CS3133 - A Term 2009: Foundations of Computer Science	Prof. Carolina Ruiz
Homework 2	
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Chapter 4

Problem 1: (10 Points) Exercise 4.3

Solution 1:

S is self–recursive, so we need to introduce a new non–recursive start symbol S' with the associated rule $S' \to S$. Note that S, A, and B are nullable variables.

$$S' \to S \mid \lambda$$
$$S \to BSA \mid BS \mid SA \mid BA \mid B \mid S \mid A$$
$$B \to Bba \mid ba$$
$$A \to aA \mid a$$

The regular expression is $(ba)^*a^*$.

Problem 2: (10 Points) Exercise 4.9

Solution 2:

Note that:

X	CHAIN(X)
S	$\{S, A, C, B\}$
A	$\{A, B\}$
B	$\{B\}$
C	$\{C, B\}$

Eliminating chain rules from the grammar results in:

$$S \to aA \mid a \mid cC \mid c \mid bB \mid b$$
$$A \to aA \mid a \mid bB \mid b$$
$$B \to bB \mid b$$
$$C \to cC \mid c \mid bB \mid b$$

The regular expression for this grammar is:

$$a^+b^* \cup c^+b^* \cup b^+$$

Problem 3: (10 Points) Exercise 4.15

Solution 3:

$TERM$	$PREV$
$\{D\ ,\ F\ ,\ G\}$	$\emptyset$
$\{D\ ,\ F\ ,\ G\ ,\ A\}$	$\{D\ ,\ F\ ,\ G\}$
$\{D\ ,\ F\ ,\ G\ ,\ A\ ,\ S\}$	$\{D\ ,\ F\ ,\ G\ ,\ A\}$
$\{D\ ,\ F\ ,\ G\ ,\ A\ ,\ S\}$	$\{D\ ,\ F\ ,\ G\ ,\ A\ ,\ S\}$

$$TERM = \{S\ ,\ A\ ,\ F\ ,\ G\ ,\ D\}$$

Intermediate Grammar:

$$\begin{array}{l} S \rightarrow aA \\ A \rightarrow aA \mid aD \\ D \rightarrow bD \mid b \\ F \rightarrow aF \mid aG \mid a \\ G \rightarrow a \mid b \end{array}$$

$REACH$	$PREV$	NEW
$\{S\ \}$	$\emptyset$	
$\{S\ ,\ A\}$	$\{S\ \}$	$\{S\ \}$
$\{S\ ,\ A\ ,\ D\}$	$\{S\ ,\ A\}$	$\{A\ \}$
$\{S\ ,\ A\ ,\ D\}$	$\{S\ ,\ A\ ,\ D\ \}$	$\{D\ \}$

$$REACH = \{S\ ,\ A\ ,\ D\}$$

Final Grammar:

$$\begin{array}{l} S \rightarrow aA \\ A \rightarrow aA \mid aD \\ D \rightarrow bD \mid b \end{array}$$

Regular Expression : aa^+b^+

Problem 4: (10 Points) Exercise 4.19

Solution 4:

Transform the rules in G to rules of the form $S \rightarrow \lambda$, $A \rightarrow a$, $A \rightarrow w$ where w is a string with variables only

$$\begin{array}{l} S \rightarrow A'AB'B \mid ABC \mid a \\ A \rightarrow A'A \mid a \\ B \rightarrow B'BC'C \mid b \\ C \rightarrow A'B'C' \\ A' \rightarrow a \\ B' \rightarrow b \\ C' \rightarrow c \end{array}$$

Then rewrite the above grammar such that each rule has exactly 2 variables on the right-hand side

$$
\begin{aligned}
&S \to A'T_1 \mid AT_3 \mid a\\
&T_1 \to AT_2\\
&T_2 \to B'B\\
&T_3 \to BC\\
&A \to A'A \mid a\\
&B \to B'T_4 \mid b\\
&T_4 \to BT_5\\
&T_5 \to C'C\\
&C \to A'T_6\\
&T_6 \to B'C'\\
&A' \to a\\
&B' \to b\\
&C' \to c
\end{aligned}
$$

Problem 5: (10 Points) Exercise 4.23

Solution 5:

Grammar after removing λ–rules:

$$
\begin{aligned}
&S \to \lambda \mid A \mid ABa \mid Ba \mid AbA \mid bA \mid Ab \mid b\\
&A \to Aa \mid a\\
&B \to Bb \mid BC\\
&C \to CB \mid CA \mid bB
\end{aligned}
$$

Grammar after removing chain rules: CHAIN(S) = $\{S, A\}$

$$
\begin{aligned}
&S \to \lambda \mid Aa \mid a \mid ABa \mid Ba \mid AbA \mid bA \mid Ab \mid b\\
&A \to Aa \mid a\\
&B \to Bb \mid BC\\
&C \to CB \mid CA \mid bB
\end{aligned}
$$

Now, let's look for useless symbols:

$TERM$	$PREV$
$\{S, A\}$	$\emptyset$
$\{S, A\}$	$\{S, A\}$

$$TERM = \{S, A\}$$

Grammar G_T:

$$
\begin{aligned}
&S \to \lambda \mid Aa \mid a \mid AbA \mid bA \mid Ab \mid b\\
&A \to Aa \mid a
\end{aligned}
$$

$REACH$	$PREV$	NEW
$\{S\}$	$\emptyset$	
$\{S, A\}$	$\{S\}$	$\{S\}$
$\{S, A\}$	$\{S, A\}$	$\{A\}$

$$REACH = \{S, A\}$$

Grammar G_U:

$$S \to \lambda \mid Aa \mid a \mid AbA \mid bA \mid Ab \mid b$$
$$A \to Aa \mid a$$

Final Grammar in Chomsky Normal Form:

$$S \to \lambda \mid AA' \mid a \mid AT_1 \mid B'A \mid AB' \mid b$$
$$T_1 \to B'A$$
$$A \to AA' \mid a$$
$$A' \to a$$
$$B' \to b$$

Problem 6: (10 Points) Exercise 4.27

Solution 6: **a.)** $aab^*(aba)b^*(ab \cup ba)^* \cup b^*abab^*$

b.)

$$S \to A \mid B$$
$$A \to aaB \mid aaBZ$$
$$Z \to abZ \mid baZ \mid ab \mid ba$$
$$B \to bB \mid aba \mid bBY \mid abaY$$
$$Y \to bY \mid b$$

Problem 7: (10 Points) Exercise 4.30

Solution 7:

$$S \to aAB' \mid a$$
$$A \to aAB'A'AB' \mid aAB'A' \mid aA'AB' \mid aA' \mid b$$
$$A' \to a$$
$$B' \to b$$

Chapter 5

Problem 8: (10 Points) Exercise 5.2

Solution 8: **part a**

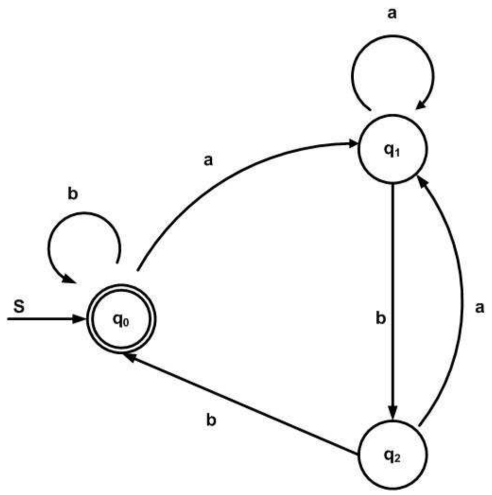

Figure 1: Chap 5 Question 2.a

part b

$$\begin{array}{ll} & [q_0, babaab] \\ \vdash & [q_0, abaab] \\ \vdash & [q_1, baab] \\ \vdash & [q_2, aab] \\ \vdash & [q_1, ab] \\ \vdash & [q_1, b] \\ \vdash & [q_2, \lambda] \end{array}$$

part c

$$[b \cup (a(a \cup ba)^* bb)]^*$$

part d

$$[b \cup (a(a \cup ba)^* bb)]^* (\lambda \cup [a(a \cup ba) *])$$

Problem 9: (10 Points) Exercise 5.3

Solution 9: **part a**

δ	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_0	q_1

part b

Accepted strings are $baab$, $abaaab$.

part c

$$[b \cup [aa^*b(ba^*b)^*a]]^*aa^*b(ba^*b)^*$$

Problem 10: (10 Points) Exercise 5.13

Solution 10:

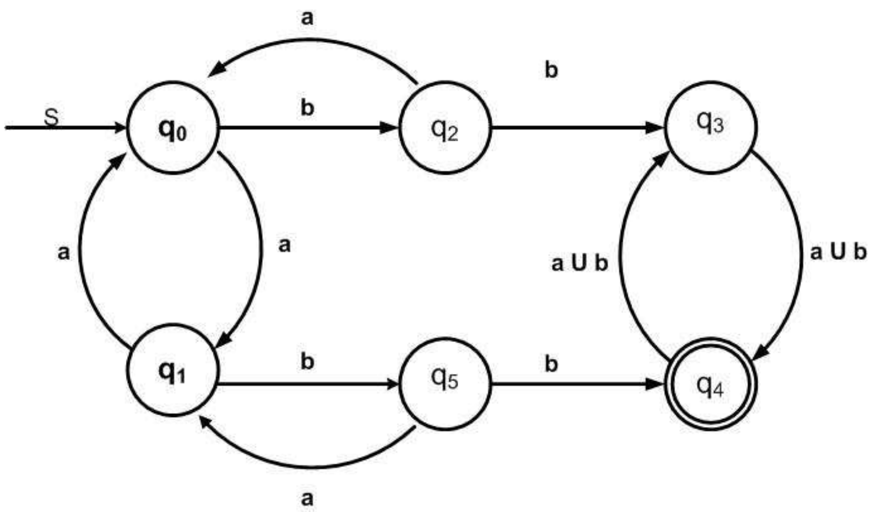

Figure 2: Chap5 Question 13

Problem 11: (10 Points) Exercise 5.23

Solution 11:

part a

δ	a	b
q_0	$\{q_0, q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_1, q_2\}$
q_2	$\{q_0, q_1\}$	$\emptyset$

part b

$[q_0, aaabb]$	$[q_0, aaabb]$	$[q_0, aaabb]$	$[q_0, aaabb]$	$[q_0, aaabb]$	$[q_0, aaabb]$
$\vdash [q_0, aabb]$	$\vdash [q_0, aabb]$	$\vdash [q_0, aabb]$	$\vdash [q_0, aabb]$	$\vdash [q_0, aabb]$	$\vdash [q_1, aabb]$
$\vdash [q_0, abb]$	$\vdash [q_0, abb]$	$\vdash [q_0, abb]$	$\vdash [q_0, abb]$	$\vdash [q_1, abb]$	
$\vdash [q_0, bb]$	$\vdash [q_1, bb]$	$\vdash [q_1, bb]$	$\vdash [q_1, bb]$		
	$\vdash [q_1, b]$	$\vdash [q_1, b]$	$\vdash [q_2, b]$		
	$\vdash [q_1, \lambda]$	$\vdash [q_2, \lambda]$			
fails	rejects	accepts	fails	fails	fails

See the parallel execution tree in Figure 3

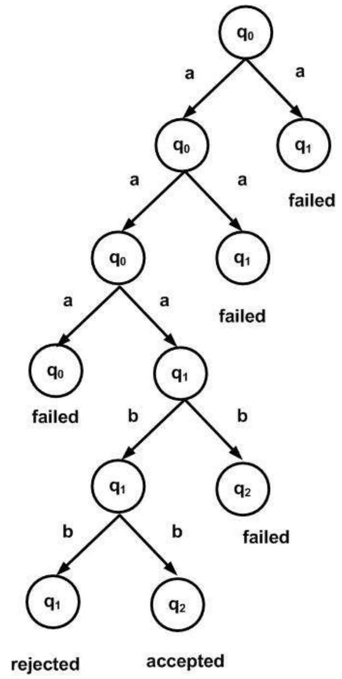

Figure 3: Chap5 Question 23(b): Parallel Execution Tree for input $aaabb$

part c
YES.
part d

$$[a \cup [ab^*b(ab^*b)^*a]]^*ab^*b(ab^*b)^*$$

Problem 12: (10 Points) Exercise 5.25 (d)

Solution 12:
See Figure 4.

Problem 13: (10 Points) Exercise 5.36

Solution 13:
part a

$$\lambda\text{–closure}(q_0) = \{q_0, q_2\}$$
$$\lambda\text{–closure}(q_1) = \{q_1\}$$
$$\lambda\text{–closure}(q_2) = \{q_2\}$$

part b
Input transition function t for M.

The input transition function of a state q on input symbol a is given by the following formula:

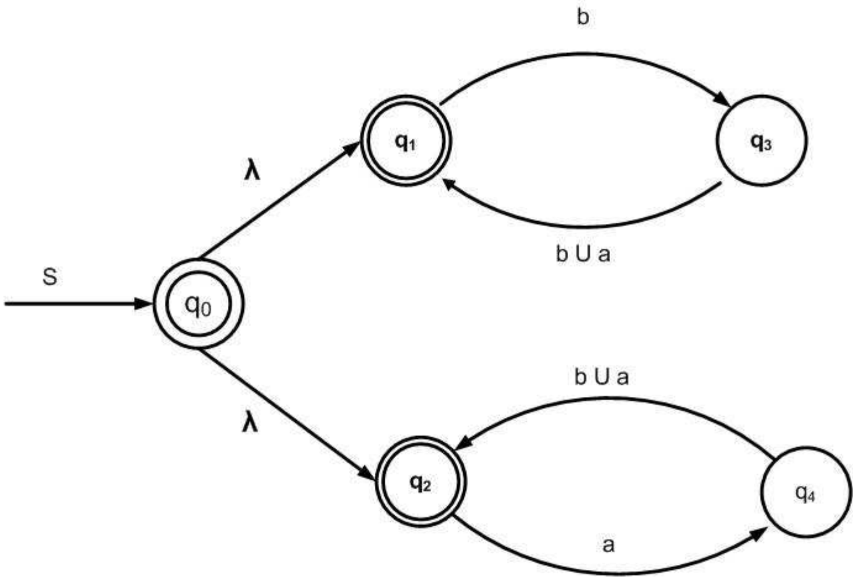

Figure 4: Chap5 Question 25 (d)

$$t(q,a) = \bigcup_{q_j \in \lambda\text{–closure}(q)} \lambda\text{–closure}(\delta(q_j, a))$$

Hence,

$$\begin{aligned}
t(q_0,a) &= \lambda\text{–closure}(\delta(q_0,a)) \cup \lambda\text{–closure}(\delta(q_2,a)) \\
&= \lambda\text{–closure}(q_0) \cup \lambda\text{–closure}(\emptyset) \\
&= \{q_0,q_2\} \cup \emptyset \\
&= \{q_0,q_2\} \\
t(q_0,b) &= \lambda\text{–closure}(\delta(q_0,b)) \cup \lambda\text{–closure}(\delta(q_2,b)) \\
&= \lambda\text{–closure}(\emptyset) \cup \lambda\text{–closure}(q_1,q_2) \\
&= \emptyset \cup \{q_1,q_2\} \\
&= \{q_1,q_2\} \\
t(q_0,c) &= \lambda\text{–closure}(\delta(q_0,c)) \cup \lambda\text{–closure}(\delta(q_2,c)) \\
&= \lambda\text{–closure}(q_1) \cup \lambda\text{–closure}(\emptyset) \\
&= \{q_1\} \cup \emptyset \\
&= \{q_1\}
\end{aligned}$$

$$\begin{aligned}
t(q_1,a) &= \lambda\text{–closure}(\delta(q_1,a)) = \emptyset \\
t(q_1,b) &= \lambda\text{–closure}(\delta(q_1,b)) = \emptyset \\
t(q_1,c) &= \lambda\text{–closure}(\delta(q_1,c)) = \lambda\text{–closure}(\{q_1\}) = \{q_1\}
\end{aligned}$$

$$\begin{aligned}
t(q_2,a) &= \lambda\text{–closure}(\delta(q_2,a)) = \emptyset \\
t(q_2,b) &= \lambda\text{–closure}(\delta(q_2,b)) = \lambda\text{–closure}(\{q_1,q_2\}) = \{q_1,q_2\} \\
t(q_2,c) &= \lambda\text{–closure}(\delta(q_2,c)) = \emptyset
\end{aligned}$$

part c

State	Symbol	λ–closure of NFA Transition	Next State
λ–closure($\{q_0\}$) $= \{q_0, q_2\}$	a	λ–closure($\delta(q_0, a)) = \lambda$–closure($\{q_0\}) = \{q_0, q_2\}$ λ–closure($\delta(q_2, a)) = \lambda$–closure($\emptyset) = \emptyset$	$\{q_0, q_2\} \cup \emptyset = \{q_0, q_2\}$ [Same as $t(q_0, a) \cup t(q_2, a)$]
	b	λ–closure($\delta(q_0, b)) = \lambda$–closure($\emptyset) = \emptyset$ λ–closure($\delta(q_2, b)) = \lambda$–closure($\{q_1, q_2\}) = \{q_1, q_2\}$	$\emptyset \cup \{q_1, q_2\} = \{q_1, q_2\}$ [Same as $t(q_0, b) \cup t(q_2, b)$]
	c	λ–closure($\delta(q_0, c)) = \lambda$–closure($\{q_1\}) = \{q_1\}$ λ–closure($\delta(q_2, c)) = \lambda$–closure($\emptyset) = \emptyset$	$\{q_1\} \cup \emptyset = \{q_1\}$ [Same as $t(q_0, c) \cup t(q_2, c)$]
$\{q_1, q_2\}$	a	λ–closure($\delta(q_1, a)) = \lambda$–closure($\emptyset) = \emptyset$ λ–closure($\delta(q_2, a)) = \lambda$–closure($\emptyset) = \emptyset$	$\emptyset \cup \emptyset = \emptyset$ [Same as $t(q_1, a) \cup t(q_2, a)$]
	b	λ–closure($\delta(q_1, b)) = \lambda$–closure($\emptyset) = \emptyset$ λ–closure($\delta(q_2, b)) = \lambda$–closure($\{q_1, q_2\}) = \{q_1, q_2\}$	$\emptyset \cup \{q_1, q_2\} = \{q_1, q_2\}$ [Same as $t(q_1, b) \cup t(q_2, b)$]
	c	λ–closure($\delta(q_1, c)) = \lambda$–closure($\{q_1\}) = \{q_1\}$ λ–closure($\delta(q_2, c)) = \lambda$–closure($\emptyset) = \emptyset$	$\{q_1\} \cup \emptyset = \{q_1\}$ [Same as $t(q_1, c) \cup t(q_2, c)$]
$\{q_1\}$	a	λ–closure($\delta(q_1, a)) = \lambda$–closure($\emptyset) = \emptyset$	$\emptyset$ [Same as $t(q_1, a)$]
	b	λ–closure($\delta(q_1, b)) = \lambda$–closure($\emptyset) = \emptyset$	$\emptyset$ [Same as $t(q_1, b)$]
	c	λ–closure($\delta(q_1, c)) = \lambda$–closure($\{q_1\}) = \{q_1\}$	$\{q_1\}$ [Same as $t(q_1, c)$]

See the state diagram of the resulting DFA in Figure 5.

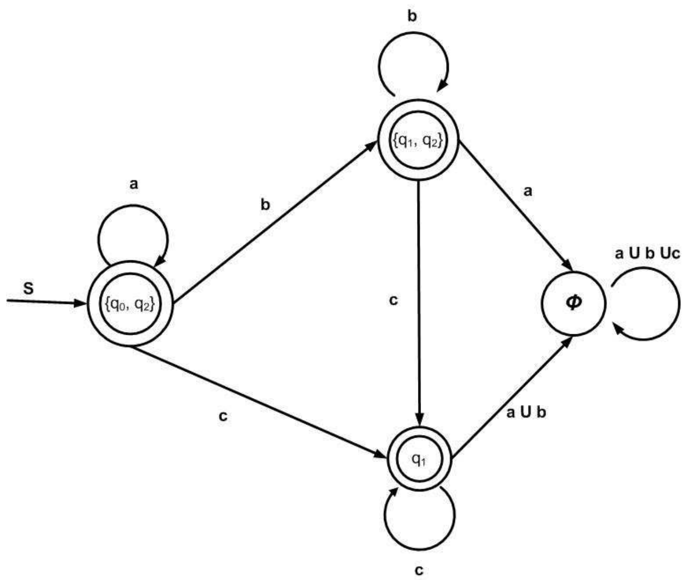

Figure 5: Chap5 Question 36 (c)

part d $a^*[b^* \cup [c \cup (b^*b)]c^*]$
A good exercise is to determine whether or not the regular expression above is equivalent to $a^*b^*c^*$.

Problem 14: (10 Points) Exercise 5.41

Solution 14:
See the constructed NFA in Figure 6.

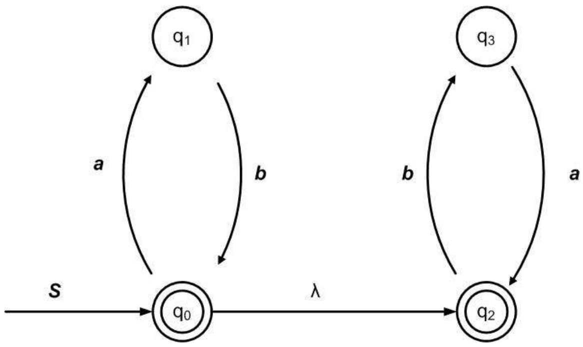

Figure 6: Chap5 Question 41 NFA

$$\lambda\text{–closure}(q_0) = \{q_0, q_2\}$$
$$\lambda\text{–closure}(q_1) = \{q_1\}$$
$$\lambda\text{–closure}(q_2) = \{q_2\}$$
$$\lambda\text{–closure}(q_3) = \{q_3\}$$

Input transition function t for the NFA

$$t(q_0, a) = \{q_1\}, \quad t(q_1, a) = \emptyset, \quad t(q_2, a) = \emptyset, \quad t(q_3, a) = \{q_2\}$$
$$t(q_0, b) = \{q_3\}, \quad t(q_1, b) = \{q_0, q_2\}, \quad t(q_2, b) = \{q_3\}, \quad t(q_3, b) = \emptyset$$

Constructing the states and transitions of an equivalent DFA:

State	Symbol	Next State
λ–closure$(\{q_0\})$	a	$t(q_0, a) \cup t(q_2, a) = \{q_1\} \cup \emptyset = \{q_1\}$
$= \{q_0, q_2\}$	b	$t(q_0, b) \cup t(q_2, b) = \{q_3\} \cup \{q_3\} = \{q_3\}$
$\{q_1\}$	a	$t(q_1, a) = \emptyset$
	b	$t(q_1, b) = \{q_0, q_2\}$
$\{q_3\}$	a	$t(q_3, a) = \{q_2\}$
	b	$t(q_3, b) = \emptyset$
$\{q_2\}$	a	$t(q_2, a) = \emptyset$
	b	$t(q_2, b) = \{q_3\}$

The resulting DFA, equivalent to the original NFA, is depicted in Figure 7.

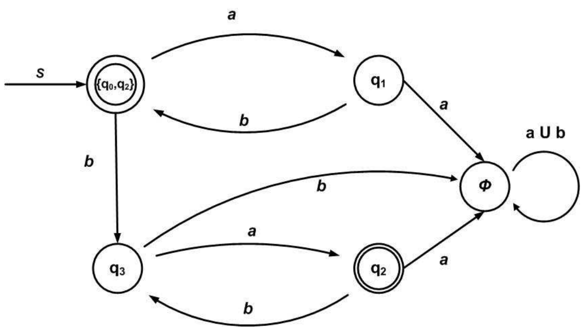

Figure 7: Chap5 Question 41 DFA