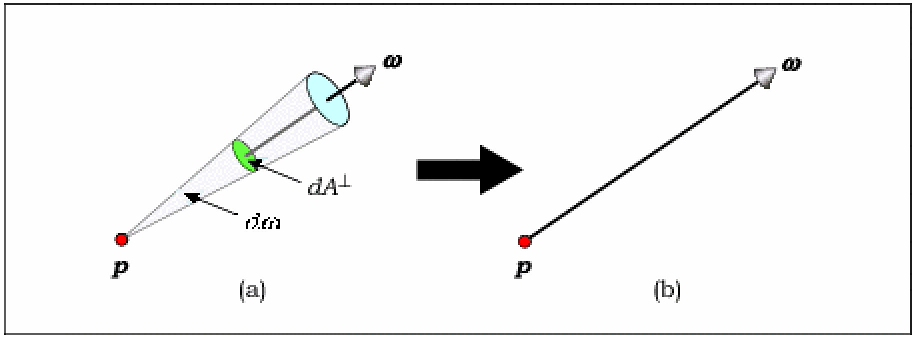
$$L = \frac{d^2 \Phi}{dA\, d\omega}$$

- Q: Why d^2 ???

Background Terminology Cont.

Specific to raytracing, a few properties stand out:

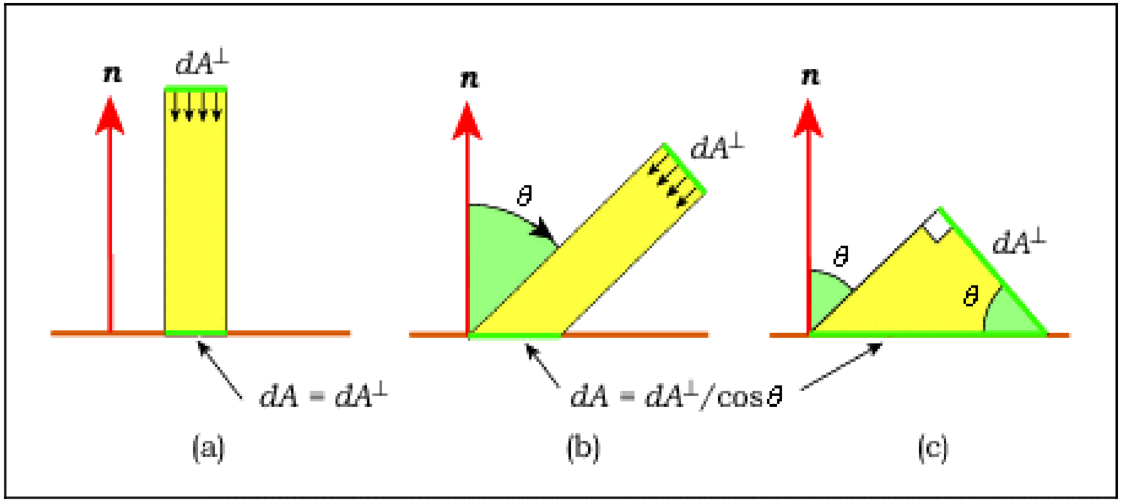
- Constant along rays in empty space
- Can be defined at any point in space (eye point)
- For points on a surface it makes no difference whether flux is directed towards or away from the surface.



Angular Dependence of Irradiance

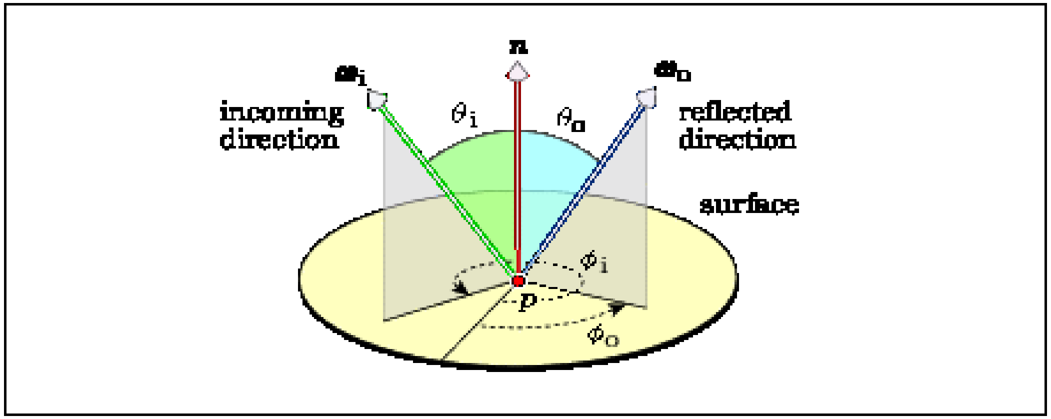
- *Irradiance Angular dependence*

$$dA^{\perp} = \cos\Theta \, dA$$



Notation and Direction

- $\boldsymbol{p}$: Surface point where light is reflected
- ω_i : Incoming direction
- ω_o : Reflected direction
- (Θ_i, ϕ_i) : Incoming coordinate angles
- (Θ_o, ϕ_o) : Reflected coordinate angles
- $\boldsymbol{n}$: Normal to surface



Notation and Direction

- Since …

$$dA^{\perp} = \cos\Theta \, dA$$

Substituting for *dA* in … $$L = \frac{d^2\Phi}{dA^{\perp}\, d\omega}$$

Yields … $$L = \frac{d^2\Phi}{\cos\Theta \, dA \, d\omega}$$

Radiance and Irradiance (cont)

- Using the following two equations ....

$$E = \frac{d\Phi}{dA} \qquad L = \frac{d^2\Phi}{\cos\Theta \, dA \, d\omega}$$

... we can relate *incident radiance* and *irradiance :*

$$dE_i(p,\omega_i) = L_i(p,\omega_i)\cos\Theta_i \, d\omega_i$$

$dE_i(p,\omega_i)$: *Irradiance in a cone with differential solid angle*

$L_i(p,\omega_i)$: *incident radiance at p*

Radiance and Irradiance (cont)

- By integrating *dE* over the different solid angle, the irradiance from the solid angle Ω_i can be obtained:

$$E_i(p) = \int_{\Omega_i} L_i(p, d\omega_i) \cos\Theta_i \, d\omega_i$$

Spectral Representation

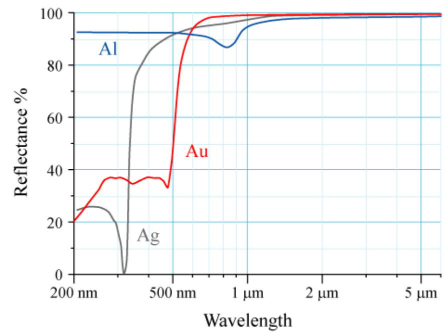
- Spectral... dependent on wavelength
- For Radiance :

$$L(p,\omega_i) = \int_{\lambda=0}^{\lambda=\infty} L_\lambda(p,\omega_i,\lambda)\, d\lambda$$

- Ray tracing primarily uses red, green, and blue (RGB) components, instead of the entire visible spectrum.

Spectral Representation(cont)

- Different materials reflect the visible light spectrum differently...



- Commercially available high reflectance material... “Spectralon” (notice 350 - 750nm)

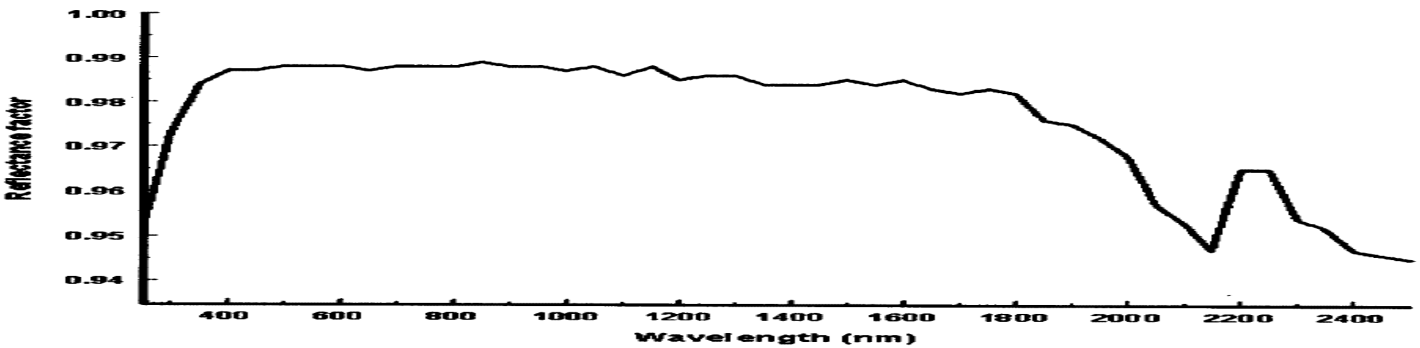
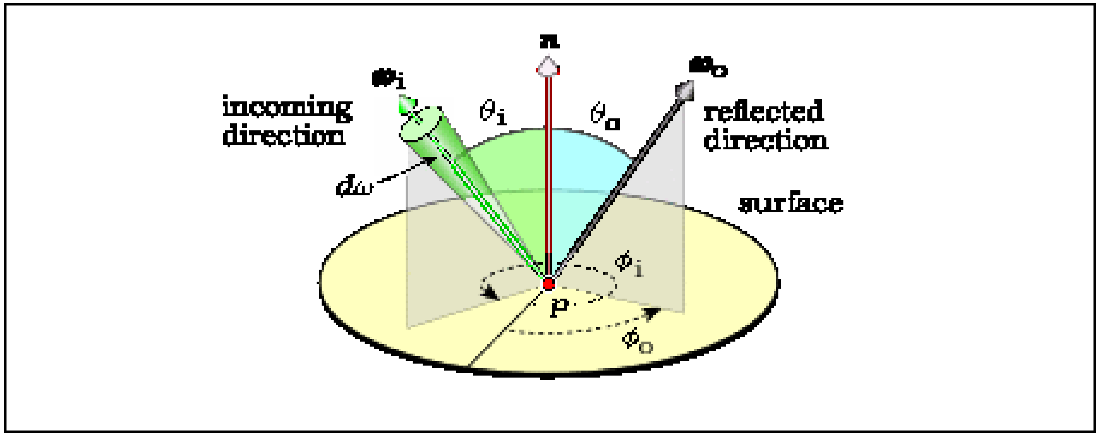


Figure 1. 8° /Hemispherical Spectral Reflectance Factor for SRS-99-020.

- **B**idirection **R**eflectance **D**istribution **F**unction (*f*)
  - Provides a means to describe how light is reflected at a surface point p , it relates reflectance in ω_i direction to irradiance in ω_o direction



BRDFs (cont)

- Due to linearity of materials... irradiance and radiance are proportional :

$$dL_o(p,\omega_o) \propto dE_i(p,\omega_i)$$

- The constant of proportionality is denoted with ***f*** :

$$dL_o(p,\omega_o) = f_r(p,\omega_i,\omega_o)\, dE_i(p,\omega_i)$$

- In terms of the incoming radiance, ***L*** :

$$dL_o(p,\omega_o) = f_r(p,\omega_i,\omega_o)\, L_i(p,d\omega_i)\cos\Theta_i\, d\omega_i$$

- Finally, solving for ***f*** :

$$f_r(p,\omega_i,\omega_o) = \frac{dL_o(p,\omega_o)}{L_i(p,d\omega_i)\cos\Theta_i\, d\omega_i}$$

BRDFs (cont)

- The Reflected Radiance in the ω_o direction from irradiance in a solid angle Ω_i is obtained by integrating over Ω_i :

$$L_o(p,\omega_o)=\int_{\Omega_i} f_r(p,\omega_i,\omega_o)L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

- The total reflected Radiance in the ω_o direction is computer by integrating over the hemisphere about p... the *reflectance equation :*

$$L_o(p,\omega_o)=\int_{2\pi^+} f_r(p,\omega_i,\omega_o)L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

BRDFs (cont)

- BRDF properties:
  - *Reciprocity*: (swap omegas)

$$f_r(p, \omega_i, \omega_o) = f_r(p, \omega_o, \omega_i)$$

  - *Linearity*: Add BRDFs..

Reflectance

- Defined as the ratio of reflected flux to incident flux. (reflected power versus incident power)
- Radiant flux on a differential surface element :

$$d\,\Phi_i = dA \int_{\Omega_i} L_i(p\,,d\omega_i)\cos\Theta_i\,d\omega_i$$

- Likewise , the reflected flux is:

$$d\,\Phi_o = dA \int_{\Omega_o} L_o(p\,,d\omega_o)\cos\Theta_o\,d\omega_o$$

Reflectance (cont)

- We can determine the reflectance ($\boldsymbol{\rho}$) by substituting this equation:

$$L_o(p,\omega_o)=\int_{\Omega_i} f_r(p,\omega_i,\omega_o)\,L_i(p,d\omega_i)\cos\Theta_i\,d\omega_i$$

Into :
$$d\,\Phi_o=dA\int_{\Omega_o}\int_{\Omega_i} f_r(p,\omega_i,\omega_o)\,L_i(p,d\omega_i)\cos\Theta_i\,d\omega_i\cos\Theta_o\,d\omega_o$$

to get the reflectance equation where
$$\rho(p,\Omega_i,\Omega_o)=\frac{d\,\Phi_o}{d\,\Phi_i}$$

$$=\frac{\int_{\Omega_o}\int_{\Omega_i} f_r(p,\omega_i,\omega_o)\,L_i(p,d\omega_i)\cos\Theta_i\,d\omega_i\cos\Theta_o\,d\omega_o}{\int_{\Omega_i} L_i(p,d\omega_i)\cos\Theta_i\,d\omega_i}$$

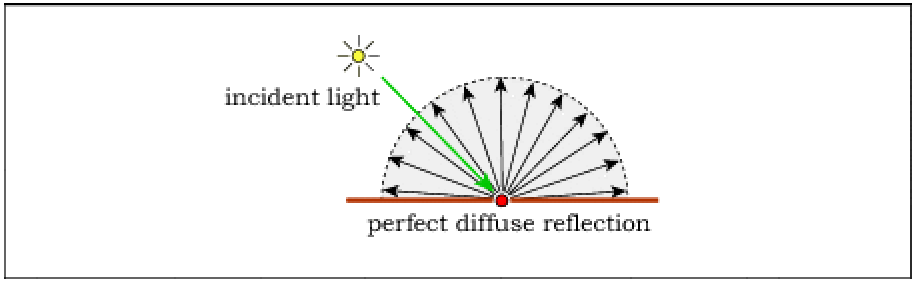
Reflectance (cont)

- Conservation of Energy... some materials absorb light and then re-radiated (blacktop), therefore the total reflectance over a whole hemisphere must be < 1.

$$\rho(p, 2\pi^{+}, 2\pi^{+}) < 1$$

Perfect Diffuse BRDF

- A simple BRDF where incidence radiance is scattered in all directions, a.k.a. *Lambertian reflection*



- *Since reflection is a perfect diffuse, reflected direction is no longer a factor:*

$$L_o(p,\omega_o)=L_{r,d}(p)$$

Perfect Diffuse BRDF

- Our *reflectance function* from an earlier slide becomes:

$$L_{r,d}(p)=\int_{\Omega_i} f_r(p,\varpi_i,\varpi_o)L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

- Since the BRDF no longer depends on the omegas, we can pull it from the integral:

$$L_{r,d}(p)=f_r(p)\int_{\Omega_i} L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

- Finally, solving for the BRDF:

$$f_r(p)=\frac{L_{r,d}(p)}{E_i(p)}$$

Perfect Diffuse BRDF

- Now, express the BRDF in terms of the perfect diffuse reflectance into the full hemisphere :

$$d\,\Phi_o = dA\,L_r(p)\int_{2\pi^+}\cos\Theta_o\,d\omega_o = dA\,L_r(p)\,\pi$$

$$d\,\Phi_i = dA\int_{\Omega_i} L_i(p\,,d\omega_i)\cos\Theta_i\,d\omega_i = dA\,E_i(p)$$

- Now get the reflectance... (only dependent on ***p***) :

$$\rho(p) = \frac{d\,\Phi_o}{d\,\Phi_i} = \frac{dA\,L_r(p)\,\pi}{dA\,E_i(p)} = f_r(p)\,\pi$$

BRDF examples....

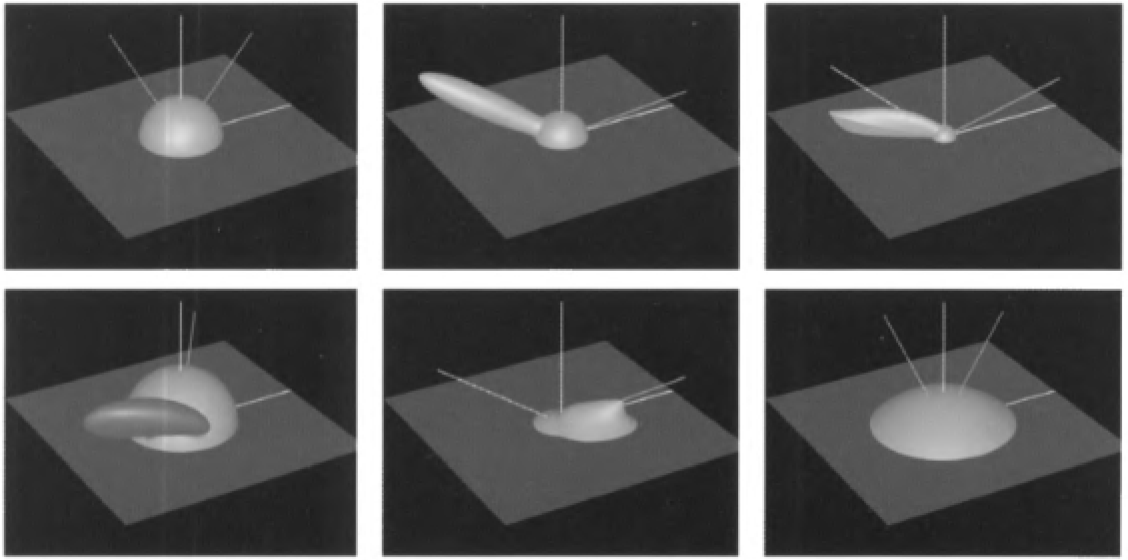


Figure 7.17. Example BRDFs. The solid green line coming from the right of each figure is the incoming light direction, and the dashed green and white line is the ideal reflection direction. In the top row, the left figure shows a Lambertian BRDF (a simple hemisphere). The middle figure shows Blinn-Phong highlighting added to the Lambertian term. The right figure shows the Cook-Torrance BRDF [192, 1270]. Note how the specular highlight is not strongest in the reflection direction. In the bottom row, the left figure shows a close-up of Ward's anisotropic model. In this case, the effect is to tilt the specular lobe. The middle figure shows the Hapke/Lommel-Seeliger "lunar surface" BRDF [501], which has strong retroreflection. The right figure shows Lommel-Seeliger scattering, in which dusty surfaces scatter light toward grazing angles. *(Images courtesy of Szymon Rusinkiewicz, from his "bv" BRDF browser.)*

The Rendering Equation

- Expression of the radiance equilibrium in a scene.
- Two Forms...
  - Hemisphere
  - Area

The rendering equation(cont)

- Hemisphere form
  - Already have a *reflection equation*:

$$L_o(p,\omega_o)=\int_{2\pi^+} f_r(p,\omega_i,\omega_o)L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

  - Now add the concept of a surface light source *(emissive surface)*

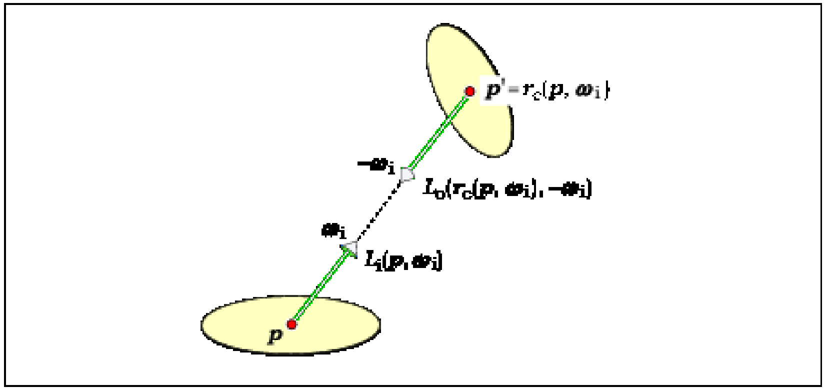
$$L_e(p,\omega_o)$$

  - Intuitively, it follows that the total radiative energy is the sum of the *emitted* and *reflected* radiance:

$$L_o(p,\omega_o)=L_e(p,\omega_o)+\int_{2\pi^+} f_r(p,\omega_i,\omega_o)L_i(p,d\omega_i)\cos\Theta_i d\omega_i$$

The rendering equation(cont)

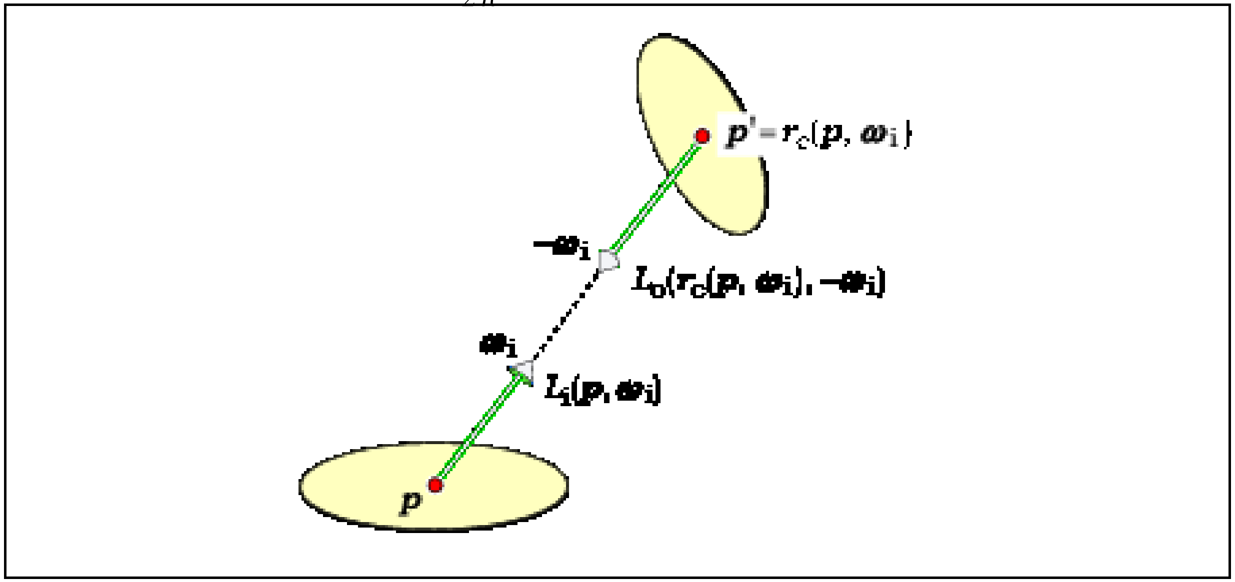
- Chicken egg problem.... For points (*p,p'*) on two facing surfaces the exitance radiance from surface *p* depends on the incoming radiance from surface *p'*... which depends of the incidence radiance from surface *p* ... Solution:
  - Trace ray from *p* to find incident radiance
  - Find nearest hit point along that ray *p'*
  - *Incident from p and exitance from p' are equal*



The rendering equation(cont)

- Use the *ray-casting operator* 'r':

$$L_o(p,\omega_o)=L_e(p,\omega_o)+\int_{2\pi^+} f_r(p,\omega_i,\omega_o)L_o(r_c(p,\omega_i,-\omega_i))\cos\Theta_i d\omega_i$$



- Can be ugly to solve (p. 233 and Ch 24-26)
  - Lo in both sides, and recursive

The rendering equation(cont)

- Area form:
  - Alternative to hemisphere
  - Expressed as an integral over all surfaces in a scene using sample points
  - Same concept of *p* and *p'* in hemisphere
  - Only matters when *p* and *p'* are in direct line of sight. Idea of a Visibility function arises:

$$A: V(p, p') = \begin{cases} 1 & \text{if } p \text{ and } p' \; can\; see\; each\; other \\ 0 & \text{if } p \text{ and } p' \; cannot\; see\; each\; other \end{cases}$$

The rendering equation(cont)

- To use area form, must recast solid angle

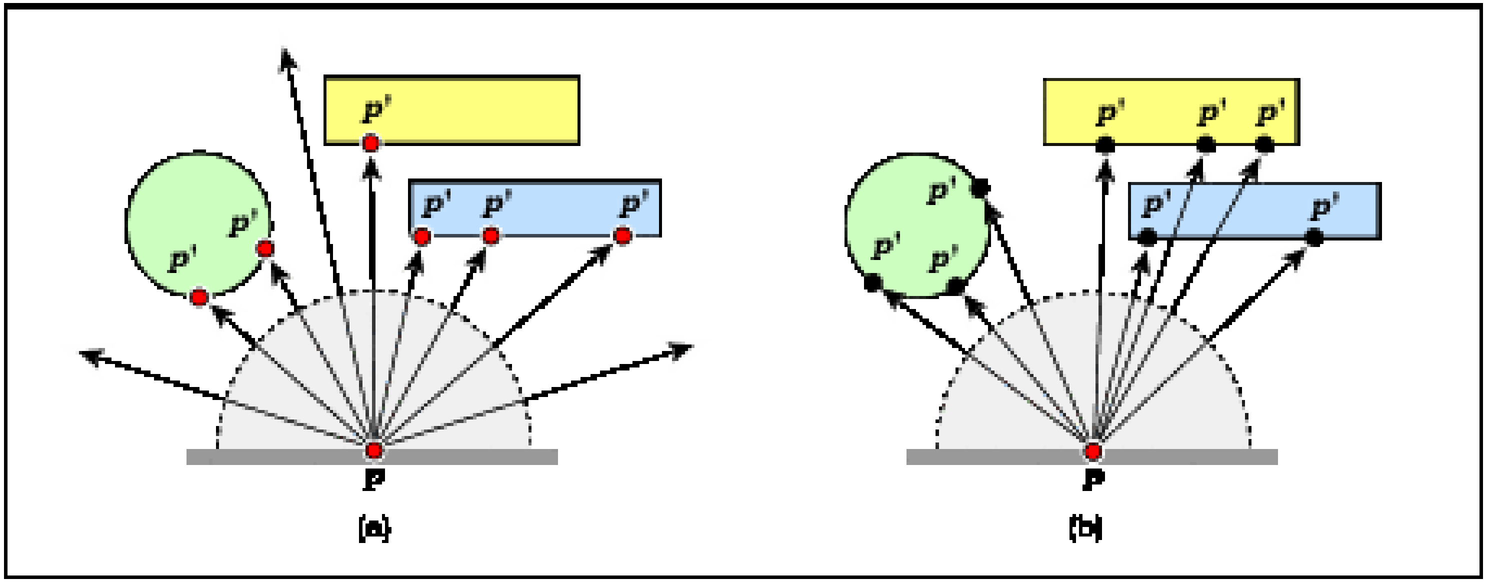
$$d\omega_i = \frac{\cos\Theta' \, dA}{\|p - p'\|^2}$$

- Ultimately, we're left with the area form of the rendering equation:
  - V term is the visibility function
  - G is the *geometry term* $G(p, p') = \frac{\cos\Theta_i \cos\Theta'}{\|p - p'\|^2}$

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_A f_r(p, \omega_i, \omega_o) L_o(p', -\omega_i) V(p, p') G(p, p') \, dA$$

The rendering equation(cont)

- Why is there no visibility function in the hemisphere form?



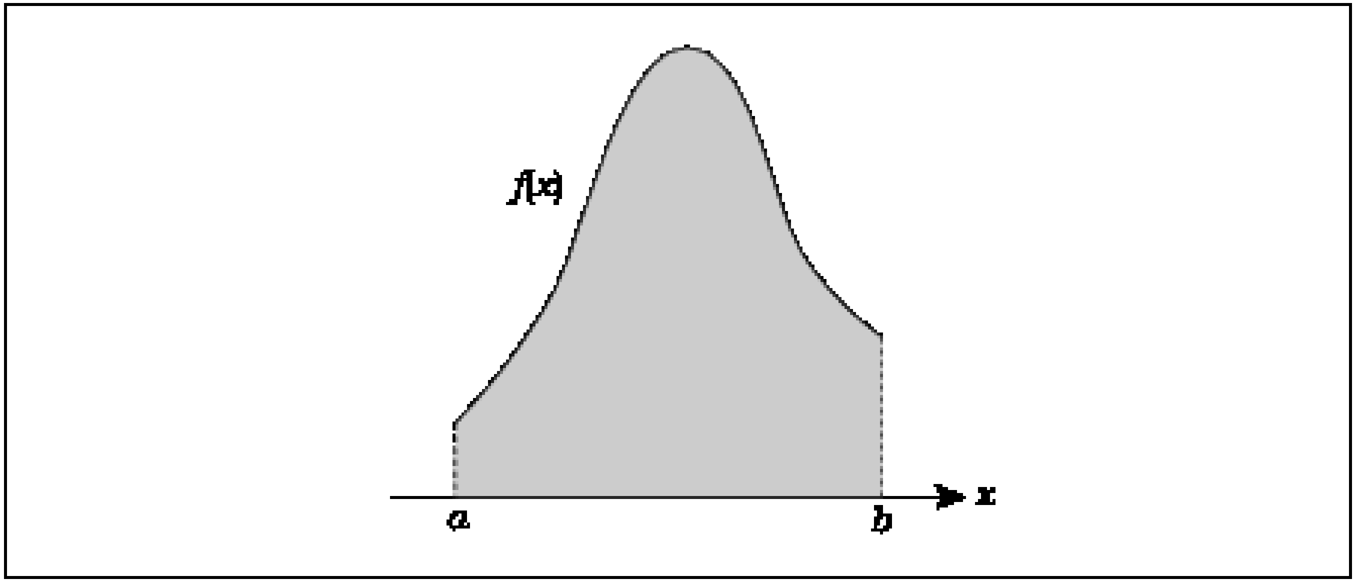
Monte Carlo integration

- The rendering equation can't be solved exactly, so we need to compute numerically. How?
  - Use Monte Carlo Integration:
    - Pick random values over some interval of interest
    - Evaluate function at each of these random values
    - Estimated Solution will be the sum of each of the evaluated solutions divided by the number of random points
    - Estimate gets better as more evaluation points are added

Monte Carlo integration (cont)

- Consider the solution for this definite integral:

$$I = \int_a^b f(x)\,dx$$

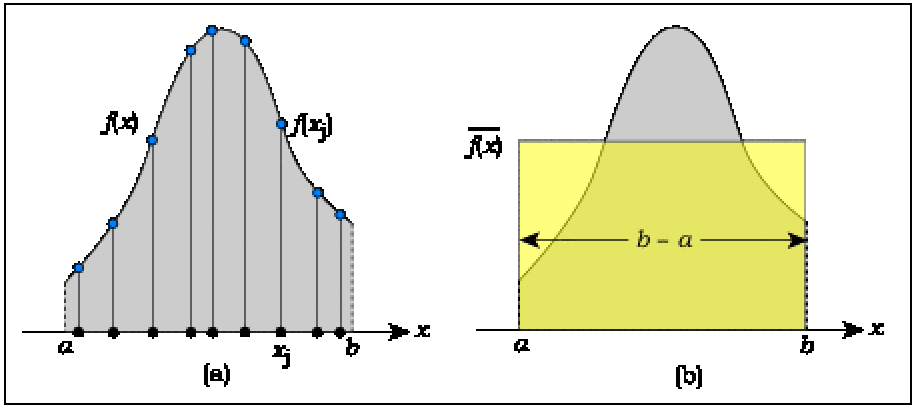


Monte Carlo integration (cont)

- By picking values between a and b and evaluating, we can start to approximate the solution to the definite integral, this leads to the *Monte Carlo estimator:*

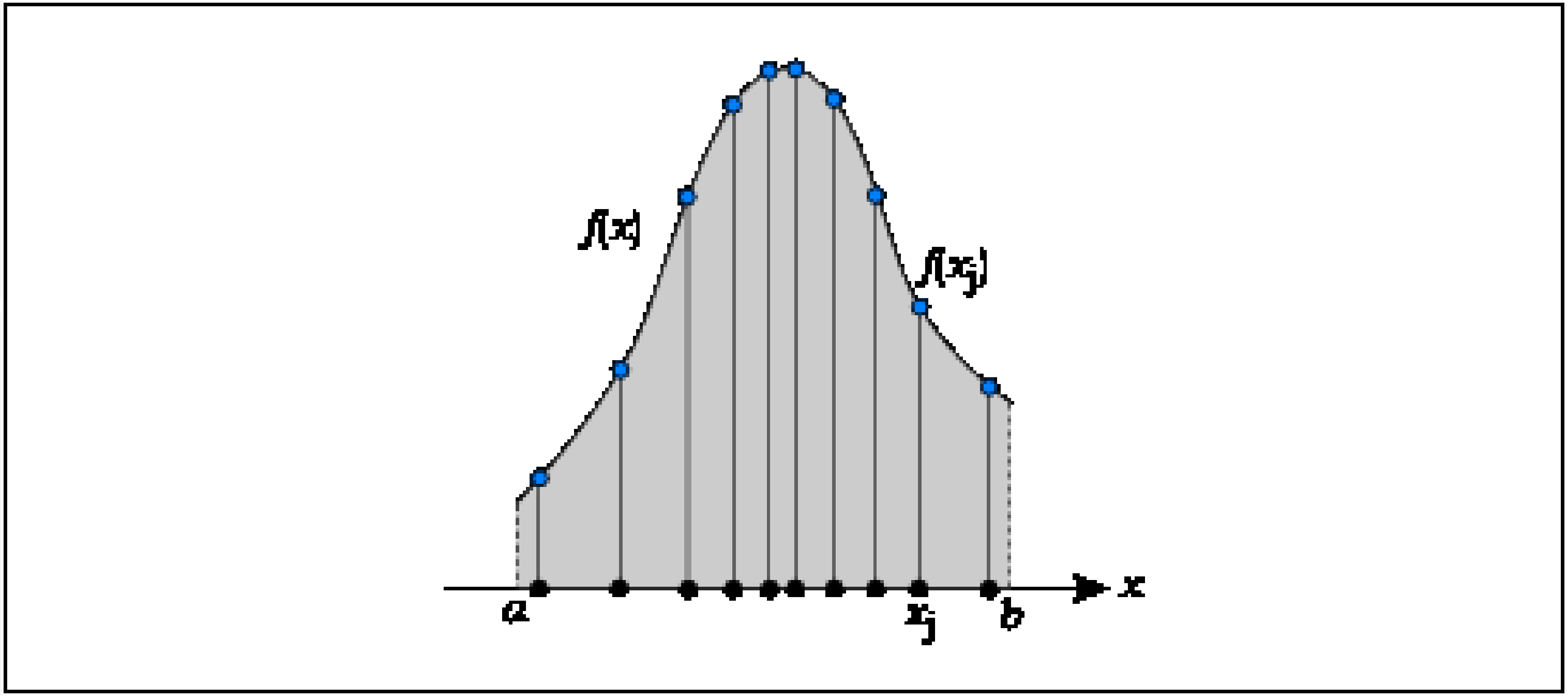
$$\langle I \rangle = \frac{b-a}{n} \sum f(x_j)$$

$$\overline{f(x)} = \frac{1}{n} \sum f(x_j)$$



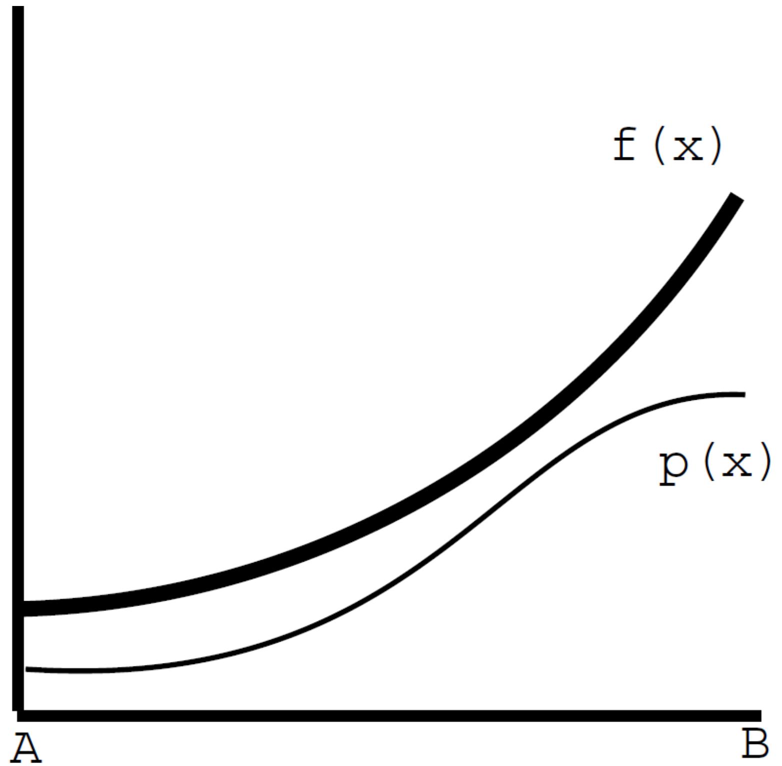
Monte Carlo integration (cont)

- Previous method works adequately, but what if there are some parts of the function contribute most of the area?
  - Use *Importance Sampling*



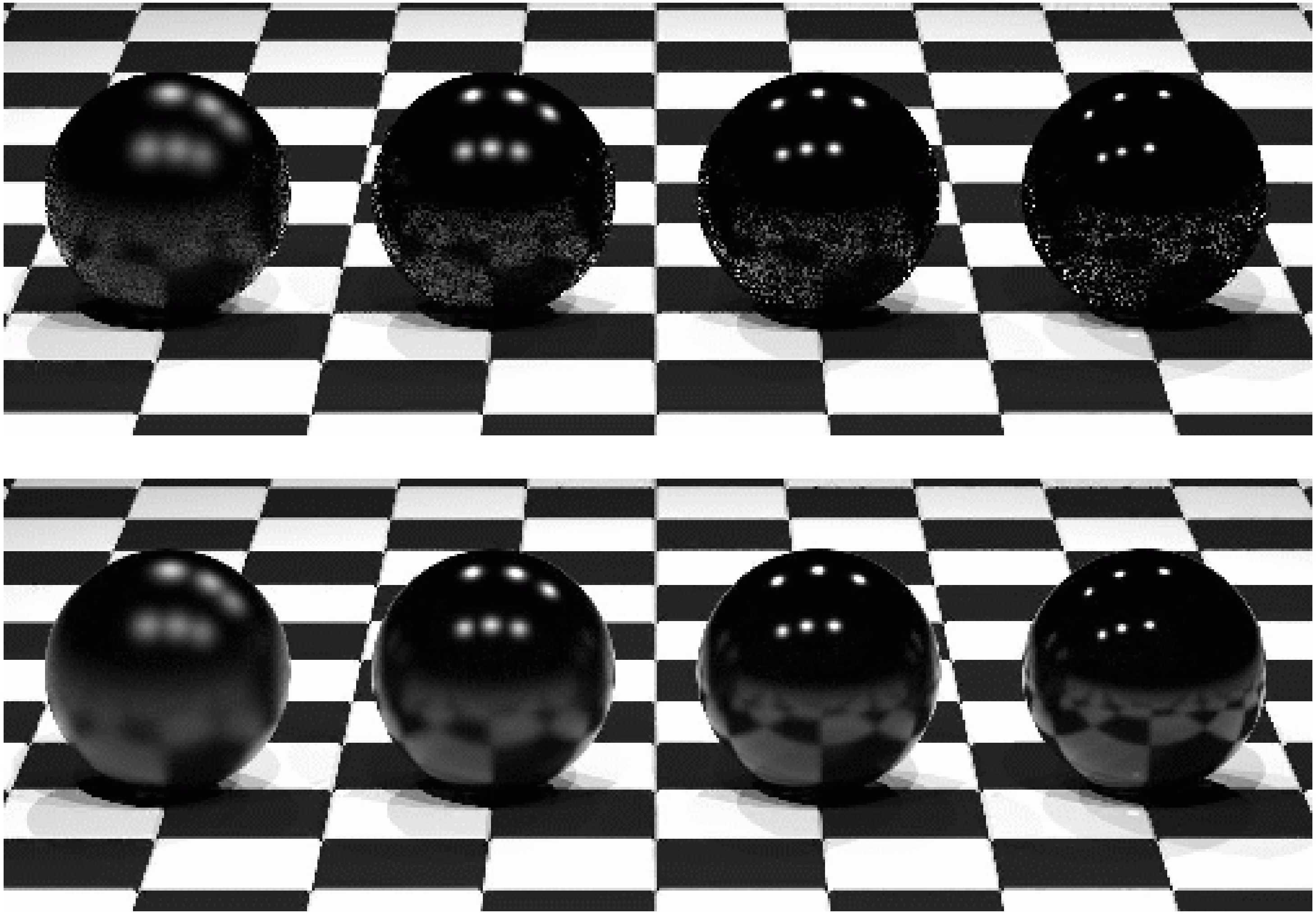
Monte Carlo integration (cont)

- We'd like to get more samples under the areas where f(x) have the greatest values. Rather than use a uniform sampling, use probability density function (pdf), p(x).



Monte Carlo integration (cont)

- Monte Carlo in action..... (regular-60 vs importance-60)



References

- Suffern, Kevin (2007). *Ray Tracing from the Ground Up*. pp. 217-242 Wellesley, MA: A K Peters, Ltd.
- F.E. Nicodemus, J.C. Richmond, J.J Hsia.(1977) *Geometric Considerations and Nomenclature for Reflectance*
- *http://en.wikipedia.org/wiki/Optical_coating*
- *http://www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA411012&Location=U2&doc=GetTRDoc.pdf#page=21*
- *http://math.nist.gov/~FHunt/appearance/obl.html*