CS3133 Foundations of CS - A Term 2009 Prof. Carolina Ruiz
Exam 2 Solutions by Prof. Ruiz and Li Feng Dept. of Computer Science
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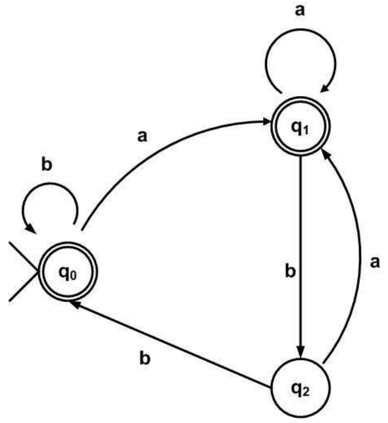
PROBLEM 1: True or False? (10 points)

Answer **true** or **false** in front of each of the statements below according to whether the statement is true or false.

	True or False?
The set of languages accepted by DFAs is the same set of languages accepted by NFAs	**true**
The set of languages accepted by deterministic PDAs is the same set of languages accepted by nondeterministic PDAs	**false**
The set of regular languages is a subset of the set of context–free languages	**true**
The particular value of k provided by the pumping lemma for CFLs is the same value for all context-free languages	**false**
Context-free languages are closed under intersection and complementation	**false**

PROBLEM 2: DFAs and Regular Expressions (25 points)

Convert the DFA below into an equivalent regular expression by following the procedure involving expression graphs explained in class and in the textbook. Show each of the steps of the process.



Solution:

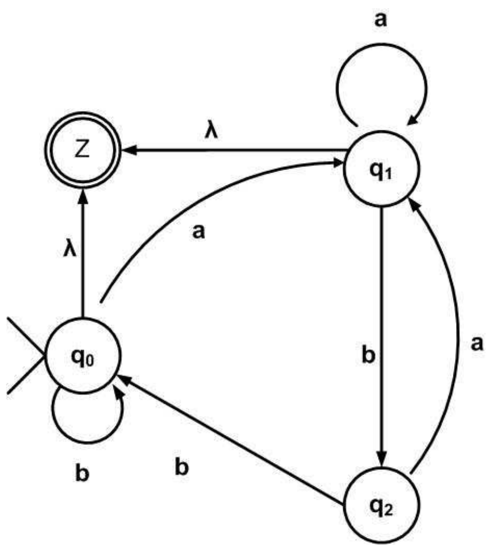


Figure 1: Adding a new accepting state Z.

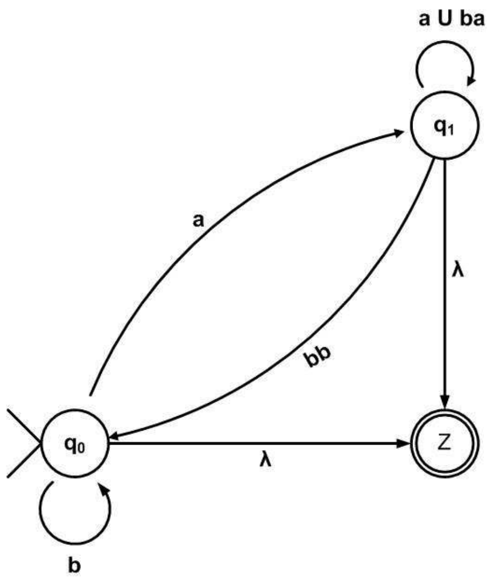


Figure 2: Removing state q_2.

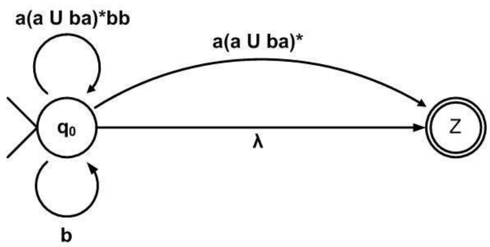


Figure 3: Removing state q_1.

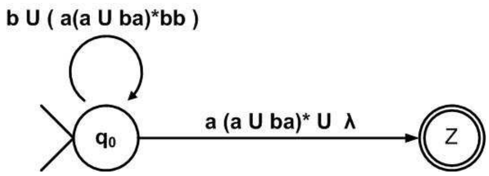


Figure 4: Reducing the edges.

The regular expression is

$$[b \cup (a(a \cup ba)^* bb)]^* (a(a \cup ba)^* \cup \lambda)$$

PROBLEM 3: Pumping Lemma for Regular Languages (25 points)

Use the pumping lemma for regular languages to prove that the following language is NOT regular:

$$L = \{w \in \{a, b\}^* \mid w \text{ has twice as many } a\text{'s as } b\text{'s}\}$$

That is, L contains strings of a's and b's where the number of a's is exactly twice the number of b's. (By the way, note that λ belongs to L). Your proof should be clear and decisive.

Solution:
By way of contradiction, let's assume that L is a regular language. Hence, there is a DFA M that accepts L. Let k the number of states of M. Consider the string $z = a^{2k}b^k$. Clearly z belongs to L since it has exactly twice as many a's as b's. The pumping lemma for regular languages guarantees that there is a decomposition of z as uvw such that:

1. length$(uv) \leq k$,
2. length$(v) > 0$, and
3. For all $i \geq 0$, uv^iw belongs to L.

Since $z = a^{2k}b^k = uvw$ and length$(uv) \leq k$, then v consists of only a's. Also, since length$(v) > 0$, then v consists of at least one a. Hence, when you pump v say one more time, the resulting string uv^2w cannot belong to L because it will contain more a's than twice the number of b's. This is a contradiction, hence L does not satisfy the pumping lemma for regular languages, and so L is not a regular language.

PROBLEM 4: Pushdown Automata (25 points)

Give the state diagram of a pushdown automaton (PDA) whose language consists of the set of strings over $\{a, b\}$ that contain more b's than a's. You can use a standard PDA or any equivalent variation of PDAs. Just make sure to state explicitly in your solution what kind of PDA variation you are using.

> **Examples:** The strings *aabababbbabba* and *baabbbab* belong to this language, but λ, *abbababa*, *bbaaaaba*, and *aaa* do not.

Solution:

We provide two alternate solutions: One using an extended PDA and the other one using a standard PDA.

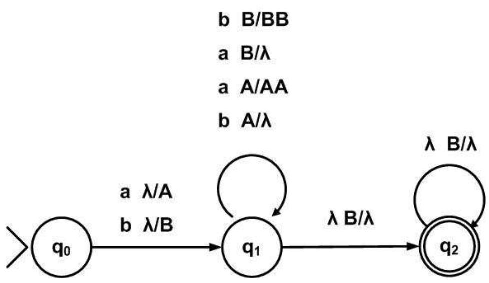


Figure 5: Extended PFA that accepts string that contain more b's than a's.

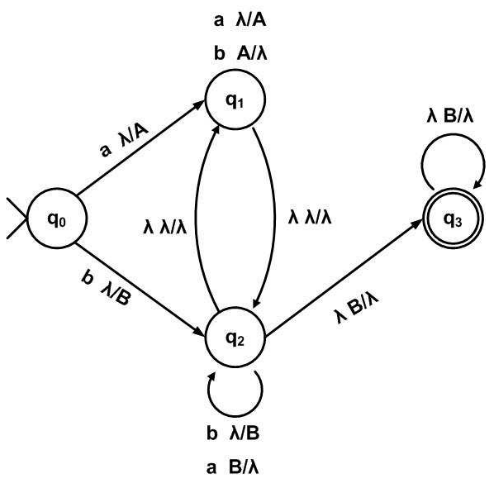


Figure 6: Standard PFA that accepts strings that contain more b's than a's.

PROBLEM 5: Context–Free Grammars and Pushdown Automata (25 points)

Convert the following context–free grammar in Greibach normal form into an equivalent pushdown automaton by following the procedure described in class and in the textbook.

$$S \to aAB' \mid a$$
$$A \to aAB'A'AB' \mid aAB'A' \mid aA'AB' \mid aA' \mid b$$
$$A' \to a$$
$$B' \to b$$

Explicitly state what symbols are included in your stack alphabet Γ.

Solution:

This context–free grammar was taken from our solutions to Exercise 4.30 in Homework 2, CS3133 A term 2009. The resulting PDA is depicted below, where $\Gamma = \{A, A', B'\}$: