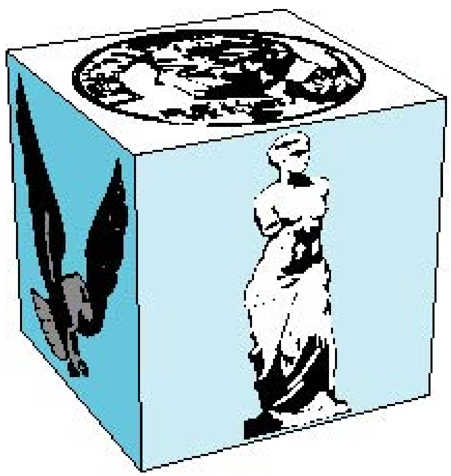
Recall: Introduction to Transformations

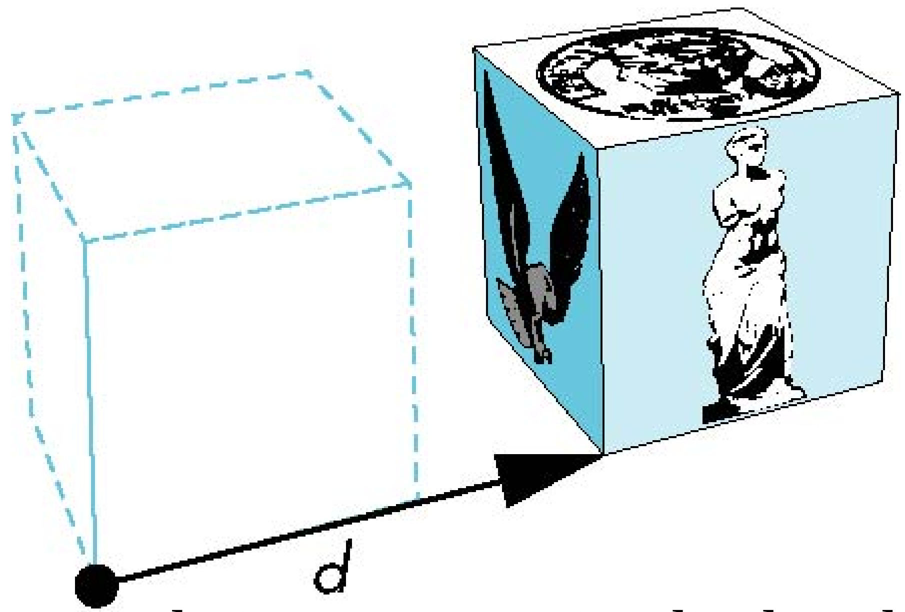
- May also want to transform objects by changing its:
  - Position (translation)
  - Size (scaling)
  - Orientation (rotation)
  - Shapes (shear)

Recall: Translation

- Move each vertex by **same** distance **d = (d_x, d_y, d_z)**



object



translation: every point displaced by same vector

Recall: Scaling

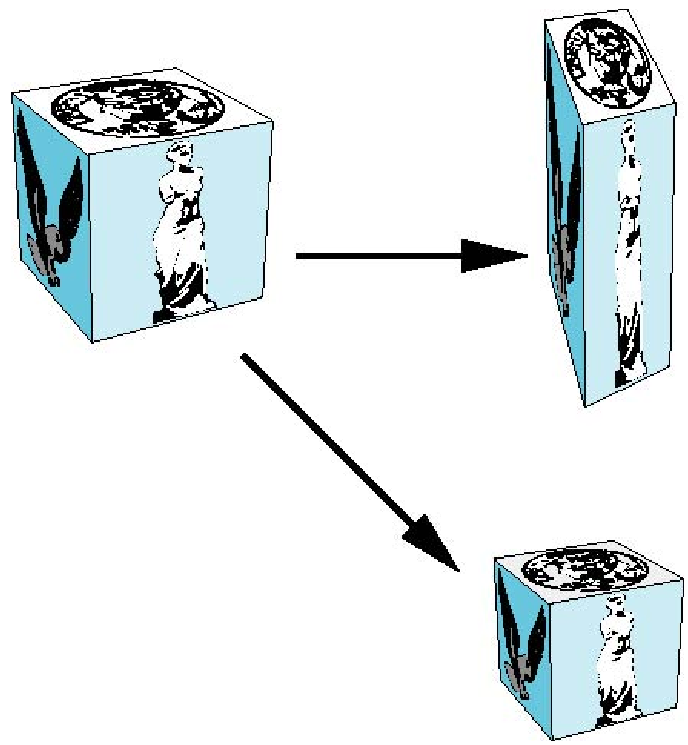
Expand or contract along each axis (fixed point of origin)

$x' = s_x x$
$y' = s_y y$
$z' = s_z z$

$$\mathbf{p}' = \mathbf{S}\mathbf{p}$$

where

$$\mathbf{S} = \mathbf{S}(s_x, s_y, s_z)$$



Introduction to Transformations

- We can transform (translation, scaling, rotation, shearing, etc) object by applying matrix multiplications to object vertices

$$\begin{pmatrix} P_x' \\ P_y' \\ P_z' \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix}$$

Transformed Vertex

Transform Matrix

Original Vertex

- Note: point (x,y,z) needs to be represented as (x,y,z,1), also called **Homogeneous coordinates**

Why Matrices?

- Multiple transform matrices can be pre-multiplied
- One final resulting matrix applied (efficient!)
- For example:

  transform 1

  transform 2 ....

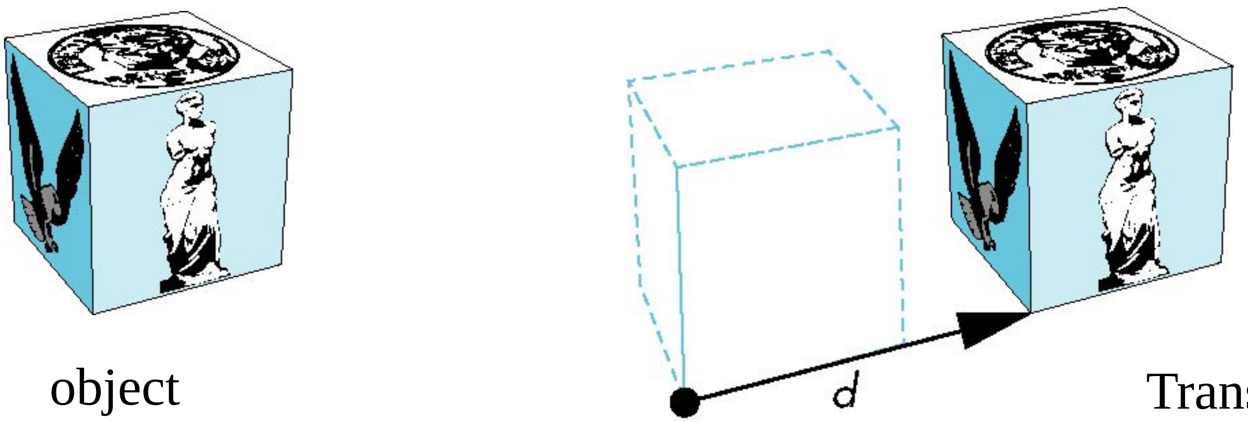
$$\begin{pmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \\ 1 \end{pmatrix}$$

Transformed Point

Transform Matrices can
Be pre-multiplied

Original Point

3D Translation Example



object

Translation of object

- **Example:** If we translate a point (2,2,2) by displacement (2,4,6), new location of point is (4,6,8)

Translate(2,4,6)

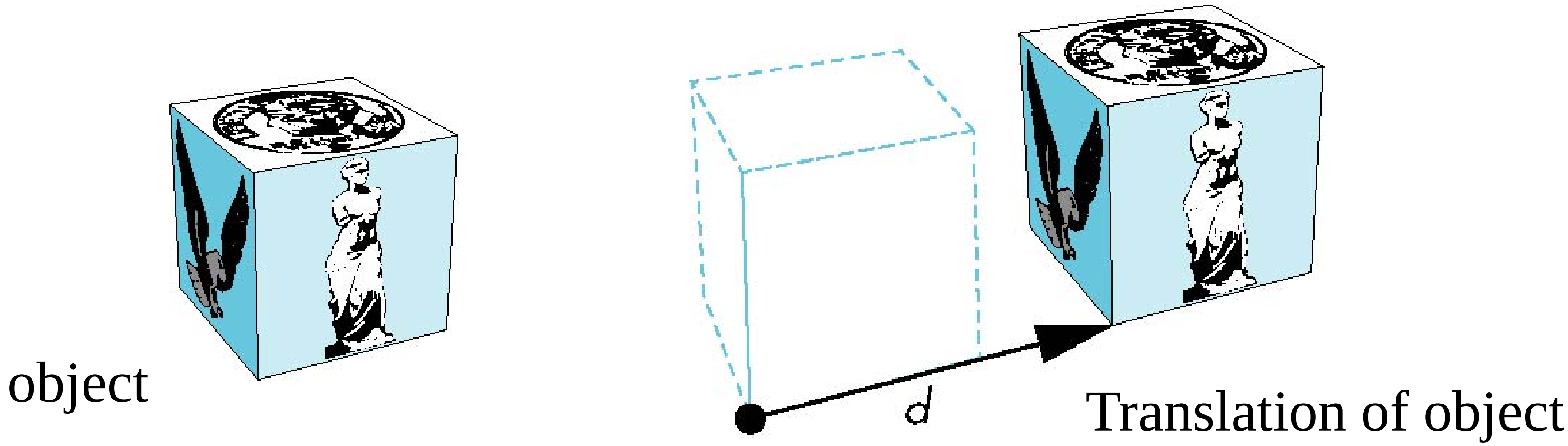
- Translated x: 2 + 2 = 4
- Translated y: 2 + 4 = 6
- Translated z: 2 + 6 = 4

$$\begin{pmatrix} 4 \\ 6 \\ 8 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

Translated point — **Translation Matrix** — **Original point**

3D Translation

- Translate object = Move each vertex by same distance **d = (d_x, d_y, d_z)**



Translate(dx,dy,dz)

▪Where:

- $x' = x + dx$
- $y' = y + dy$
- $z' = z + dz$

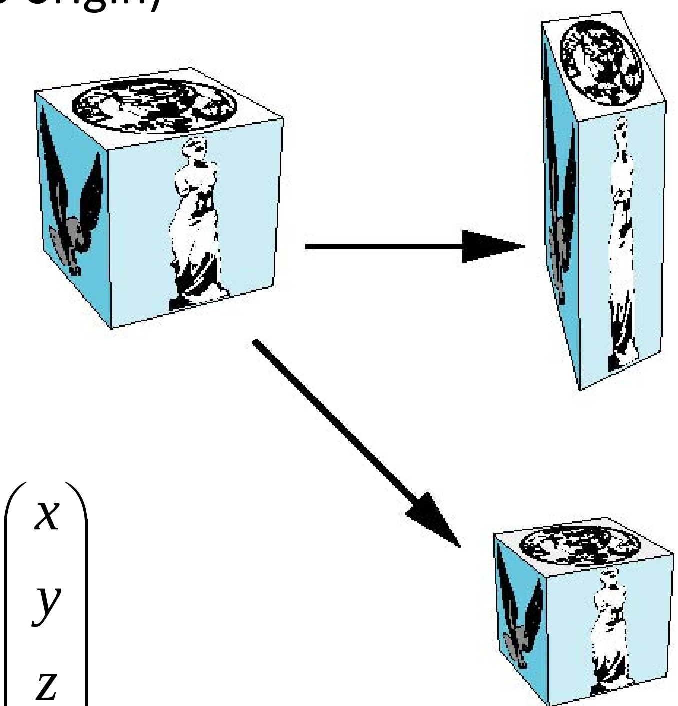
$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Translation Matrix

Scaling

Scale object = Move each object vertex by scale factor **S = (S_x, S_y, S_z)**
Expand or contract along each axis (relative to origin)

$$x' = s_x x$$
$$y' = s_y y$$
$$z' = s_z z$$



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

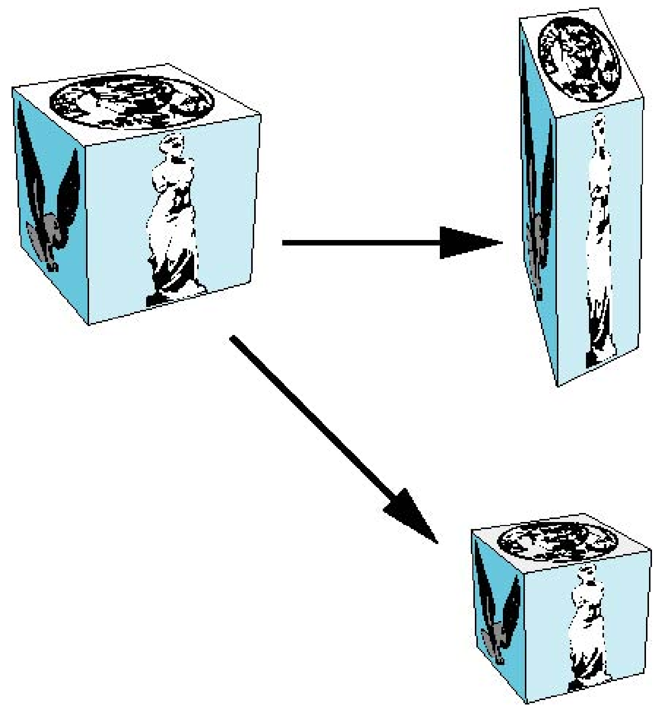
Scale Matrix

Scale(Sx,Sy,Sz)

Scaling Example

If we scale a point (2,4,6) by scaling factor (0.5,0.5,0.5)
Scaled point position = (1, 2, 3)

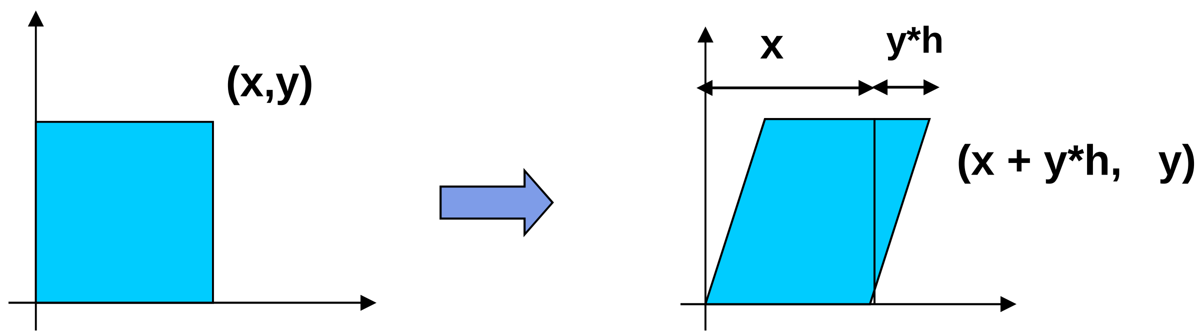
- Scaled x: 2 x 0.5 = 1
- Scaled y: 4 x 0.5 = 2
- Scaled z: 6 x 0.5 = 3



$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ 6 \\ 1 \end{pmatrix}$$

Scale Matrix for Scale(0.5, 0.5, 0.5)

Shearing

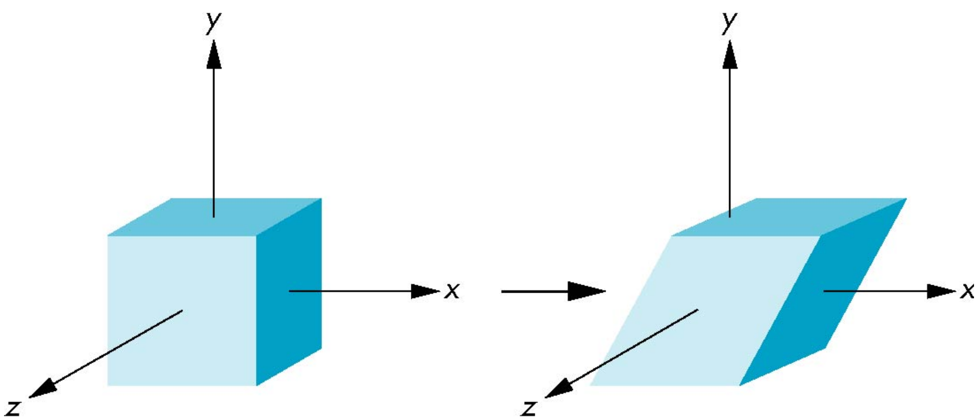


- Y coordinates are unaffected, but x cordinates are translated linearly with y
- That is:
  - **y' = y**
  - **x' = x + y * h**

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

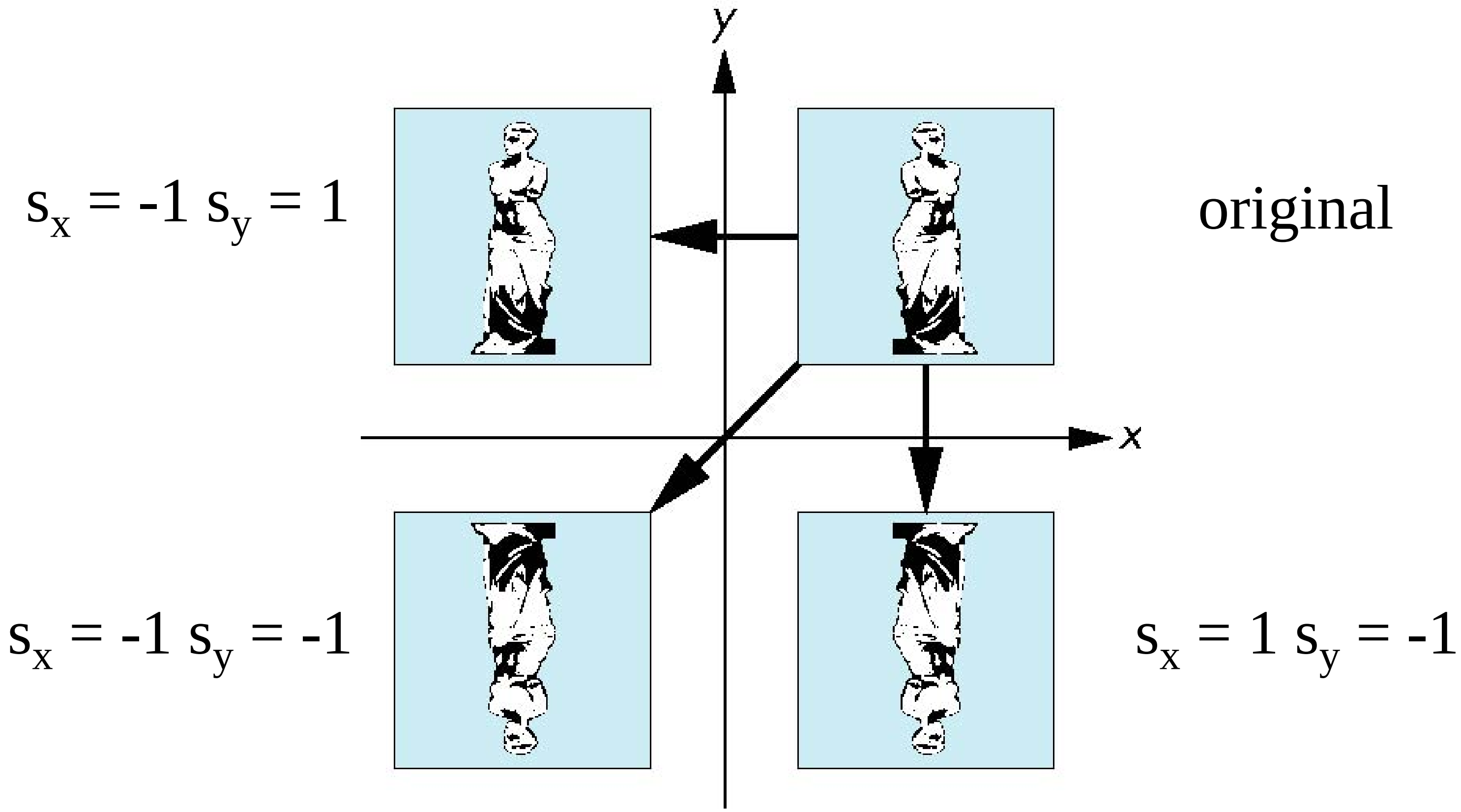
▪h is fraction of y to be added to x

3D Shear



Reflection

- corresponds to negative scale factors



Computer Graphics (CS 4731)
Lecture 9: Implementing Transformations

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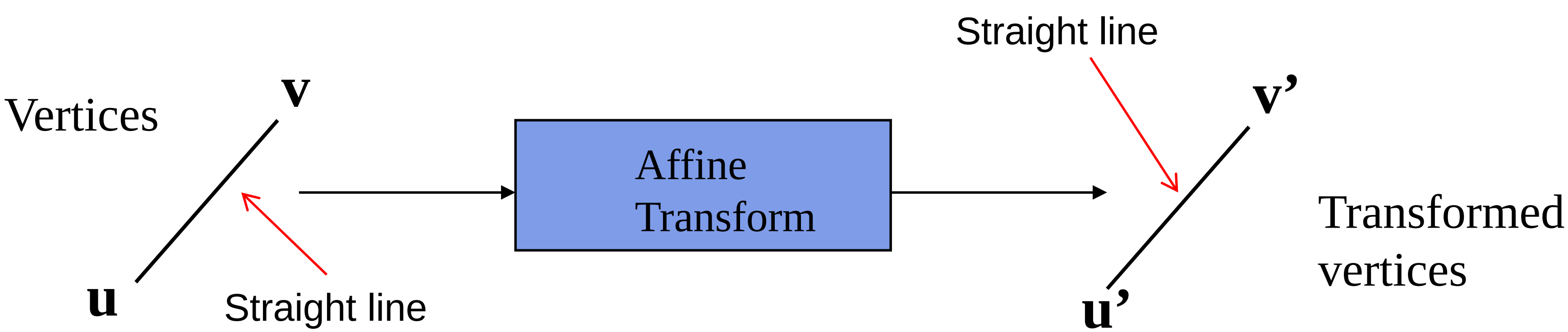
Objectives

- Learn how to implement transformations in OpenGL
  - Rotation
  - Translation
  - Scaling
- Introduce mat.h and vec.h transformations
  - Model-view
  - Projection

Affine Transformations

- Translate, Scale, Rotate, Shearing, are affine transforms
- **Rigid body transformations:** rotation, translation, scaling, shear
- **Line preserving:** important in graphics since we can
    1. Transform endpoints of line segments
    2. Draw line segment between the transformed endpoints



Previously: Transformations in OpenGL

- Pre 3.0 OpenGL had a set of transformation functions
  - glTranslate
  - glRotate( )
  - glScale( )
- Previously, OpenGL would
  - Receive transform commands (Translate, Rotate, Scale)
  - Multiply tranform matrices together and maintain transform matrix stack known as **modelview matrix**

Previously: Modelview Matrix Formed?

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

Specify transforms
In OpenGL Program

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&3\\0&1&0&6\\0&0&1&4\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0&3\\0&2&0&12\\0&0&3&12\\0&0&0&1\end{pmatrix}$$

Identity Matrix | glScale Matrix | glTranslate Matrix | Modelview Matrix

OpenGL implementations
(glScale, glTranslate, etc)
in Hardware (Graphics card)

OpenGL multiplies transforms together
To form modelview matrix
Applies final matrix to vertices of objects

Previously: OpenGL Matrices

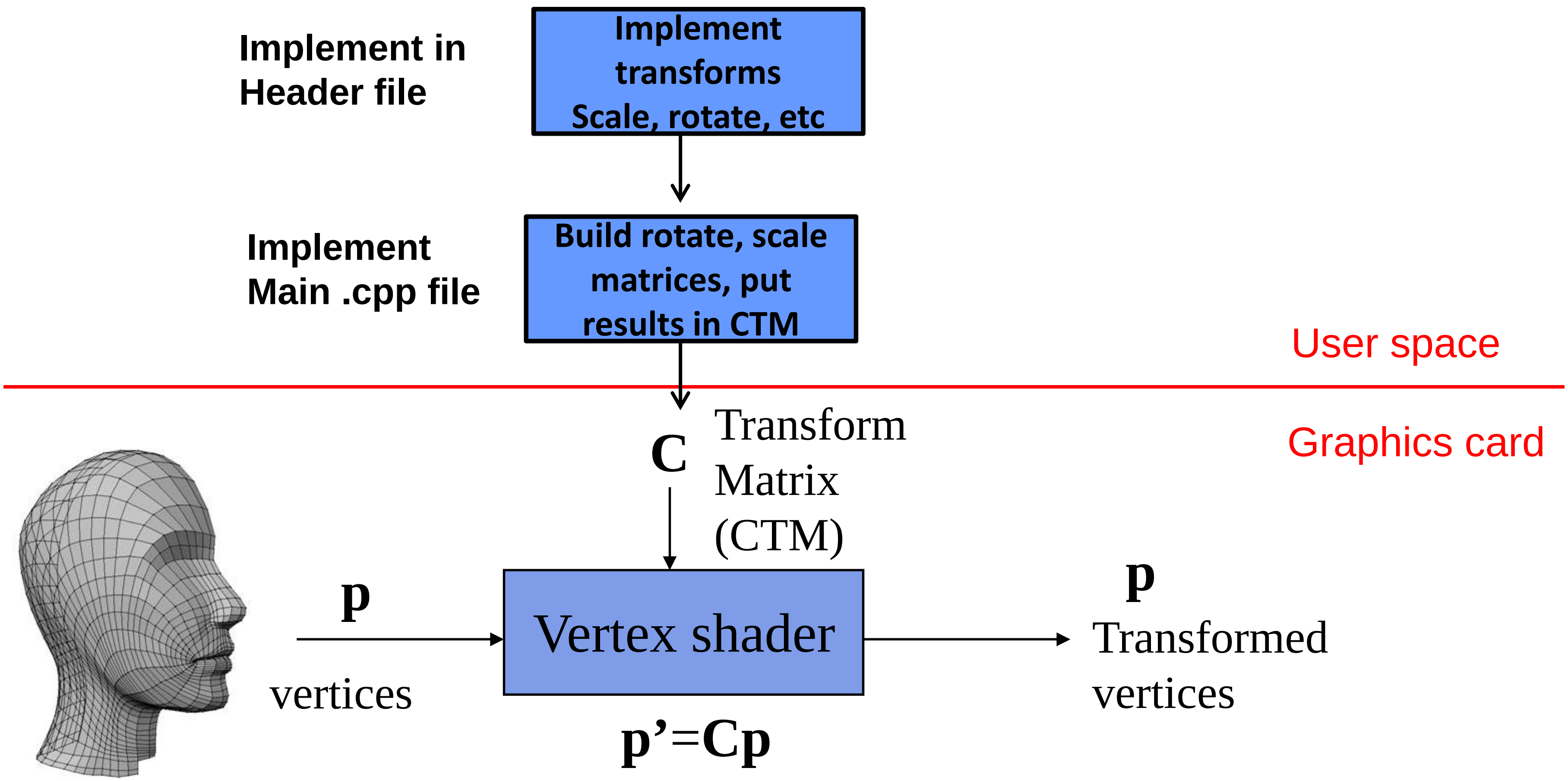
- OpenGL maintained 4 matrix stacks maintained as part of OpenGL state
  - Model-View (**GL_MODELVIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**)
  - Color(**GL_COLOR**)

Now: Transformations in OpenGL

- **From OpenGL 3.0:** No transform commands (scale, rotate, etc), matrices maintained by OpenGL!!
- glTranslate, glScale, glRotate, OpenGL modelview all deprecated!!
- If programmer needs transforms, matrices implement it!
- **Optional:** Programmer ***may*** now choose to maintain transform matrices **or NOT!**

Current Transformation Matrix (CTM)

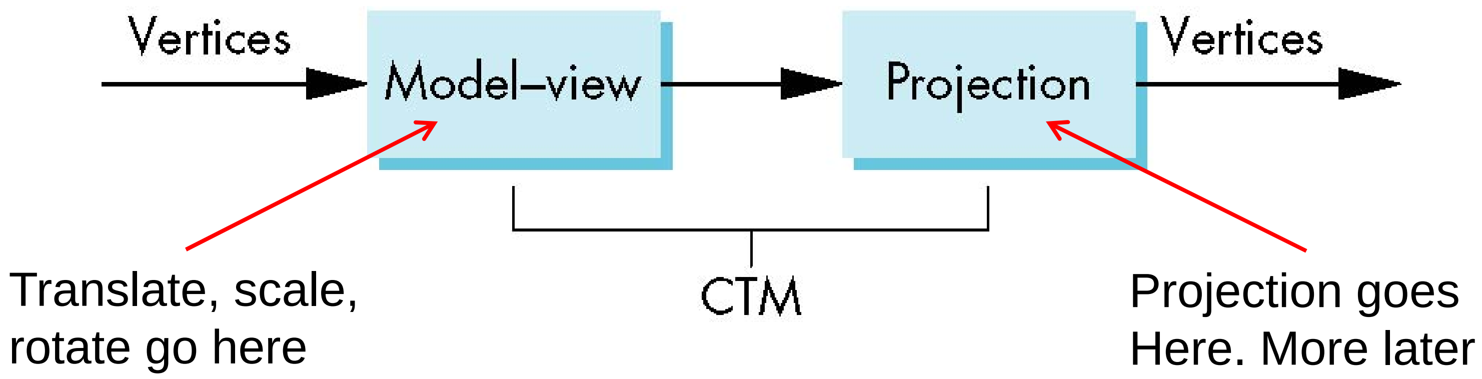
- Conceptually user can implement a 4 x 4 homogeneous coordinate matrix, the *current transformation matrix* (CTM)
- The **CTM** defined and updated in user program



CTM in OpenGL Matrices

- CTM = modelview + projection
  - Model-View (**GL_MODELVIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**)
  - Color(**GL_COLOR**)

CTM



CTM Functionality

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

1. We need to implement our own transforms

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&3\\0&1&0&6\\0&0&1&4\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0&3\\0&2&0&12\\0&0&3&12\\0&0&0&1\end{pmatrix}$$

Identity Matrix | glScale Matrix | glTranslate Matrix | Modelview Matrix

2. Multiply our transforms together to form CTM matrix
3. Apply final matrix to vertices of objects

Implementing Transforms and CTM

- Where to implement transforms and CTM?
- We implement CTM in 3 parts
  1. mat.h (Header file)
     - Implementations of translate( ) , scale( ), etc
  2. Application code (.cpp file)
     - Multiply together translate( ) , scale( ) = final CTM matrix
  3. GLSL functions (vertex and fragment shader)
     - Apply final CTM matrix to vertices

Implementing Transforms and CTM

- We just have to include mat.h **(#include "mat.h")**, use it
- **Uniformity: mat.h** syntax resembles GLSL language in shaders
- **Matrix Types: mat4** (4x4 matrix), **mat3** (3x3 matrix).

```
class mat4 {
    vec4  _m[4];
    ……..
}
```

- Can declare CTM as mat4 type

```
mat4 ctm = Translate(3,6,4);
```

CTM ← $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ **Translation Matrix**

- **mat.h also has transform functions**: Translate, Scale, Rotate, etc.

```
mat4 Translate(const GLfloat x, const GLfloat y, const GLfloat z )
mat4 Scale( const GLfloat x, const GLfloat y, const GLfloat z )
```

CTM operations

- The CTM can be altered either by loading a new CTM or by postmutiplication

Load identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Load arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{C}\ \mathbf{R}$
Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{C}\ \mathbf{S}$

Example: Rotation, Translation, Scaling

Create an identity matrix:

```
mat4 m = Identity();
```

Form Translate and Scale matrices, multiply together

```
mat4 s = Scale( sx, sy, sz)
mat4 t = Transalate(dx, dy, dz);
m = m*s*t;
```

Example: Rotation about a Fixed Point

- We want $\mathbf{C} = \mathbf{T}\,\mathbf{R}\,\mathbf{T}^{-1}$
- Be careful with order. Do operations in following order

$\mathbf{C} \leftarrow \mathbf{I}$
$\mathbf{C} \leftarrow \mathbf{CT}$
$\mathbf{C} \leftarrow \mathbf{CR}$
$\mathbf{C} \leftarrow \mathbf{CT}^{-1}$

- Each operation corresponds to one function call in the program.
- **Note:** last operation specified is first executed

Transformation matrices Formed?

- Converts all transforms (translate, scale, rotate) to 4x4 matrix
- We put 4x4 transform matrix into **CTM**
- Example

```
mat4 m = Identity();
```

mat4 type stores 4x4 matrix
Defined in mat.h

→

CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

```
mat4 m = Identity();
mat4 t = Translate(3,6,4);
m = m*t;
```

Identity Matrix		Translation Matrix		CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

- Consider following code snipet

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
```

Identity Matrix **Scaling Matrix** **CTM Matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

- What of translate, then scale, then ….
- Just multiply them together. Evaluated in ***reverse order***!! E.g:

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
```

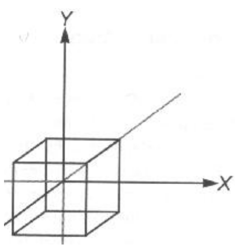
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix **Scale Matrix** **Translate Matrix** **Final CTM Matrix**

How are Transform matrices Applied?

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

1. In application:
Load object vertices into points[ ] array -> VBO
Call glDrawArrays



Application code

Object Vertices **CTM**

Vertex shader

CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. CTM built in application, passed to vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

Transformed vertex

3. In vertex shader: Each vertex of object (cube) is multiplied by CTM to get transformed vertex position

```
gl_Position = model_view*vPosition;
```

Passing CTM to Vertex Shader

- Build CTM (modelview) matrix in application program
- Pass matrix to shader

```
void display( ){
    .....
    mat4 m = Identity();
    mat4 s = Scale(1,2,3);
    mat4 t = Translate(3,6,4);
    m = m*s*t;

      // find location of matrix variable "model_view" in shader
      // then pass matrix to shader

      matrix_loc = glGetUniformLocation(program, "model_view");
      glUniformMatrix4fv(matrix_loc, 1, GL_TRUE, m);
        .....
}
```

Build CTM in application

CTM matrix **m** in application is same as **model_view** in shader

Implementation: Vertex Shader

- On glDrawArrays( ), vertex shader invoked with different vPosition per shader
- E.g. If colorcube( ) generates 8 vertices, each vertex shader receives a vertex stored in vPosition
- Shader calculates modified vertex position, stored in gl_Position

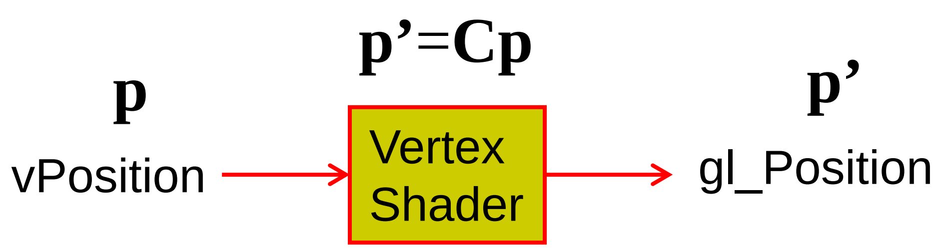
```
in vec4 vPosition;
uniform mat4 model_view;

void main( )
{
   gl_Position = model_view*vPosition;
}
```

Transformed vertex **position**

Contains **CTM**

Original vertex **position**



What Really Happens to Vertex Position Attributes?

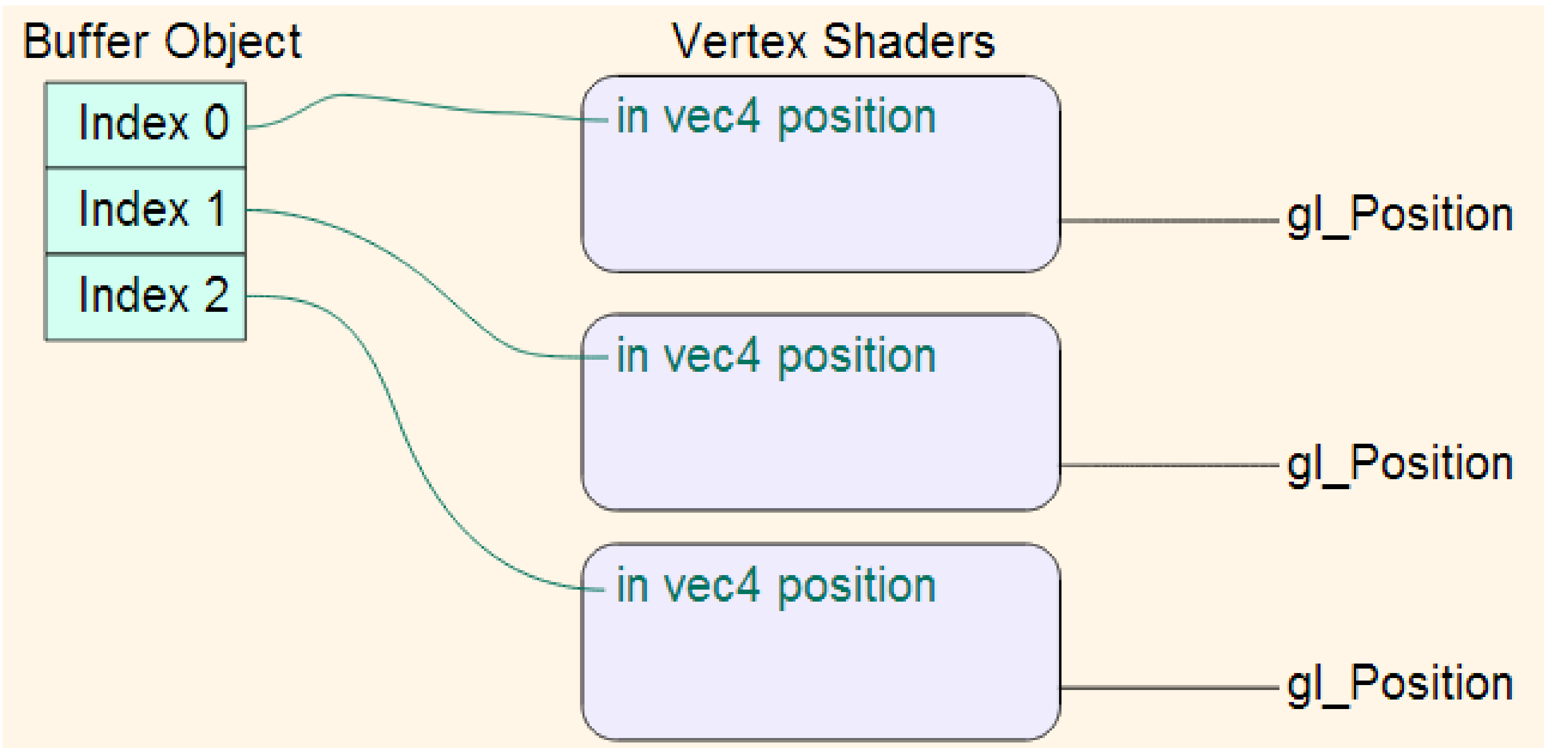


Image credit: Arcsynthesis tutorials

What About Multiple Vertex Attributes?

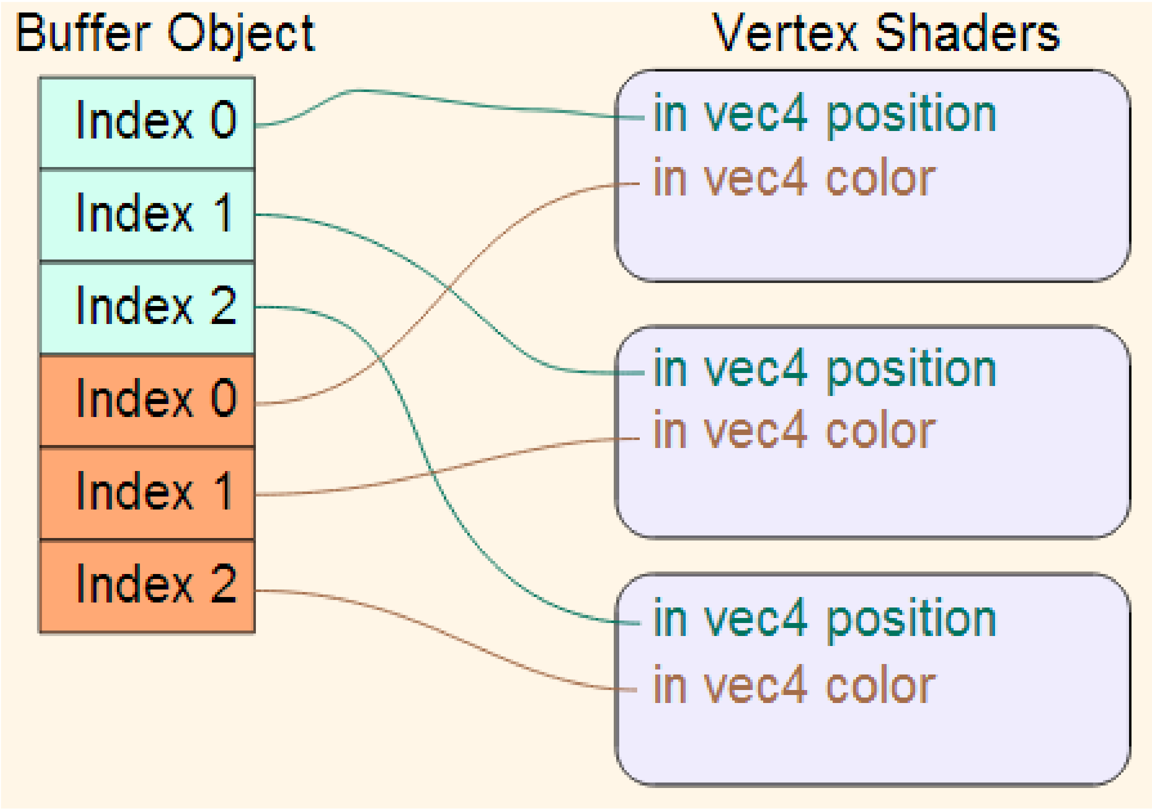


Image credit: Arcsynthesis tutorials

Transformation matrices Formed?

- Example: Vertex (1, 1, 1) is one of 8 vertices of cube

In application

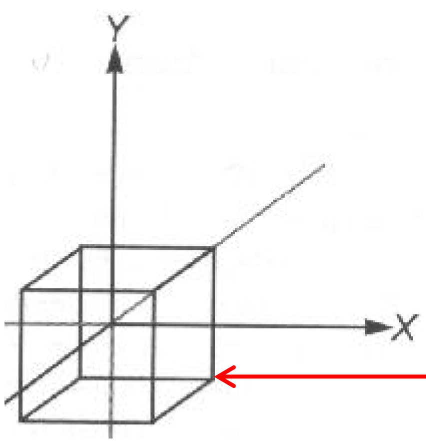
```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
colorcube( );
```

In vertex shader

CTM (m)   p   p'

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

Original vertex (p) — Transformed vertex (p')



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

Transformation matrices Formed?

- **Another example:** Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

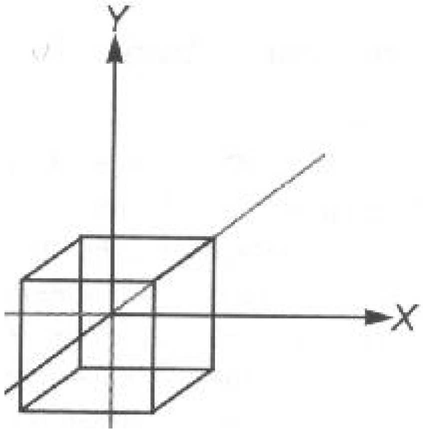
In vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

CTM Matrix

Original vertex

Transformed vertex



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

References

- Angel and Shreiner, Chapter 3
- Hill and Kelley, appendix 4