CS3133 - A Term 2009: Foundations of Computer Science	Prof. Carolina Ruiz

Homework 3

WPI — *By Li Feng, Shweta Srivastava, and Carolina Ruiz*

Chapter 6

Problem 1:

For the regular expressions:
$(a \cup bc \cup c)^*$ in our posted solutions to Exercise 25 of Chapter 2 in Homework 1
$(b^*ab^*ab^*ab^*)^* \cup b^*$ in our posted solutions to Exercise 26 of Chapter 2 in Homework 1.

1. Construct a finite automaton.
2. Convert your finite automaton into an equivalent regular grammar.

Solution 1:

For regular expression: $(a \cup bc \cup c)^*$
part 1

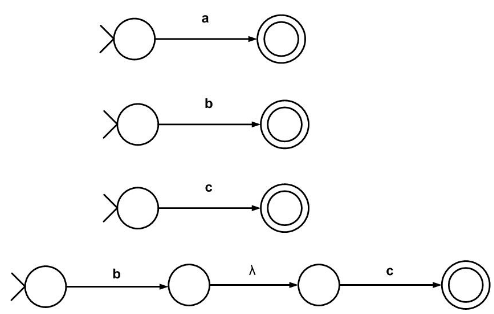


Figure 1: Basic NFAs for a, b, c and bc

By combining the NFAs above with λ-transitions we get the NFA below:

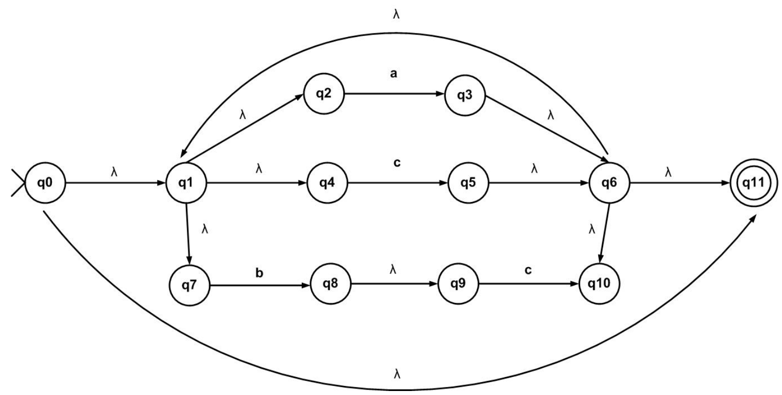


Figure 2: Combining NFAs to create NFA with λ-transitions

A reduced NFA can be obtained by the following steps:
q0, q1, q2, q7, and q4 are merged into one state called Q_0
q8 and q9 are merged into one state called Q_1
q3, q5, q6, q10, and q11 into one state called Q_2

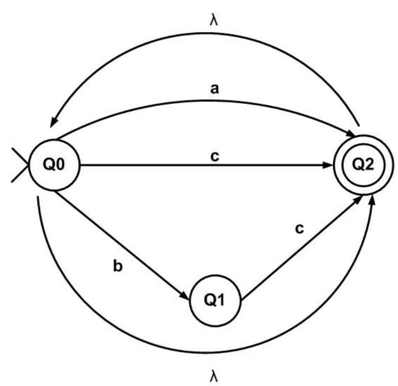


Figure 3: Reduced NFA

part 2
Based on the NFA in Figure 3, we construct the grammar:

$$S \to aQ_2 \mid cQ_2 \mid bQ_1 \mid Q_2$$
$$Q_1 \to cQ_2$$
$$Q_2 \to S \mid \lambda$$

On removing the chain rules we get:

$$S \to aQ_2 \mid cQ_2 \mid bQ_1 \mid \lambda$$
$$Q_1 \to cQ_2$$
$$Q_2 \to aQ_2 \mid cQ_2 \mid bQ_1 \mid \lambda$$

For regular expression: $(b^*ab^*ab^*ab^*)^* \cup b^*$

part 1

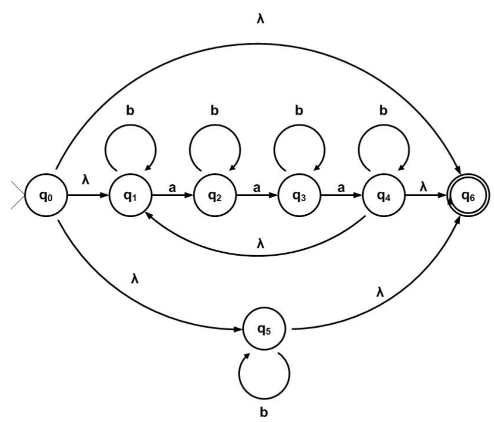


Figure 4: NFA for Chap2 26

part 2

Based on the NFA in Figure 4, we construct the grammar:

$$
\begin{aligned}
&S \to Q_1 \mid Q_5 \mid Q_6 \\
&Q_1 \to bQ_1 \mid aQ_2 \\
&Q_2 \to bQ_2 \mid aQ_3 \\
&Q_3 \to bQ_3 \mid aQ_4 \\
&Q_4 \to bQ_4 \mid Q_1 \mid Q_6 \\
&Q_5 \to bQ_5 \mid Q_6 \\
&Q_6 \to \lambda
\end{aligned}
$$

Removing **chain rules** we obtain the following grammar which is in regular form.

$$
\begin{aligned}
&S \to bQ_1 \mid aQ_2 \mid \lambda \\
&Q_1 \to bQ_1 \mid aQ_2 \\
&Q_2 \to bQ_2 \mid aQ_3 \\
&Q_3 \to bQ_3 \mid aQ_4 \\
&Q_4 \to bQ_4 \mid bQ_1 \mid aQ_2 \mid \lambda \\
&Q_5 \to bQ_5 \mid \lambda \\
&Q_6 \to \lambda
\end{aligned}
$$

Problem 2: For the NFAs from:
Exercise 23 of Chapter 5 and
Exercise 36 of Chapter 5

1. Convert the finite automaton into an equivalent regular expression.
2. Convert your finite automaton into an equivalent regular grammar.

Solution 2:
Exercise 23 of Chapter 5

part a

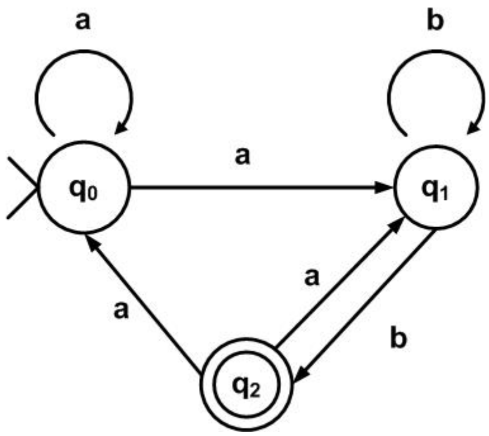


Figure 5: NFA for Chap5 Question 23

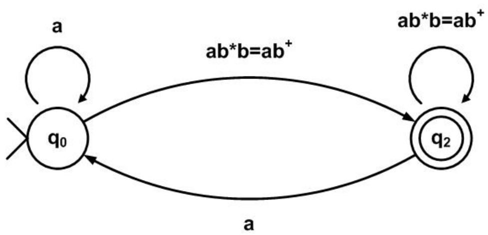


Figure 6: Step1: Remove q_1 from NFA Chap 5-23

Start to eliminate the state q_1, and the result is shown in Figure 6. The regular expression is

$$a^*(ab^+)(ab^+ \cup aa^*ab^+)^*$$

Note: this regular expression is equivalent to $(a^+b^+)^+$.

$$\begin{aligned} & a^*(ab^+)(ab^+ \cup aa^*ab^+)^* \\ \equiv\ & a^+b^+(ab^+ \cup aa^+b^+)^* \\ \equiv\ & a^+b^+(a^+b^+)^* \\ \equiv\ & (a^+b^+)^+ \end{aligned}$$

part b

Based on the NFA in Figure 5, we can construct the regular grammar:

$$\begin{aligned} & S \rightarrow aS \mid aQ_1 \\ & Q_1 \rightarrow bQ_1 \mid bQ_2 \\ & Q_2 \rightarrow aQ_1 \mid aS \mid \lambda \end{aligned}$$

Exercise 36 of Chapter 5

part a

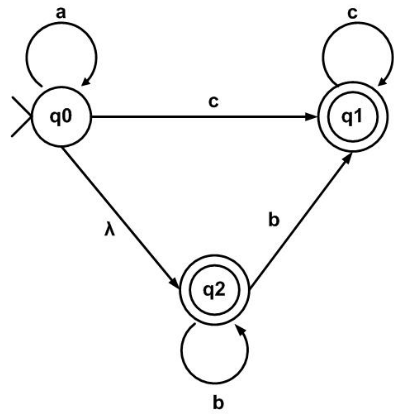


Figure 7: NFA for Chap5 Question 36

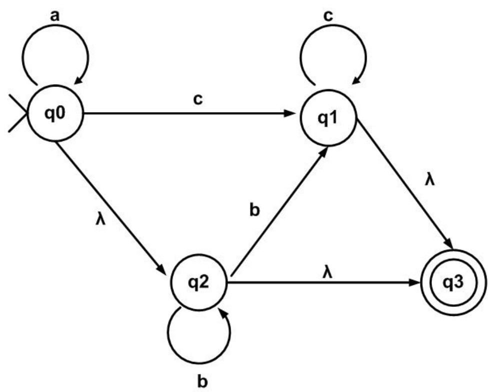


Figure 8: Step 1: Create a new accepting state

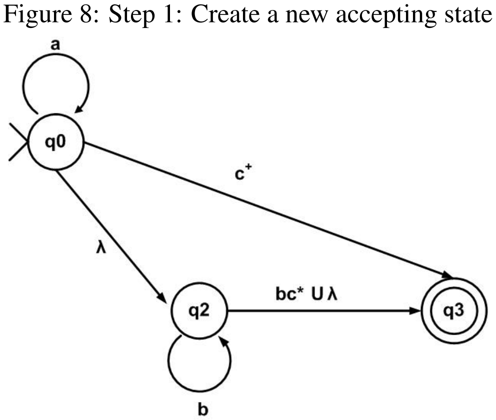


Figure 9: Step 2: remove State q_1

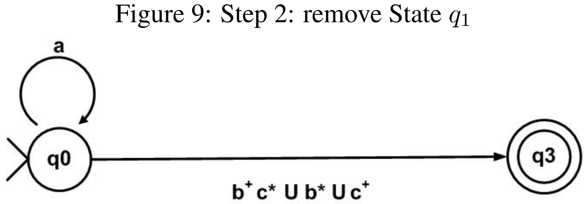


Figure 10: Step 3: remove State q_2

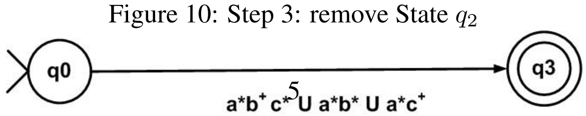


Figure 11: Step 3: removing the loop on q_0

The regular expression is: $a^*b^+c^* \cup a^*b^* \cup a^*c^+$

part b

Based on the NFA in Figure 7, we can construct the grammar:

$$\begin{aligned} &S \rightarrow aS \mid cQ_1 \mid Q_2 \\ &Q_1 \rightarrow cQ_1 \mid \lambda \\ &Q_2 \rightarrow bQ_2 \mid bQ_1 \mid\lambda \end{aligned}$$

On removing the chain rule we get the regular grammar:

$$\begin{aligned} &S \rightarrow aS \mid cQ_1 \mid bQ_2 \mid bQ_1 \mid\lambda \\ &Q_1 \rightarrow cQ_1 \mid \lambda \\ &Q_2 \rightarrow bQ_2 \mid bQ_1 \mid\lambda \end{aligned}$$

Problem 3:

For the regular grammar in our posted solutions of Exercise 9 of Chapter 4 in Homework 2. and the regular grammar for solution of Exercise 25 of Chapter 3 in Homework 1.

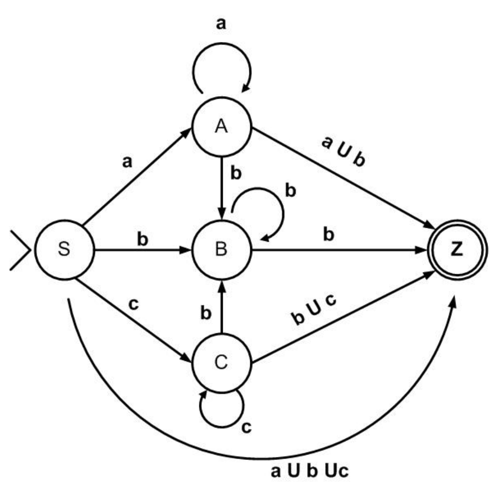
1. Construct a finite automaton based on the grammar .
2. Convert your finite automaton into an equivalent regular expression.

Solution 3:

Regular grammar in our posted solutions of Exercise 9 of Chapter 4 in Homework 2:

$$\begin{aligned} &S \rightarrow aA \mid a \mid cC \mid c \mid bB \mid b \\ &A \rightarrow aA \mid a \mid bB \mid b \\ &B \rightarrow bB \mid b \\ &C \rightarrow cC \mid c \mid bB \mid b \end{aligned}$$

part a



part b

We eliminate the state A first, and then state C, and state B to get the regular expression. The detailed steps are shown in Figure 12 to 20.

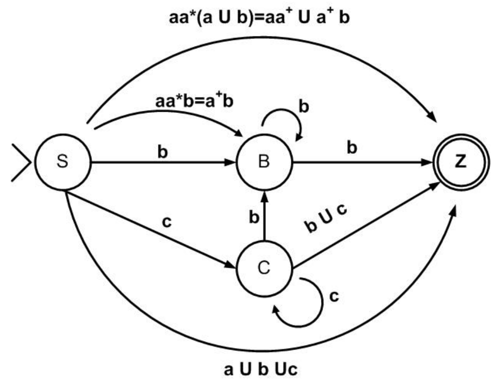


Figure 12: Step 1: remove State A

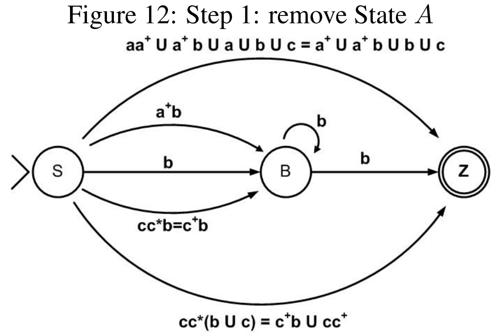


Figure 13: Step 2: remove State C

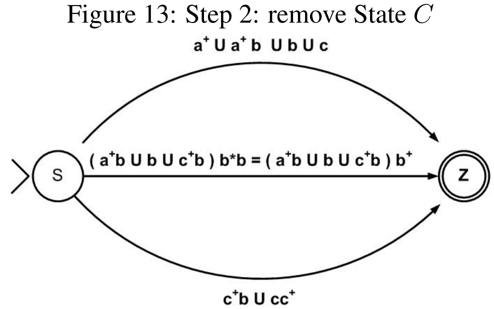


Figure 14: Step 3: remove State B

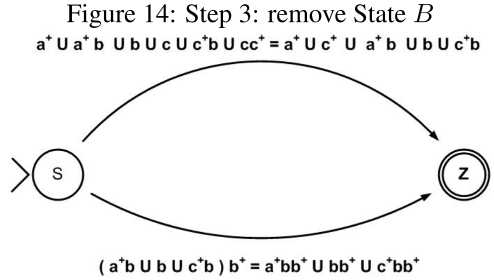


Figure 15: Step 3: reduce the number of arcs

We can see the regular expression is

$$a^+ \cup c^+ \cup a^+b \cup b \cup c^+b \cup a^+bb^+ \cup bb^+ \cup c^+bb^+$$

Actually, it is equivalent to the regular expression $a^+ \cup c^+ \cup (a^+b \cup b \cup c^+b)b^*$ in our HW2 solution, because:

$$\begin{aligned} & a^+ \cup c^+ \cup a^+b \cup b \cup c^+b \cup a^+bb^+ \cup bb^+ \cup c^+bb^+ \\ \equiv\ & a^+ \cup c^+ \cup b^+ \cup a^+b \cup c^+b \cup a^+bb^+ \cup c^+bb^+ \\ \equiv\ & a^+ \cup c^+ \cup a^+bb^* \cup bb^* \cup c^+bb^* \\ \equiv\ & a^+ \cup c^+ \cup (a^+b \cup b \cup c^+b)b^* \end{aligned}$$

Regular grammar for solution of Exercise 25 of Chapter 3 in Homework 1

$$\begin{aligned} & S \to aA \mid bC \mid aB \mid bD \mid \lambda \\ & C \to aA \mid bC \mid \lambda \\ & A \to aC \mid bA \\ & D \to aD \mid bB \mid \lambda \\ & B \to aB \mid bD \end{aligned}$$

part a

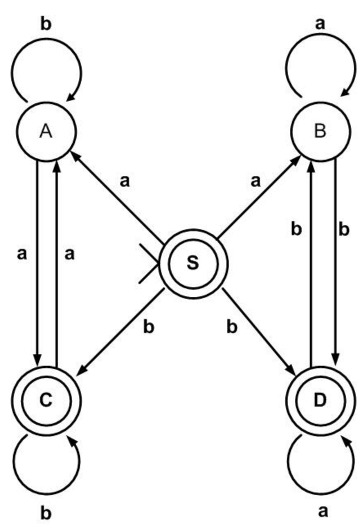


Figure 16: NFA obtained from the grammar

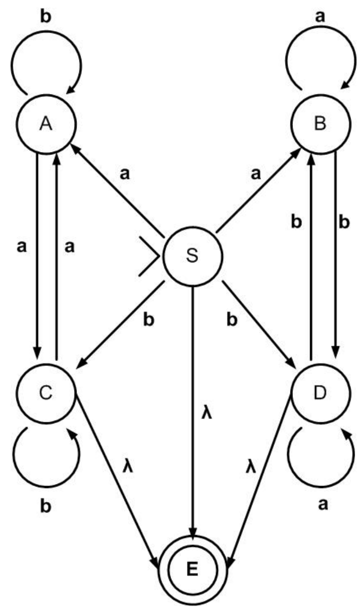


Figure 17: Step 1: Adding a new accepting state

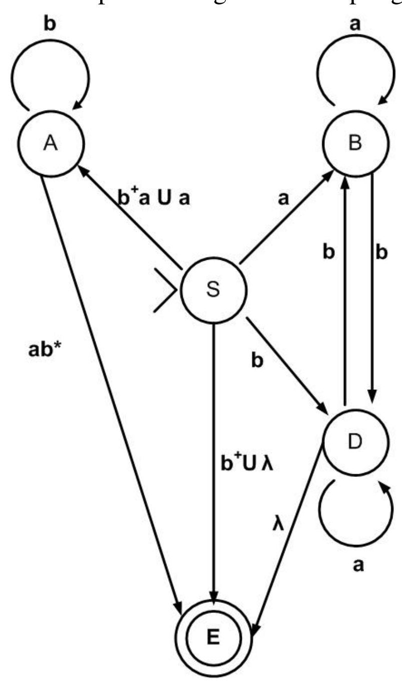


Figure 18: Step 2: remove State C

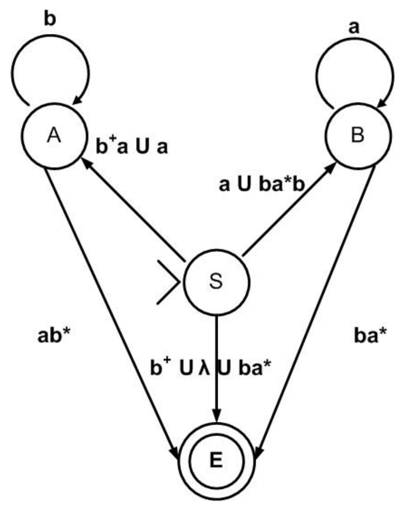


Figure 19: Step 3: remove State D

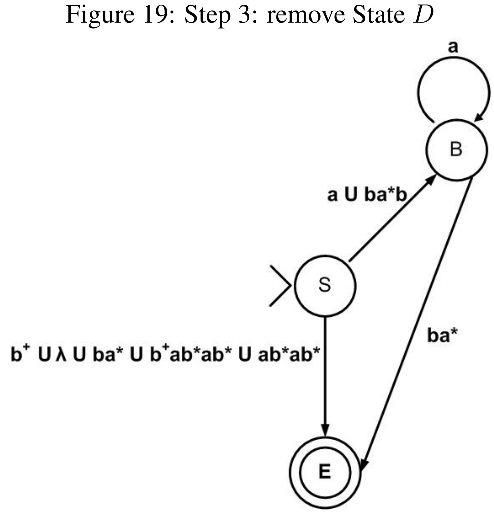


Figure 20: Step 4: remove State A

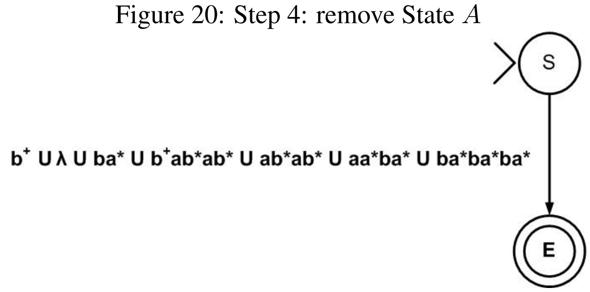


Figure 21: Step 5: remove State B

The regular expression is:
$b^+ \cup \lambda \cup ba^* \cup b^+ab^*ab^* \cup ab^*ab^* \cup aa^*ba^* \cup ba^*ba^*ba^*$
$\equiv b^+ \cup \lambda \cup ba^* \cup b^+ab^*ab^* \cup ab^*ab^* \cup a^+ba^* \cup ba^*ba^*ba^*$

Problem 4: **Solution 4**:

Chap 6.7.a
Let $H = \{w | w \in L$ and w ends with $aa\}$

Let L_1 be the language over $\{a, b, c\}$ that contains strings ending with aa. L_1 is described by the regular expression $(a \cup b \cup c)^*aa$. And so L_1 is regular.
A language that contains all strings that belong to both L and L_1 can be obtained by the intersection of the two languages. Therefore $H = L \cap L_1$. The regularity of H then follows from the closure of the regular languages under intersection.

Chap 6.7.b
Let $H = \{w | w \in L$ or w contains an $a\}$

Let L_1 be the language over $\{a, b, c\}$ of strings that contain an a. L_1 is described by the regular expression $(a \cup b \cup c)^*a(a \cup b \cup c)^*$. And so L_1 is regular.
A language that contains any string that belongs to either L or L_1 or both, can be obtained by the union of the two languages. Therefore $H = L \cup L_1$. The regularity of H then follows from the closure of the regular languages under union.

Chap 6.7.c
Let $H = \{w | w \notin L$ and w does not contain an $a\}$

Any $w \notin L$ belongs to $\bar{L}$. We know that $\bar{L}$ is regular as regular languages are closed under complement. Let L_1 be the language over $\{a, b, c\}$ of strings that contain an a. We have shown in the previous part(b) that this language is regular. Any w that does not contain an a then belongs to $\bar{L_1}$. We know that $\bar{L_1}$ is regular as regular languages are closed under complement.
A language that contains all strings that belong to both $\bar{L}$ AND $\bar{L_1}$, can be obtained by the intersection of the two languages. Therefore $H = \bar{L} \cap \bar{L_1}$. The regularity of H then follows from the closure of the regular languages under complement and intersection.

Chap 6.7.d
Let $H = \{uv | u \in L$ and $v \notin L\}$

Any $v \notin L$ belongs to $\bar{L}$. We know that $\bar{L}$ is regular as regular languages are closed under complement.
A language that contains strings formed by the concatenation of two strings belonging to two separate languages, can be obtained by the concatenation of the two languages. Therefore $H = L\bar{L}$. The regularity of H then follows from the closure of the regular languages under complement and concatenation.

Chap 6.14.a

By way of contradiction, we assume $L = \{w \mid w$ is a palindromes over $\{a, b\}\}$ is regular. Let M be a DFA that accepts L, and k be the number of states in M. Consider the string z equal to a^kba^k. Clearly, $z \in L$.

By the pumping lemma, z can be written as uvw where:

1. $v \neq \lambda$
2. $length(uv) \leq k$
3. $uv^iw \in L$ for all $i \geq 0$

However, by condition 2, v must consist of only a's. Pumping v would produce the string uv^2w where the number of a before the b is more than the number of a after the b. Therefore, uv^2w is not a palindrome, and $uv^2w \notin L$, yielding a contradiction.

Thus, L is not regular.

Chap 6. 14. b

By way of contradiction, we assume $L = \{a^nb^m | n < m\}$ is regular. Let M be a DFA that accepts L, and k be the number of states in M. Consider the string z equal to a^kb^{k+1}. Clearly, $z \in L$.

By the pumping lemma, z can be written as uvw where:

1. $v \neq \lambda$
2. $length(uv) \leq k$
3. $uv^iw \in L$ for all $i \geq 0$

However, by condition 2, v must consist of only a's. Pumping v would produce the string uv^2w which contains at least as many a's and b's. Therefore, $uv^2w \notin L$, yielding a contradiction. Thus, L is not regular.

Chap 6. 14. c

By way of contradiction, we assume $L = \left\{a^ib^jc^{2j} | i \geq 0, j \geq 0\right\}$ is regular. Let M be a DFA that accepts L, and k be the number of states in M. Consider the string z equal to b^kc^{2k}. Clearly, $z \in L$.

By the pumping lemma, z can be written as uvw where:

1. $v \neq \lambda$
2. $length(uv) \leq k$
3. $uv^iw \in L$ for all $i \geq 0$

However, by condition 2, v must consist of only b's. Pumping v would produce the string uv^2w which could not contain as twice as many as c's as b's. Therefore, $uv^2w \notin L$, yielding a contradiction. Thus, L is not regular.

Chap 6. 14. d

By way of contradiction, we assume $L = \left\{ww | w \in \{a, b\}^*\right\}$ is regular. Let M be a DFA that accepts L, and k be the number of states in M. Consider the string z equal to a^kba^kb. Clearly, $z \in L$.

By the pumping lemma, z can be written as uvw where:

1. $v \neq \lambda$
2. $length(uv) \leq k$
3. $uv^iw \in L$ for all $i \geq 0$

However, by condition 2, v must consist of only a's. Pumping v would produce the string uv^2w where the number of a's before the first b is greater than the number of a's between the two bs. Therefore, $uv^2w \notin L$, yielding a contradiction. Thus, L is not regular.

Chap 6. 14. f

L is the set of string over $\{a, b\}^*$ in which the number of a's is a perfect cube. By way of contradiction, we assume L is regular. Let M be a DFA that accepts L, and k be the number of states in M. Consider the string z equal to a^{k^3}. Clearly, $z \in L$, because $number\_of\_a(z) = k^3$, .

By the pumping lemma, z can be written as uvw where:

1. $v \neq \lambda$
2. $length(uv) \leq k$
3. $uv^iw \in L$ for all $i \geq 0$

However, by condition 1, v must not be λ. It means that $0 < length(v) \leq k$. Because v consists of as, we have $number\_of\_a(v) = length(v)$, and $0 < number\_of\_a(v) \leq k$. This observation can be used to compute the upper bound of $number\_of\_a(uv^2w)$:

$$\begin{aligned} number\_of\_a(uv^2w) &= number\_of\_a(uvw) + number\_of\_a(v) \\ &= k^3 + length(v) \\ &\leq k^3 + k \\ &< k^3 + 3k^2 + 3k + 1 \\ &= (k+1)^3 \end{aligned}$$

Thus, uv^2w must not be in L. The assumption that L is regular yields a contradiction and therefore L is not regular.

Chap 6. 15

Prove that the set of nonpalindromes over $\{a, b\}$ is not a regular language.

We shall prove this by way of contradiction. Let us assume that H be the set of nonpalidromes over $\{a, b\}$ and that H is regular. Then $\bar{H}$ that is the set of palindromes over $\{a, b\}$ will also be regular. However we have proved in Exercise 6.14, part(a) that $\bar{H}$ is not regular. This implies that the complement of $\bar{H}$ that is equal to H is also not regular. This contradicts our assumption of H being regular.

Chap 6. 16

Let L be a regular language and let $L_1 = \{uu | u \in L\}$ be the language L "doubled". Is L_1 necessarily regular? Prove your answer.

No, L_1 is not necessarily regular. Let us take the language $\{ww | w \in \{a, b\}^*\}$ in exercise 6.14, part(d). We have shown that this language is not regular even though the language $\{a, b\}^*$ is regular.