Advanced Computer Graphics
CS 563: *Area and Environmental Light*

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Outline

- Radiometry
- Area Light Source Approximation
- Ambient Light
- Environmental Mapping
  - Maps
  - Explicit
- Spherical Harmonics
- Irradiance Map

Radiometry

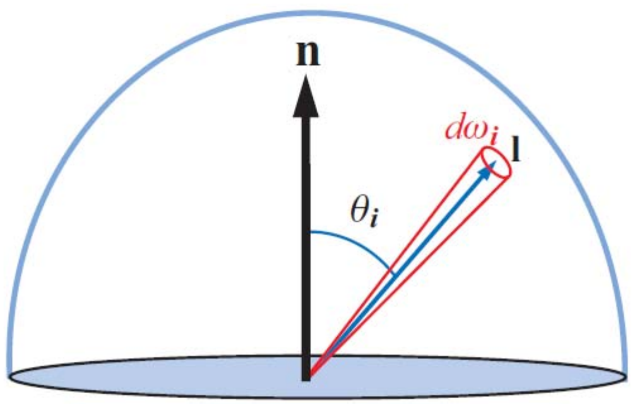
- Radiance: $\frac{W}{m^2 sr}$

$$L_i(l) = \frac{dE}{d\omega_i \cos(\theta_i)}$$

- Irradiance: $\frac{W}{m^2}$

$$E = \int_{\Omega} L_i(l) \cos(\theta) \, d\omega_i$$

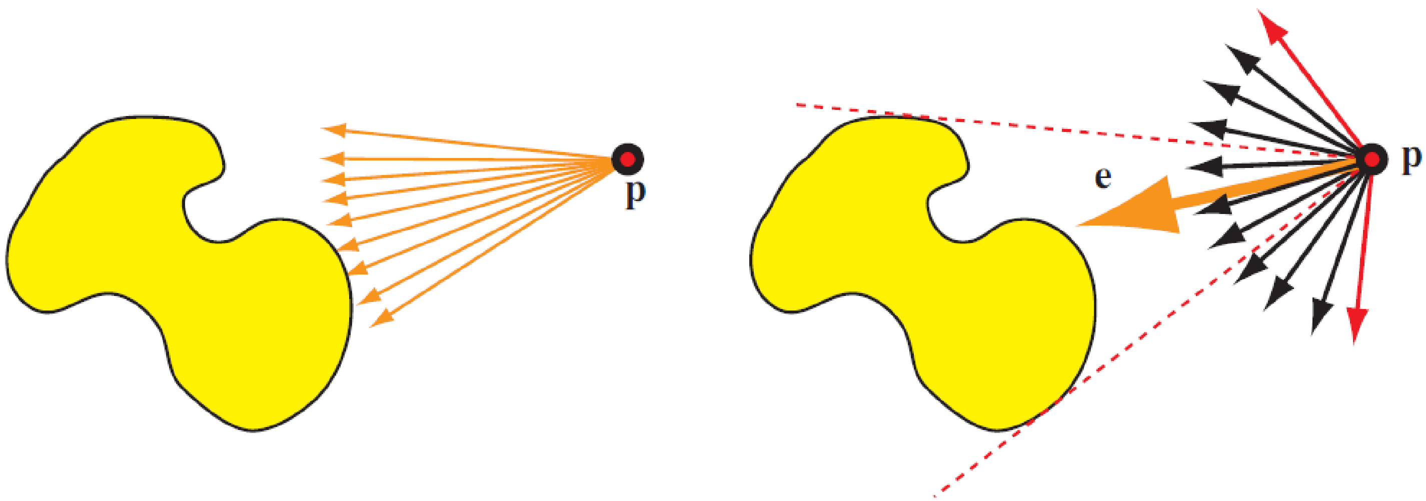
$$E = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} L_i(\theta, \phi) \cos(\theta) \sin(\phi) \, d\theta \, d\phi$$



Radiance [1]

Area Light Source

- Approximation:
  - Model the area light as point or directional
  - For any or all area sources compute a single vector e
    - This vector represents the average magnitude and direction
    - Irradiance can now be computed as $E(p,n) = n \cdot e(p)$



Light Vector [1]

Area Light Source: Alternatives

- Easy to add color to the light vector model
- Wrapping: point light -> light that covers hemisphere

$$E = E_L \, max\left(\frac{\cos(\theta_i) + c_{wrap}}{1 + c_{wrap}}, 0 \right)$$

- Implicit expression for a spherical area light
  - Assuming constant radiance

$$E_L = \pi \frac{r^2_L}{r^2} L_L$$

Ambiance

- Outgoing radiance take a simple constant term

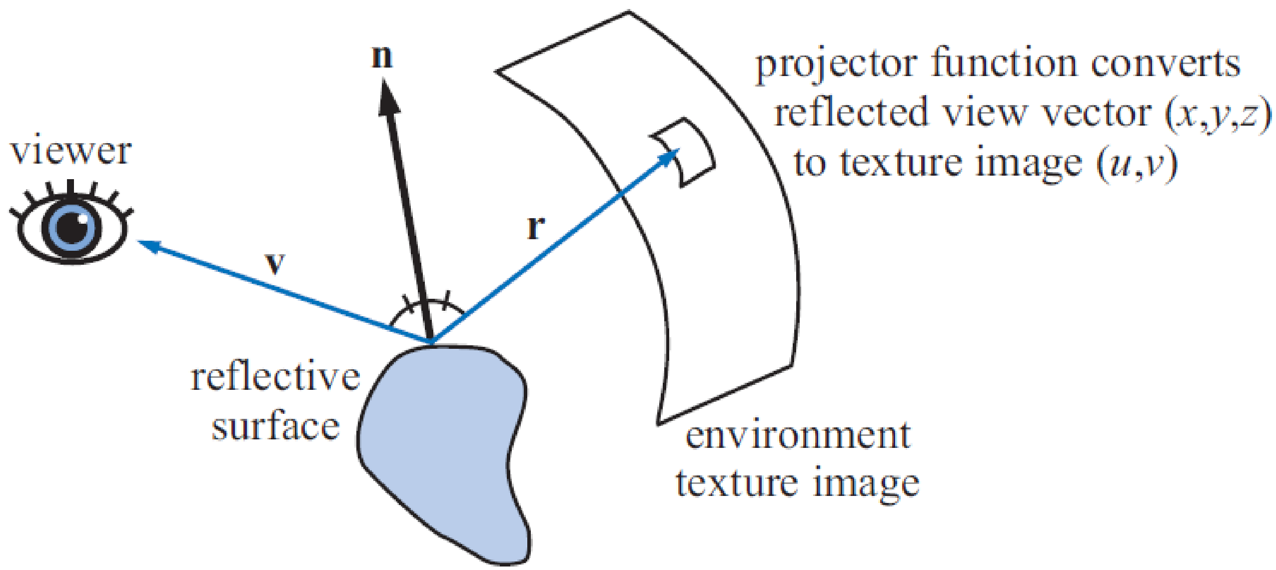
$$L_o(v) = c_{amb} L_A + \sum_{k=1}^{n} f(l_k, v) E_{L_k} \cos(\theta_{i_k})$$

- Could replace c_{amb} with ambient reflectance for view dependent, self occluding ambiance

$$R_A(v) = \int_{\Omega} f(l, v) \cos(\theta_i) d\omega_i$$

Environmental Mapping

- Model reflective surfaces
  - Project reflect vectors onto some function
  - Use function evaluation as radiance



Environment Map [1]

EM: Maps

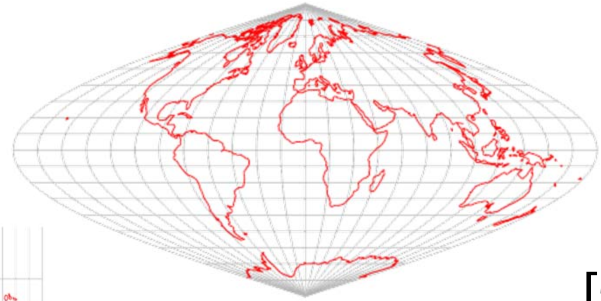
- Use components of reflect vector to sample:
  - Equirectangular: Two singularities at poles, does not preserve area

$$\rho = arcos(-r_z)$$
$$\phi = atan2(r_y, r_x)$$

[6]

  - Mercator Equal Area [5]:

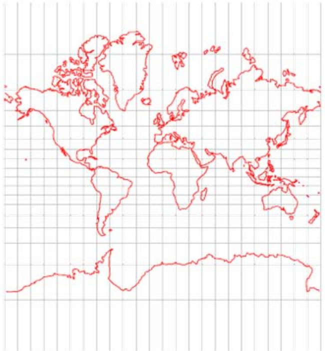
$$x = (\lambda - \lambda_0)\cos(\phi)$$
$$y = \phi$$



[5]

  - Other Maps:

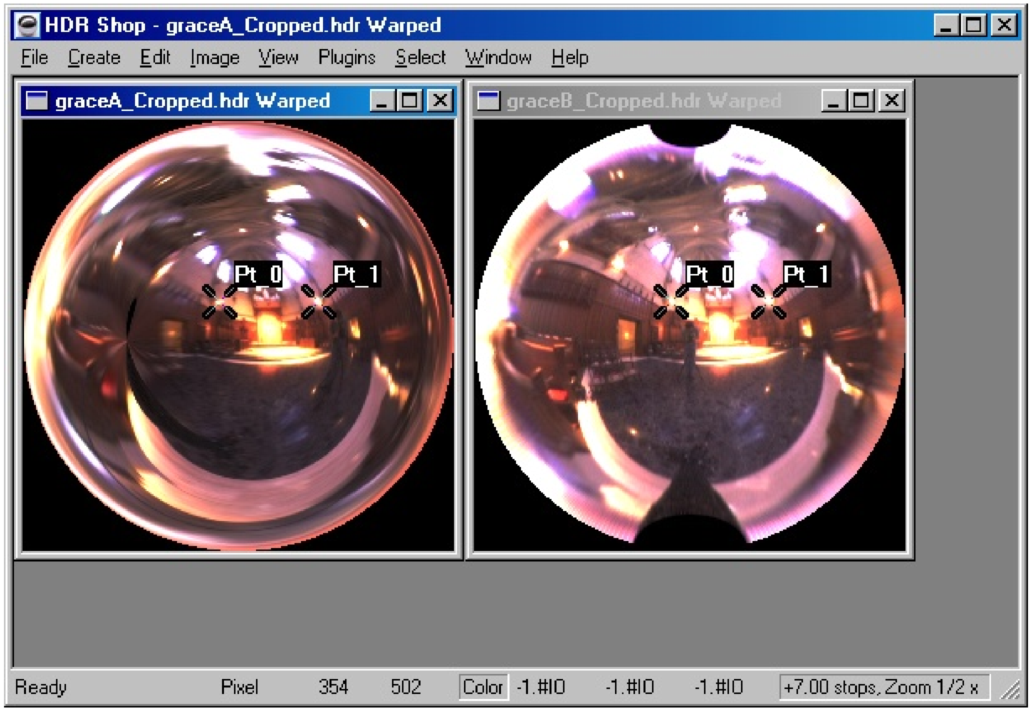
[7]



[5]

EM: Sphere Mapping

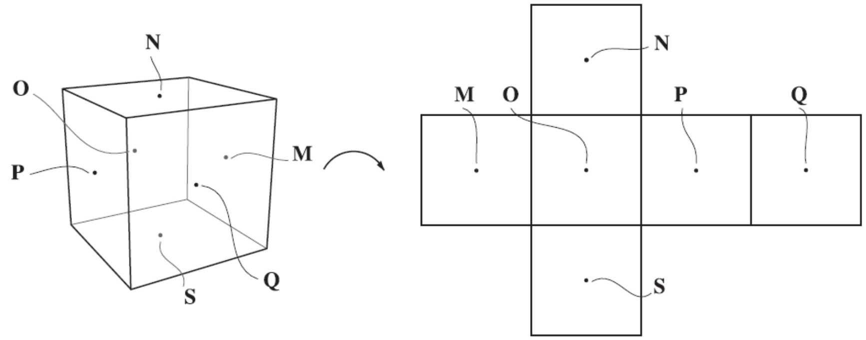
- Use light probe or generate data in
  - View Dependent, recomputed for different view
- Transform surface normal and view vector into reference frame of sphere map projection



[8]

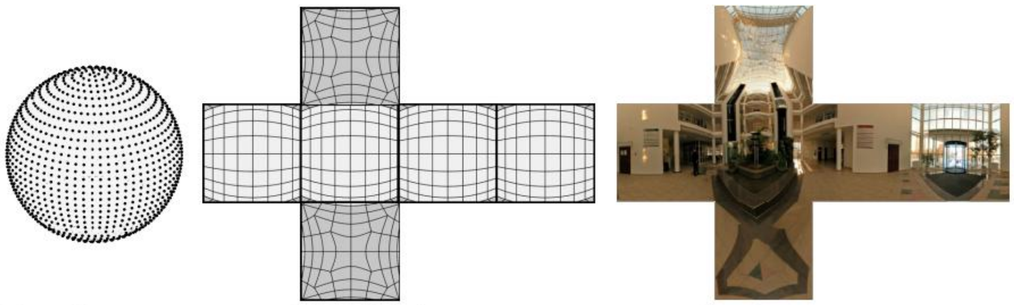
EM: Cubic Environment Mapping

- View independent
- Better uniformity in sampling



[1]

- Use isocube to achieve better distribution



(a) sampling pattern (b) unrolled view (c) environment map

[9]

EM: Parabolic Mapping

- Join two parabolic projections
- No singularities
- Decent sampling
- Difficult to generate

$$u = \frac{r_x}{2(1+r_z)} + 0.5$$

$$v = \frac{r_y}{2(1+r_z)} + 0.5$$

[10]

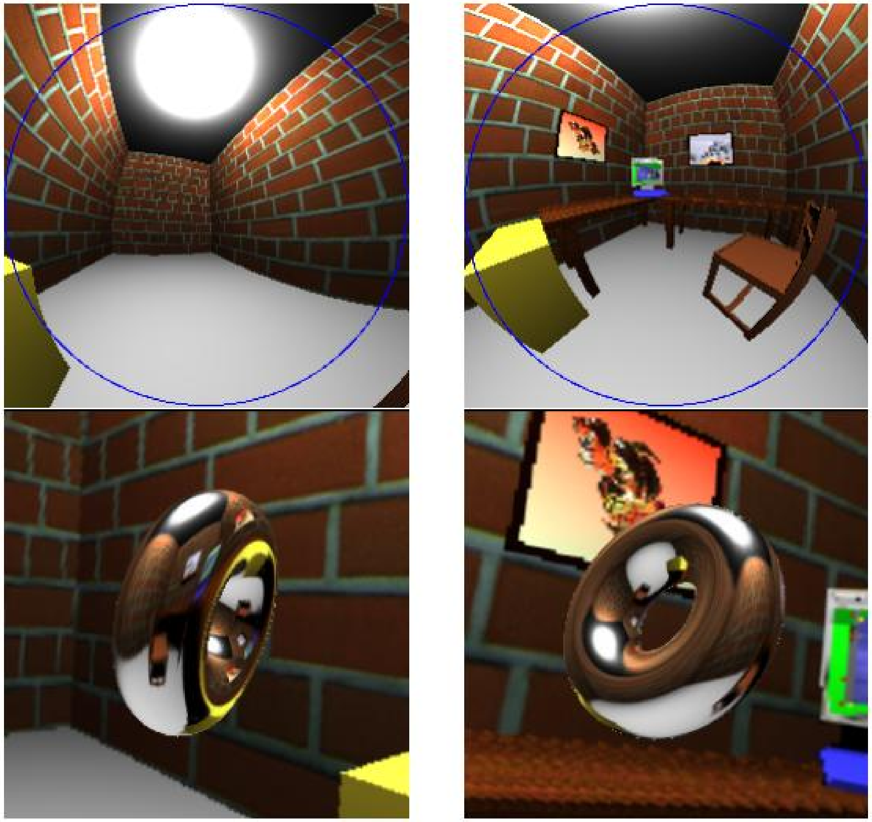


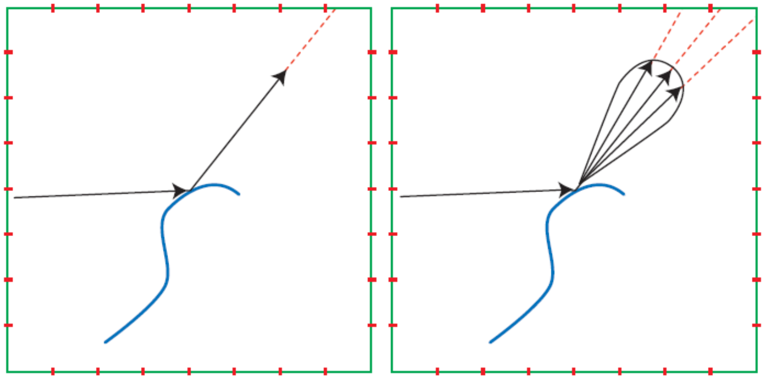
Figure 6: Top: two parabolic images comprising one environment map. Bottom: rendering of a torus using this environment map.

EM: Glossy Reflections

- Artifacts from sampling cube map

[11]

- Filter with Gaussian lobes at various resolutions
    - Not accurate but gives appearance of variable reflectivity

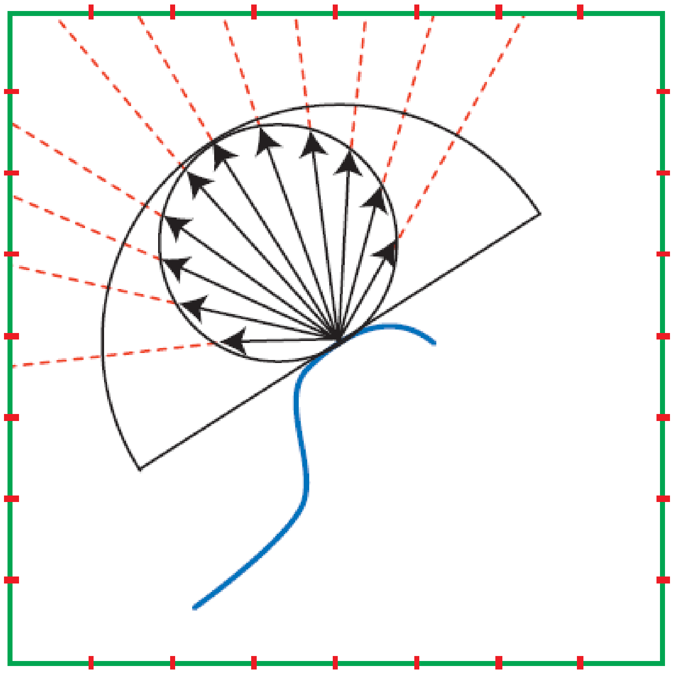


[1]

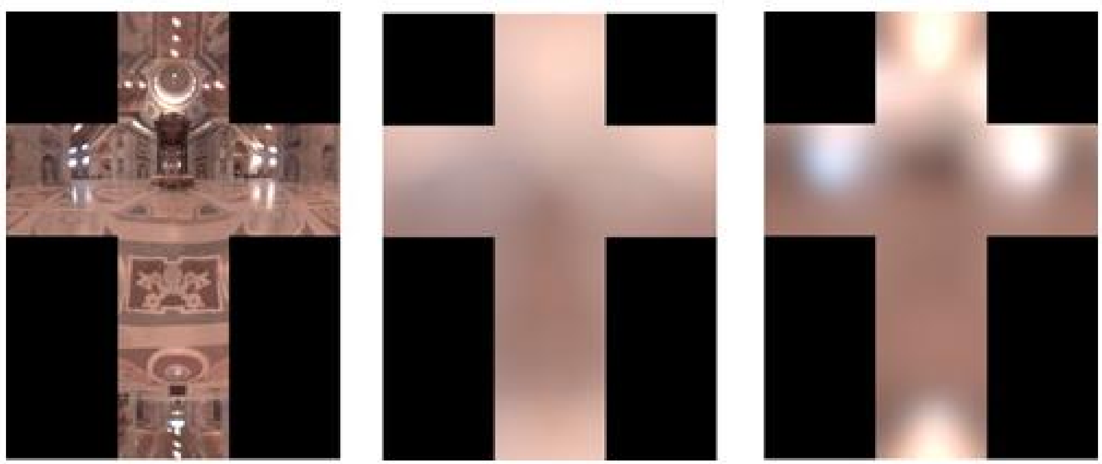
[11]

EM: Irradiance Mapping

- Map irradiance to some texture
- Generated from EM
- Addressed by the normal of surface



Generate Irradiance Map [1]



[12]

Spherical Harmonics: Impetus

- SH expression can allow for a reasonably accurate representation of low frequency objects.
- Fast to compute, small set of polynomials
- Reasonably fast to solve
- Allow for frequency domain modification
- Functions are orthonormal

$$\langle f_i(), f_j() \rangle = \begin{cases} 0, \textit{where}\ i \neq j \\ 1, \textit{where}\ i = j \end{cases}$$

Spherical Harmonics : Description

- Laplacian (divergence of gradient) expression.

$$\frac{\partial^2 x f}{\partial x^2} + \frac{\partial^2 y f}{\partial y^2} + \frac{\partial^2 z f}{\partial z^2} = \Delta f = 0$$

- Provides a frequency domain representation of some feature in spherical coordinates.
  - We look for where this expression is 0. We will fit solutions to zeros of the second derivative (essentially edge detection).

$$\frac{\Phi(\phi)}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{\sin^2\theta}\right) + \frac{\Theta(\theta)}{\sin^2\theta}\frac{d^2\Phi(\phi)}{d\phi^2} + l(l+1)\Theta(\theta)\Phi(\phi) = 0$$

Spherical Harmonics: Expression

- Two parts of the equation:
  - Zonal (perturbed only in the altitude angle [0..PI])

$$Z^l(\theta, \phi) = P_l(\cos\theta)$$

Legendre Polynomial

$$P_l(x)$$

  - Azimuthal (oscillates with altitude and azimuth)
    - More components as frequency increases.

$$Y_l^m(\theta, \phi) = \sqrt{\frac{21+1}{4\pi}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\phi}$$

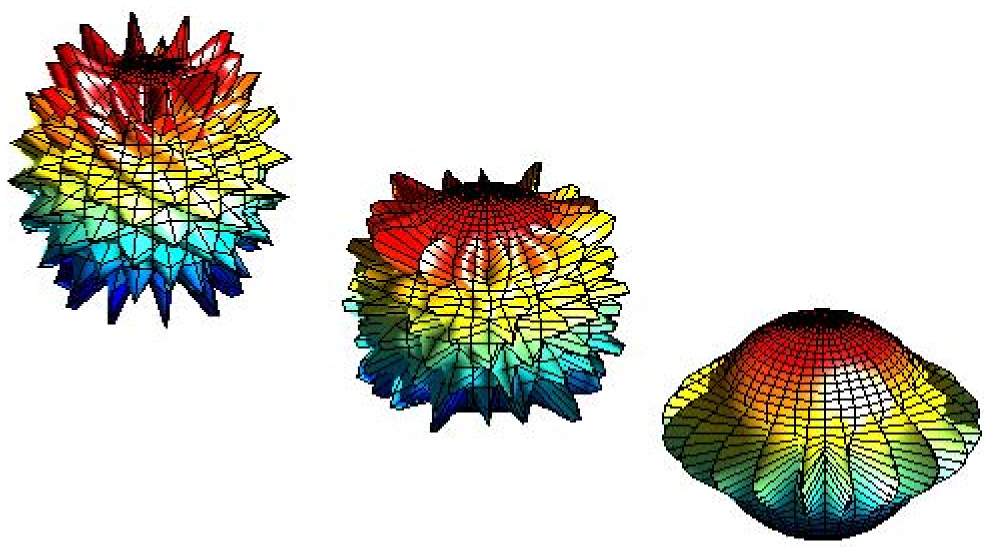
Associated Legendre Polynomial

$$P_l^m(x)$$

  - Real and imaginary components are identical but out of phase

Spherical Harmonics: Intuition

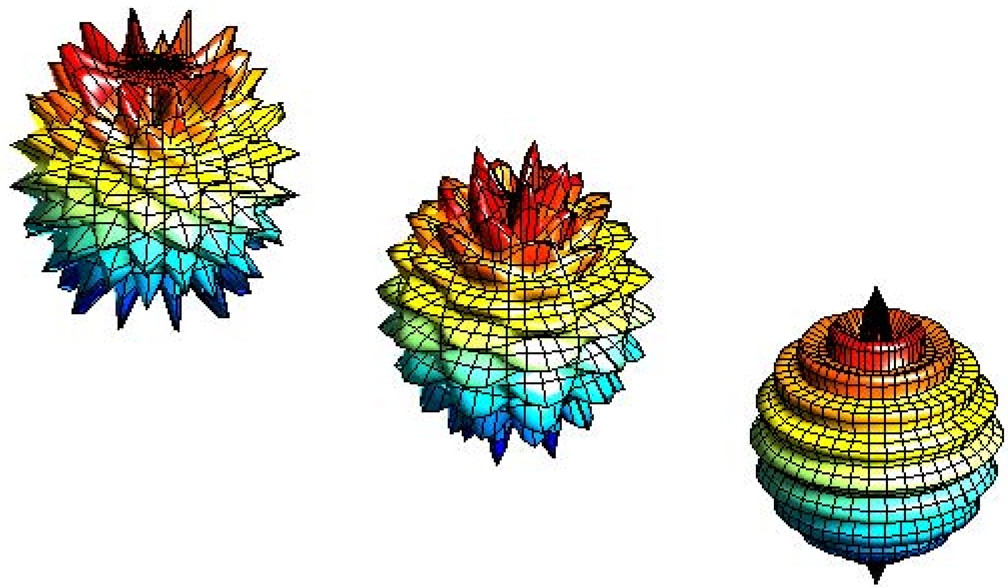
- As order index m approaches degree l, oscillations concentrate in theta angle



Left to Right: Degree 20, Order 10, 15, 20

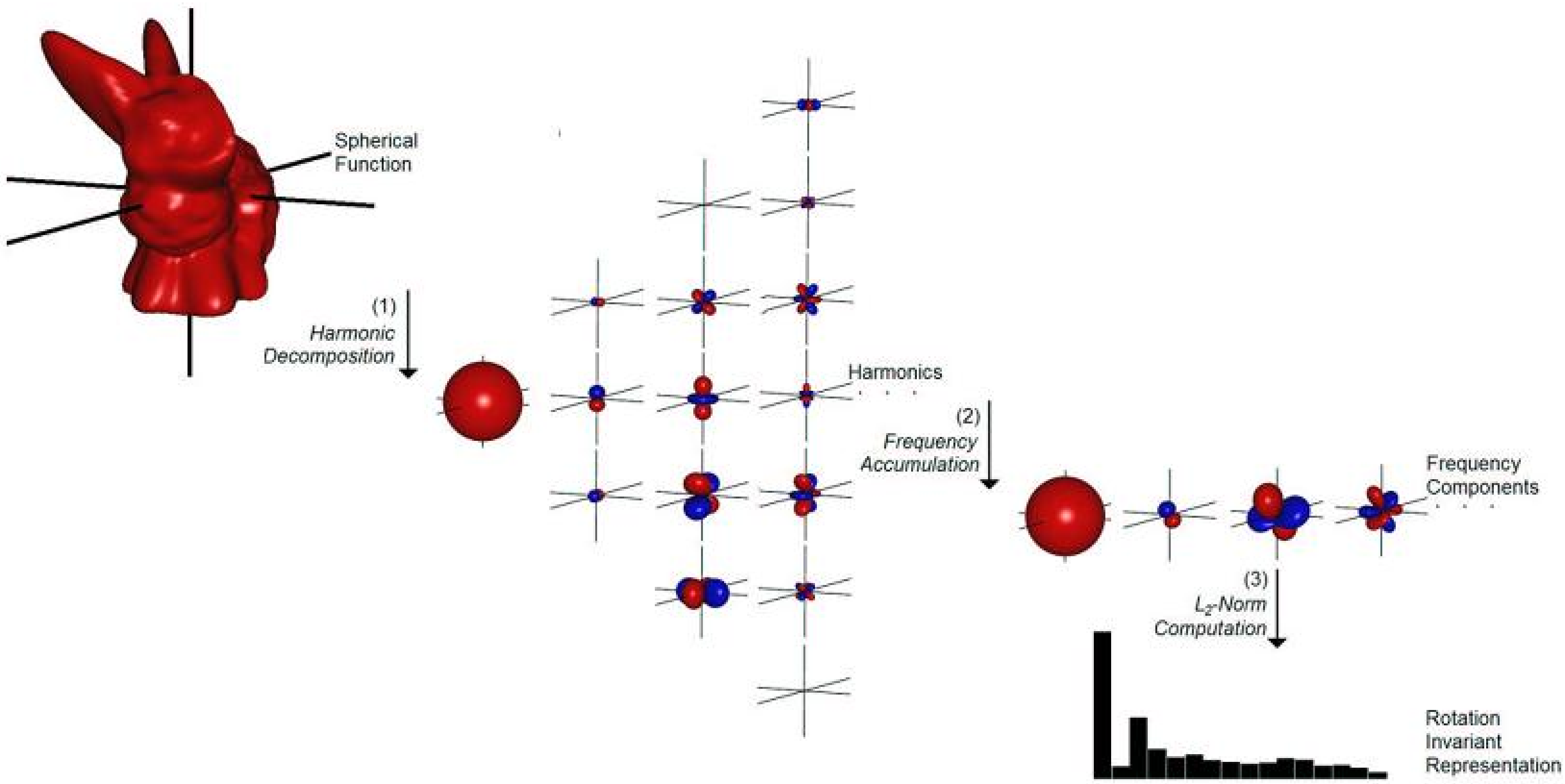
Spherical Harmonics: Intuition

- When m index is close to l, oscillations concentrate in phi angle



Left to Right: Degree 20, Order 10, 5, 0

Spherical Harmonics : Graphic



* Modified from original to fit page [2]

Spherical Harmonics: Limitations

- Requires many components to represent non-axially symmetric data
- Cannot represent all object perfectly, singularities require infinite terms
- Is not necessarily rotation invariant
  - however its power spectrum is rotation invariant

Spherical Harmonics: Solutions

- Fit SH with least squares or some other method
  - Build matrix of observed energy per (θ, ϕ)
  - Build matrix of basis functions constructed from associated Legendre polynomials
  - Use some fitting method to find function weights
- Easy to generate with MATLAB, Mathematica, Boost libraries etc.
- Some methods can solve in $O(n^2 lg(n))$ [4]

EM: Inexpensive Irradiance

- Weighted sum of ground and sky radiance

$$E = \pi (1 - \frac{1}{2}\sin\theta) L_{sky} + \frac{1}{2}\sin\theta L_{ground}, where\ \theta < \frac{\pi}{2}$$

$$E = \pi (\frac{1}{2}\sin\theta) L_{sky} + (1 - \frac{1}{2}\sin\theta L_{ground}), where\ \theta \geq \frac{\pi}{2}$$

[3]

- Ambient cube

$$E = E_x + E_y + E_z$$

  - (x,y,z) irradiance selected from cube map surfaces within the hemisphere of surface normal

References

- [1] *Real Time Rendering: Third Edition by*
- *[2]* Michael Kazhdan , Thomas Funkhouser , Szymon Rusinkiewicz, Rotation invariant spherical harmonic representation of 3D shape descriptors, Proceedings of the 2003 Eurographics/ACM SIGGRAPH symposium on Geometry processing, June 23-25, 2003, Aachen, Germany
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- [4] *Rokhlin, V. and Tygert, M., Fast algorithms for spherical harmonic expansions, SIAM J. Sci.*

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- [5]http://mathworld.wolfram.com/SinusoidalProjection.html
- [6]http://earthobservatory.nasa.gov/Features/BlueMarble/

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- [10] Wolfgang Heidrich , Hans-Peter Seidel, Realistic, hardware-accelerated shading and lighting, Proceedings of the 26th annual conference on Computer graphics and interactive techniques, p.171-178, July 1999
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