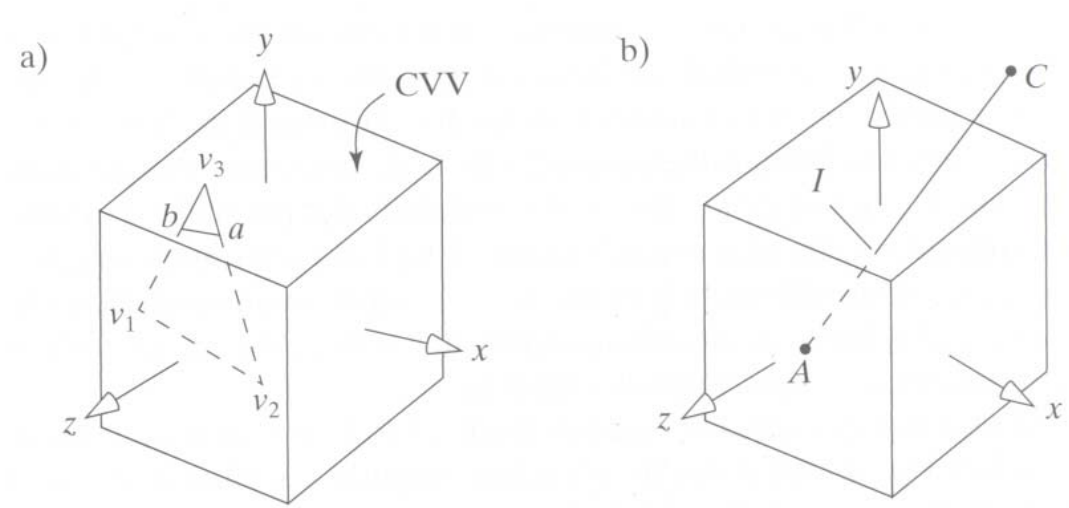
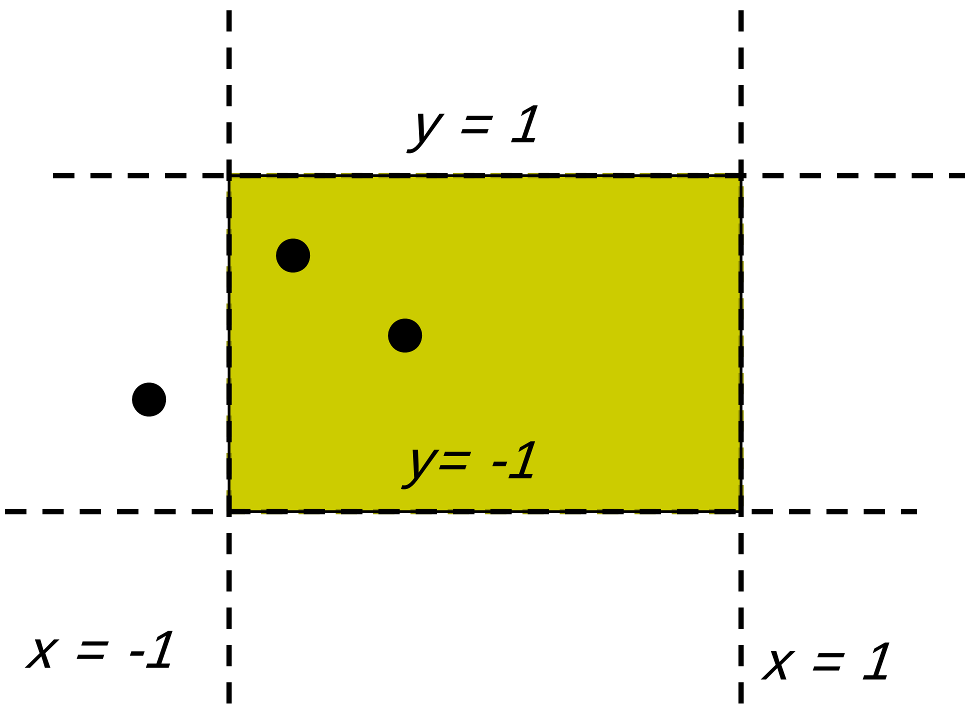
Recall: Liang-Barsky 3D Clipping

- **Goal:** Clip object edge-by-edge against Canonical View volume (CVV)
- **Problem:**
  - 2 end-points of edge: **A = (Ax, Ay, Az, Aw)** and **C = (Cx, Cy, Cz, Cw)**
  - If edge intersects with CVV, compute intersection point **I =(Ix,Iy,Iz,Iw)**



Recall: Determining if point is inside CVV



- **Problem:** Determine if point (x,y,z) is inside or outside CVV?

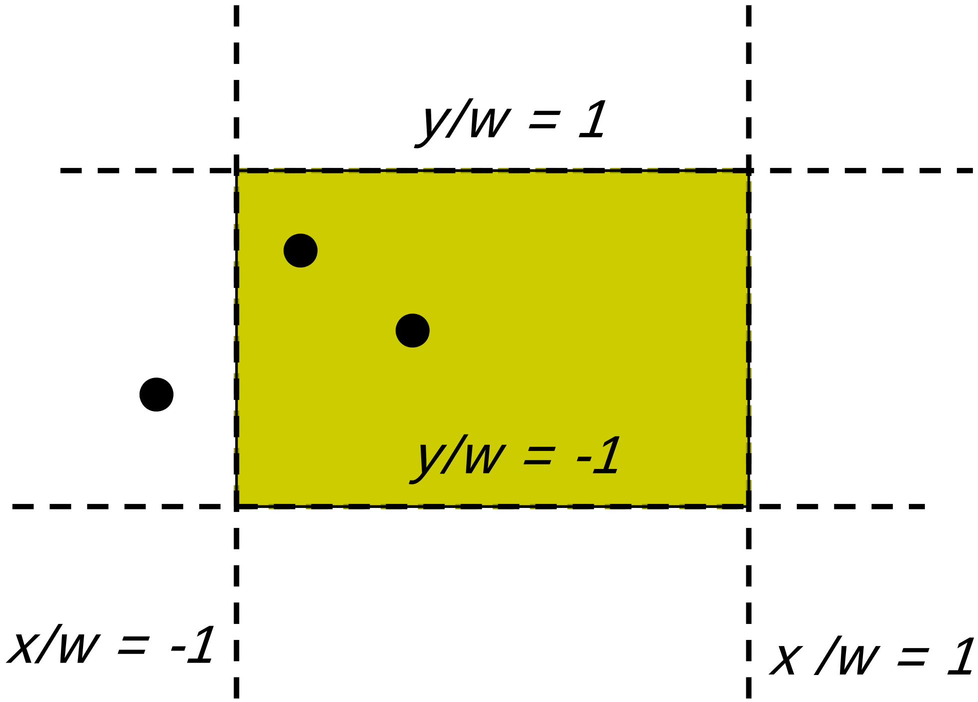
Point (x,y,z) is **inside CVV if**

(-1 <= x <= 1)
and (-1 <= y <= 1)
and (-1 <= z <= 1)

else point **is outside CVV**

- **CVV == 6 infinite planes (x=-1,1; y=-1,1; z=-1,1)**

Recall: Determining if point is inside CVV



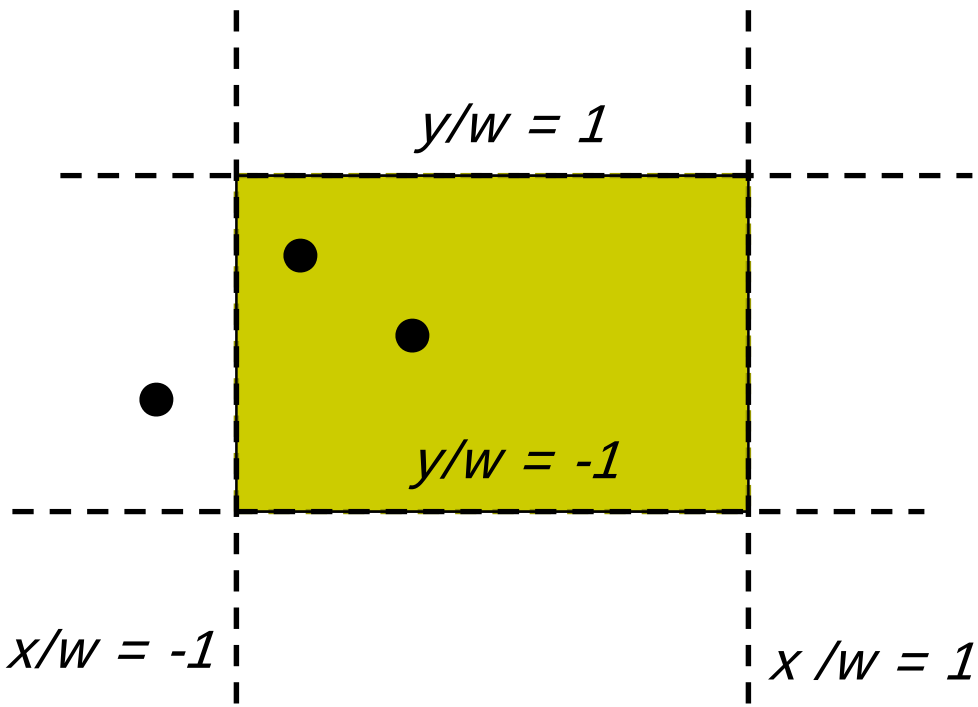
- If point specified as (x,y,z,w)
- **Test (x/w, y/w , z/w)!**

Point (x/w, y/w, z/w) is inside CVV

if (-1 <= **x/w** <= 1)
and (-1 <= **y/w** <= 1)
and (-1 <= **z/w** <= 1)

else point is outside CVV

Recall: Modify Inside/Outside Tests Slightly



Our test: ($-1 <$ **x / w** < 1)

Point (x,y,z,w) inside plane x = 1 if

$x/w < 1$

=> **w – x > 0**

Point (x,y,z,w) inside plane x = -1 if

$-1 < x/w$

=> **w + x > 0**

Recall: Numerical Example: Inside/Outside CVV Test

- Point (x,y,z,w) is
  - inside plane x=-1 **if w+x > 0**
  - inside plane x=1 **if w – x > 0**

- Example Point (0.5, 0.2, 0.7) inside planes (x = -1,1) because - 1 <= 0.5 <= 1

- If w = 10,   (0.5, 0.2, 0.7) = (5, 2, 7, 10)
- Can either **divide by w** then test: – 1 <= 5/10 <= 1 **OR**

To test if inside   x = - 1,   **w + x** =   10 + 5 = 15   >   0

To test if inside   x = 1,   **w - x** =   10 - 5 = 5   >   0

Recall: 3D Clipping

- Do same for y, z to form boundary coordinates for 6 planes as:

Boundary coordinate (BC)	**Homogenous coordinate**	**Clip plane**	**Example (5,2,7,10)**
BC0	w+x	x=-1	15
BC1	w-x	x=1	5
BC2	w+y	y=-1	12
BC3	w-y	y=1	8
BC4	w+z	z=-1	17
BC5	w-z	z=1	3

- **Consider line that goes from point A to C**
  - **Trivial accept:** 12 BCs (6 for pt. A, 6 for pt. C) > 0
  - **Trivial reject:** Both endpoints outside (-ve) for same plane

Edges as Parametric Equations

- Implicit form $F(x, y) = 0$

- Parametric forms:
  - points specified based on single parameter value
  - Typical parameter: time *t*

$$P(t) = P_0 + (P_1 - P_0) * t \qquad 0 \le t \le 1$$

- Some algorithms work in parametric form
  - Clipping: exclude line segment ranges
  - Animation: Interpolate between endpoints by varying t
- Represent each edge parametrically as A + (C – A)t
  - at time t=0, point at A
  - at time t=1, point at C

Inside/outside?

- Test A, C against 6 walls **(x=-1,1;   y=-1,1;   z=-1,1)**
- There is an intersection if BCs have opposite signs. i.e. if either
  - A is outside (< 0), C is inside (> 0)   or
  - A inside (> 0) , C outside (< 0)
- Edge intersects with plane at some t_hit between [0,1]

Calculating hit time (t_hit)

- How to calculate t_hit?
- Represent an edge t as:

$$Edge(t) = ((Ax + (Cx - Ax)t, (Ay + (Cy - Ay)t, (Az + (Cz - Az)t, (Aw + (Cw - Aw)t)$$

- E.g. If x = 1,

$$\frac{Ax + (Cx - Ax)t}{Aw + (Cw - Aw)t} = 1$$

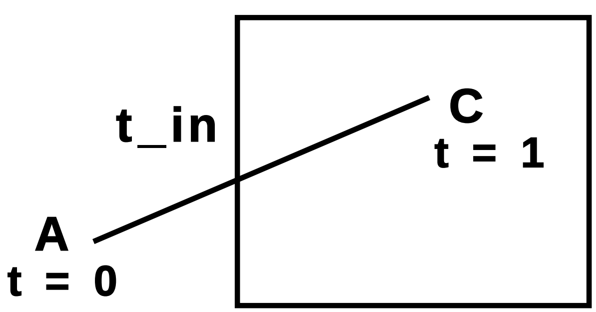
- Solving for t above,

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

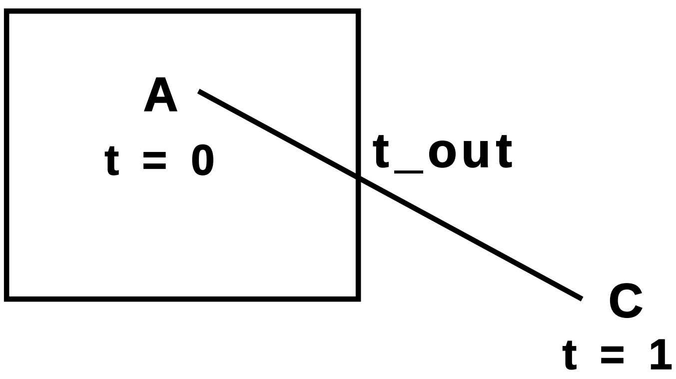
Inside/outside?

- t_hit can be **“entering (t_in) ”** or **”leaving (t_out)”**
- Define: **“entering”** if A outside, C inside
  - Why? As t goes [0-1], edge goes from outside (at A) to inside (at C)
- Define **“leaving”** if A inside, C outside
  - Why? As t goes [0-1], edge goes from inside (at A) to inside (at C)

Entering

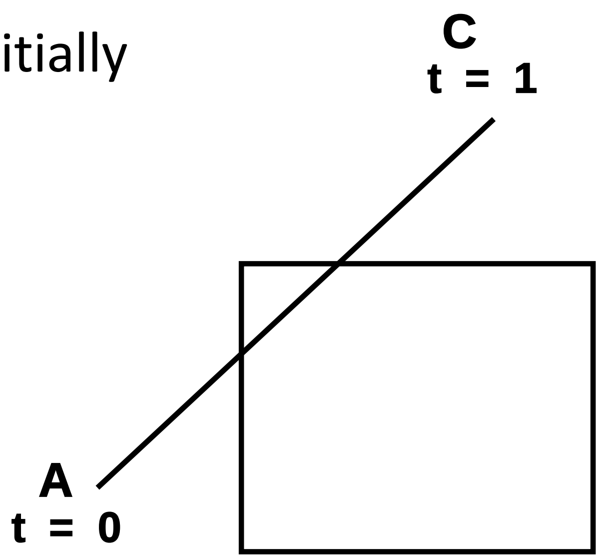


Leaving



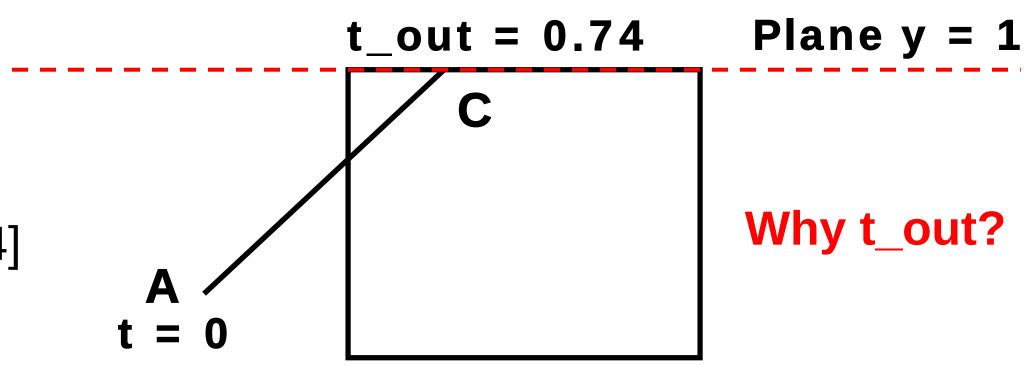
Chop step by Step against 6 planes

- Initially



t_in = 0,   t_out = 1
Candidate Interval (CI) = [0 to 1]

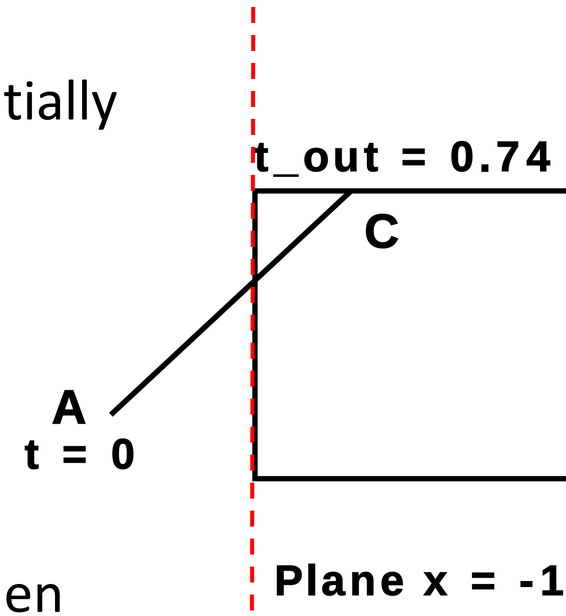
- Chop against each of 6 planes



t_in = 0,   t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]

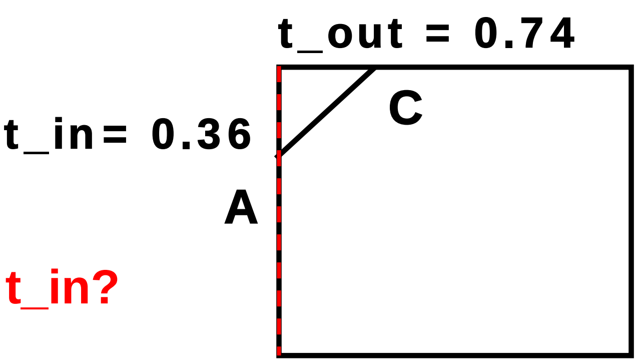
Chop step by Step against 6 planes

- Initially



t_in = 0,   t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]

- Then

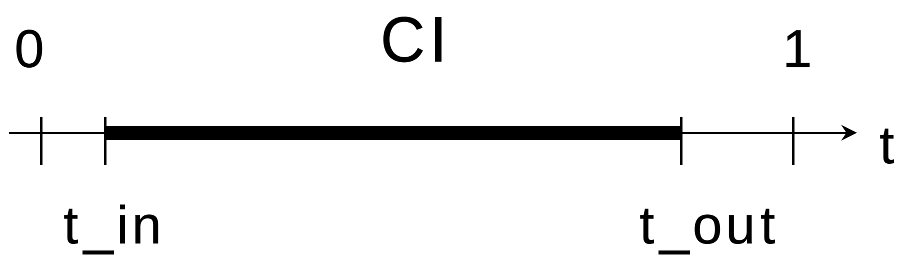


Why t_in?

t_in = 0.36,   t_out = 0.74
Candidate Interval (CI) CI = [0.36 to 0.74]

Candidate Interval

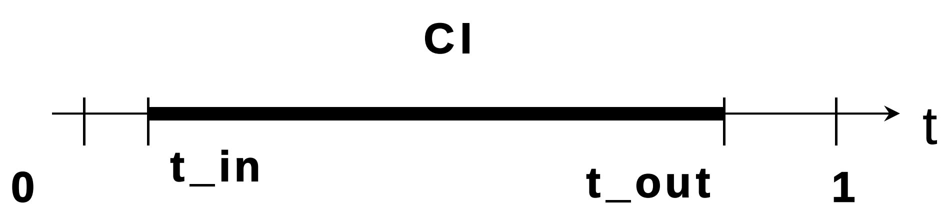
- Candidate Interval (CI): time interval during which edge might still be inside CVV. i.e. CI = t_in to t_out
- Initialize CI to [0,1]
- For each of 6 planes, calculate **t_in** or **t_out**, shrink CI



- Conversely: values of t outside CI = edge is outside CVV

Shortening Candidate Interval

- **Algorithm:**
  - Test for trivial accept/reject (stop if either occurs)
  - Set CI to [0,1]
  - For each of 6 planes:
    - Find hit time t_hit
    - If t_in,   new t_in = max(t_in,t_hit)
    - If t_out, new t_out = min(t_out, t_hit)
    - If t_in > t_out => exit (no valid intersections)

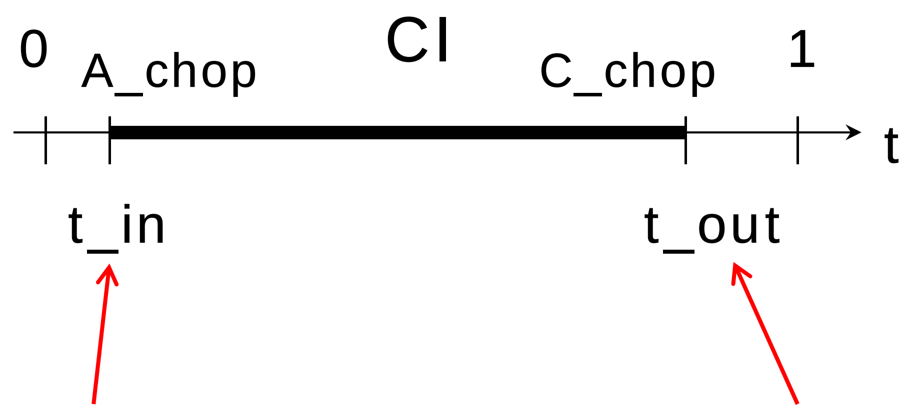


Note: seeking smallest valid CI without t_in crossing t_out

Calculate choppped A and C

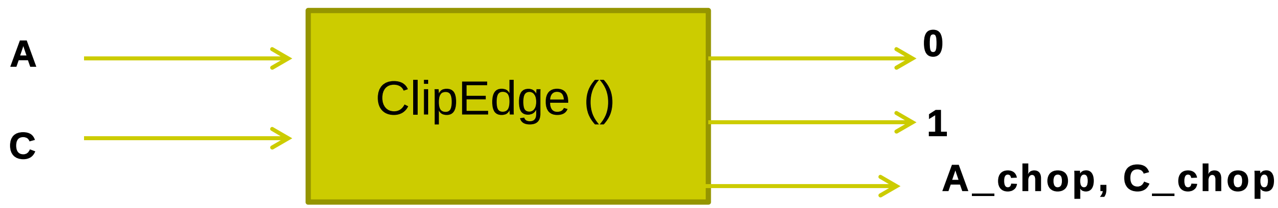
- If valid t_in, t_out, calculate adjusted edge endpoints A, C as

- A_chop = A + t_in (C – A) (calculate for Ax,Ay, Az)
- C_chop = A + t_out (C – A) (calculate for Cx,Cy,Cz)



3D Clipping Implementation

- Function clipEdge( )
- Input: two points A and C (in homogenous coordinates)
- Output:
  - 0, if AC lies **complete outside** CVV
  - 1, **complete inside** CVV
  - Returns clipped A and C otherwise
- Calculate 6 BCs for A, 6 for C



Store BCs as Outcodes

- Use outcodes to track in/out
  - Number walls x = +1, -1; y = +1, -1, and z = +1, -1 as 0 to 5
  - Bit *i* of A's **outcode = 1** if A is outside ith wall
  - 1 otherwise
- **Example:** outcode for point outside walls 1, 2, 5

Wall no.	0	1	2	3	4	5
OutCode	0	1	1	0	0	1

Trivial Accept/Reject using Outcodes

- **Trivial accept:** inside (not outside) any walls

Wall no.	0	1	2	3	4	5
A Outcode	0	0	0	0	0	0
C OutCode	0	0	0	0	0	0

Logical bitwise test: A | C == 0

- **Trivial reject:** point outside **same** wall. Example Both A and C outside wall 1

Wall no.	0	1	2	3	4	5
A Outcode	0	1	0	0	1	0
C OutCode	0	1	1	0	0	0

Logical bitwise test: A & C != 0

3D Clipping Implementation

- Compute BCs for A,C store as outcodes
- Test A, C outcodes for trivial accept, trivial reject
- If not trivial accept/reject, for each wall:
  - Compute tHit
  - Update t_in, t_out
  - If t_in > t_out, early exit

3D Clipping Pseudocode

```
int clipEdge(Point4& A, Point4& C)
{
    double tIn = 0.0, tOut = 1.0, tHit;
    double aBC[6], cBC[6];
    int aOutcode = 0, cOutcode = 0;

    …..find BCs for A and C
    …..form outcodes for A and C

    if((aOutCode & cOutcode) != 0) // trivial reject
       return 0;
    if((aOutCode | cOutcode) == 0) // trivial accept
       return 1;
```

3D Clipping Pseudocode

```
for(i=0;i<6;i++)  // clip against each plane
{
    if(cBC[i] < 0)  // C is outside wall i (exit so tOut)
    {
        tHit = aBC[i]/(aBC[i] – cBC[I]);     // calculate tHit
        tOut = MIN(tOut, tHit);
    }
    else if(aBC[i] < 0)  // A is outside wall I (enters so tIn)
    {
        tHit = aBC[i]/(aBC[i] – cBC[i]);     // calculate tHit
        tIn = MAX(tIn, tHit);
    }
    if(tIn > tOut) return 0; // CI is empty: early out
}
```

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

3D Clipping Pseudocode

```
Point4 tmp;  // stores homogeneous coordinates
If(aOutcode != 0) // A is outside: tIn has changed. Calculate A_chop
{
    tmp.x = A.x + tIn * (C.x – A.x);
    // do same for y, z, and w components
}
If(cOutcode != 0) // C is outside: tOut has changed. Calculate C_chop
{
    C.x = A.x + tOut * (C.x – A.x);
    // do same for y, z and w components
}
A = tmp;
Return 1; // some of the edges lie inside CVV
}
```

Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a concave polygon can yield multiple polygons

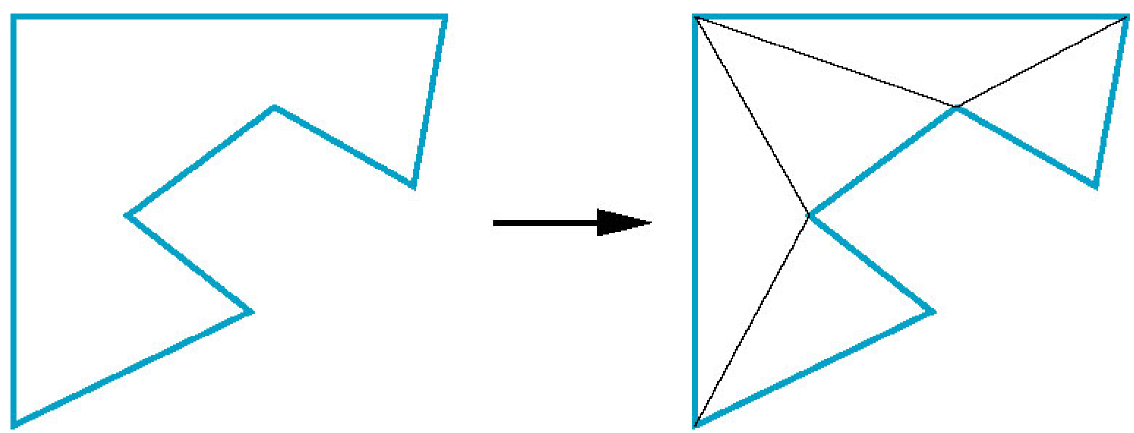
- Clipping a convex polygon can yield at most one other polygon

Clipping Polygons

- Need more sophisticated algorithms to handle polygons:
  - ***Sutherland-Hodgman:*** clip any given polygon against a convex clip polygon (or window)
  - ***Weiler-Atherton:*** Both clipped polygon and clip polygon (or window) can be concave

Tessellation and Convexity

- One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)
- Also makes fill easier



Computer Graphics (CS 4731)
Lecture 23: Viewport Transformation & Hidden Surface Removal

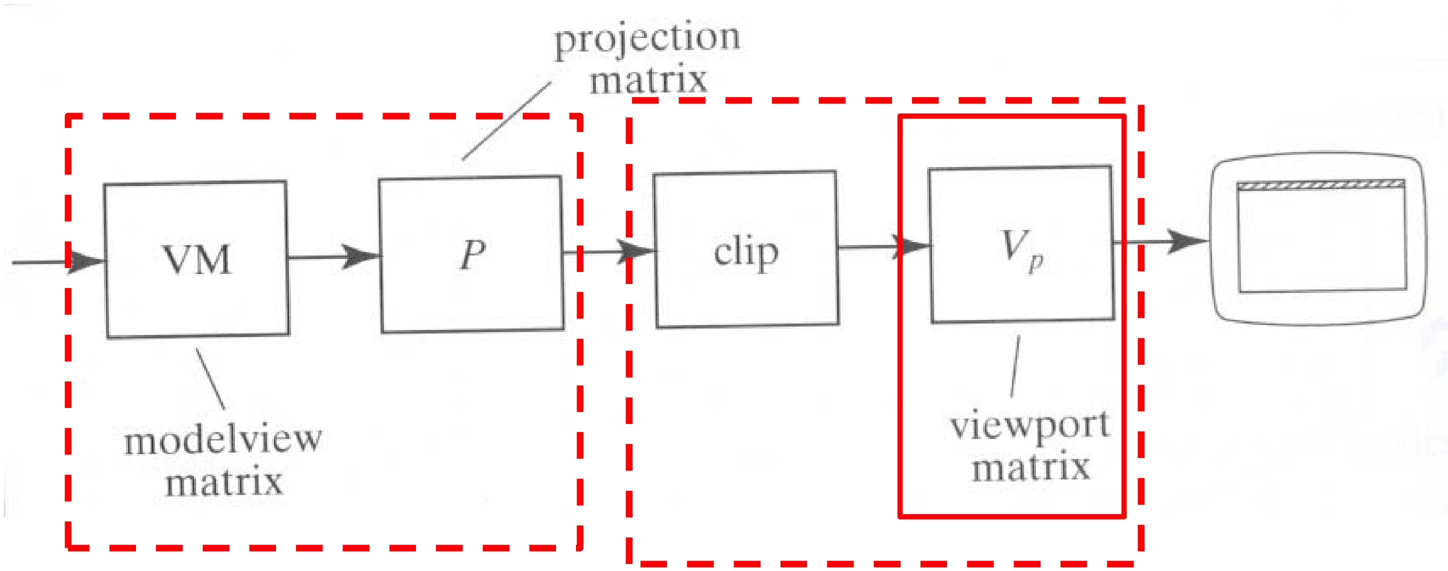
Prof Emmanuel Agu

Computer Science Dept.

Worcester Polytechnic Institute (WPI)

Viewport Transformation

- After clipping, do viewport transformation

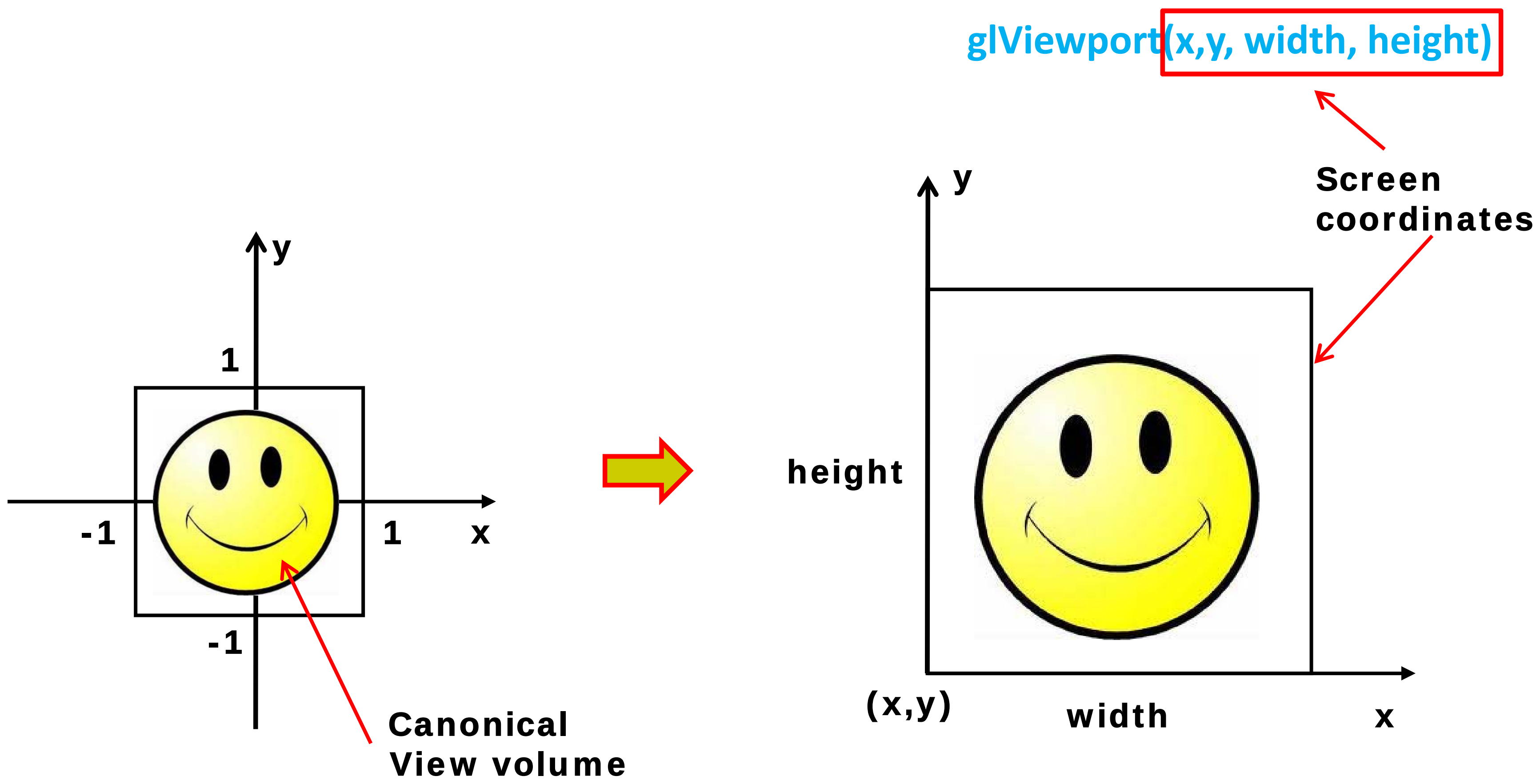


User implements in Vertex shader

Manufacturer implements In hardware

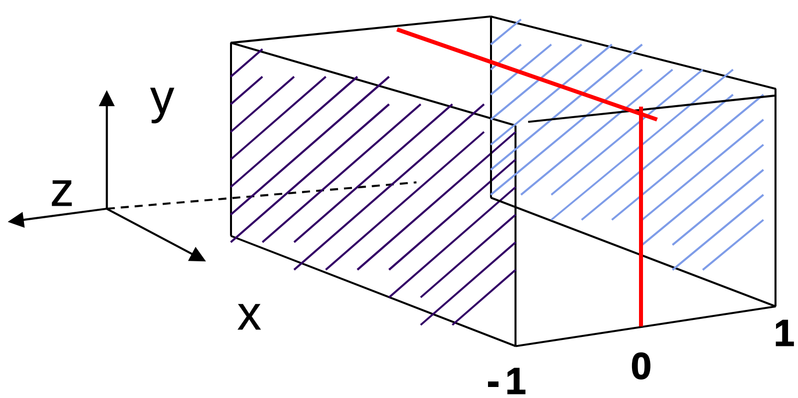
Viewport Transformation

- Maps **CVV (x, y)** -> **screen (x, y)** coordinates



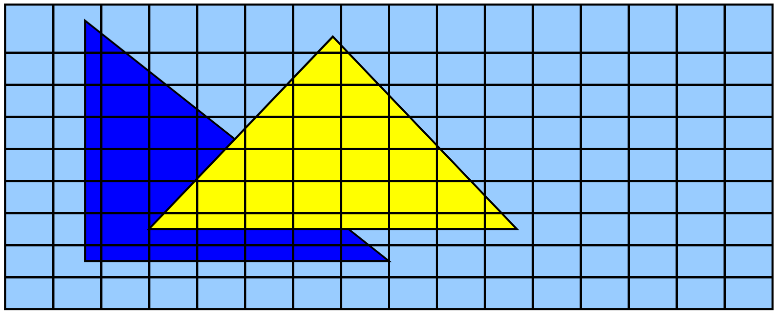
Viewport Transformation: What of z?

- Also maps z (pseudo-depth) from [-1,1] to [0,1]
- [0,1] pseudo-depth stored in depth buffer,
  - Used for Depth testing (Hidden Surface Removal)

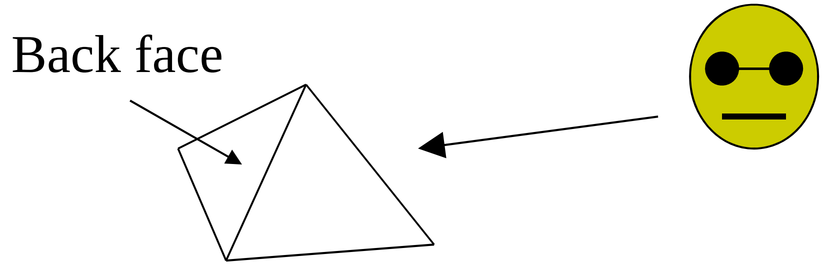


Hidden surface Removal

- Drawing polygonal faces on screen consumes CPU cycles
- User cannot see every surface in scene
- To save time, draw only surfaces we see
- Surfaces we cannot see and elimination methods?



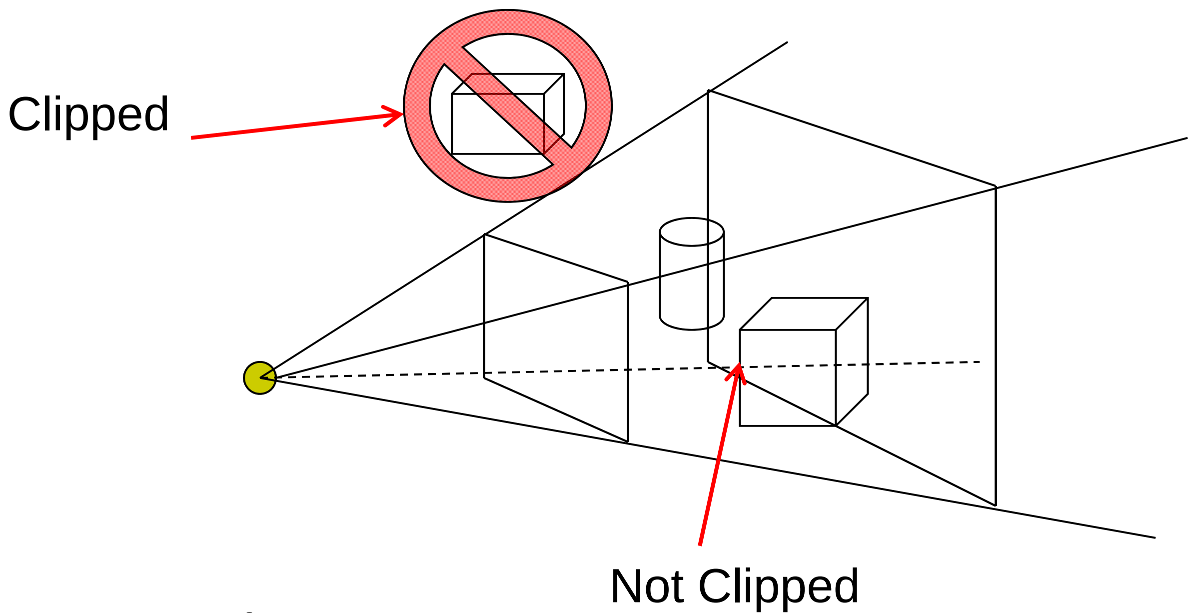
1. Occluded surfaces: hidden surface removal (visibility)



2. Back faces: back face culling

Hidden surface Removal

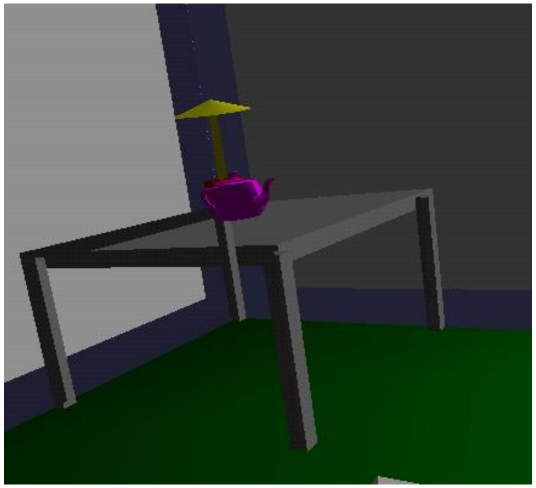
- Surfaces we cannot see and elimination methods:
  - **3. Faces outside view volume:** viewing frustrum culling



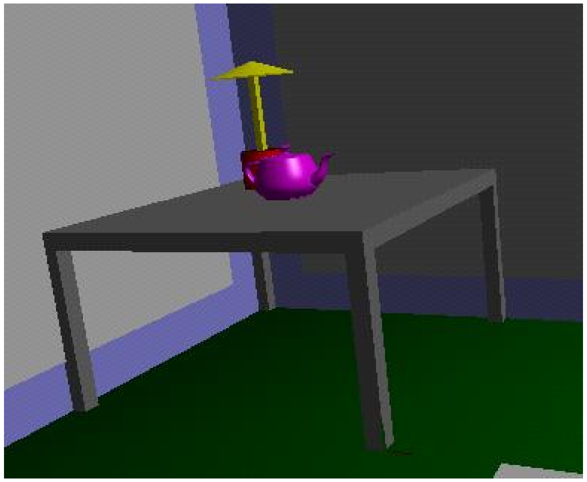
- Classes of HSR techniques:
  - **Object space techniques:** applied before rasterization
  - **Image space techniques:** applied after vertices have been rasterized

Visibility (hidden surface removal)

- Overlapping opaque polygons
- **Correct visibility?** Draw only the closest polygon
  - (remove the other hidden surfaces)



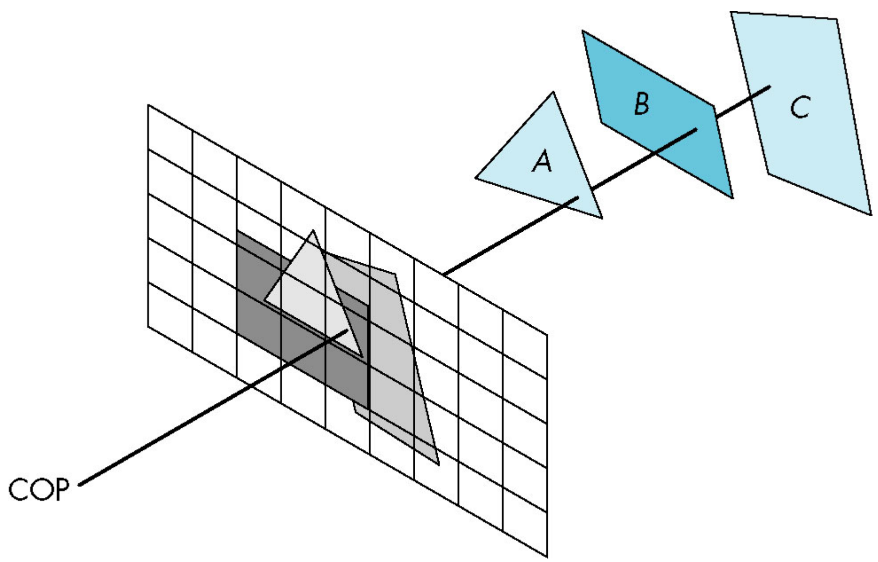
wrong visibility



Correct visibility

Image Space Approach

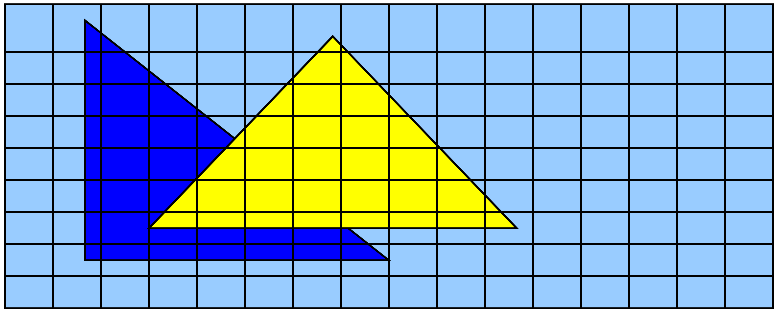
- Start from pixel, work backwards into the scene
- Through each pixel, (nm for an n x m frame buffer) find closest of k polygons
- Complexity O(nmk)
- Examples:
  - Ray tracing
  - z-buffer : OpenGL



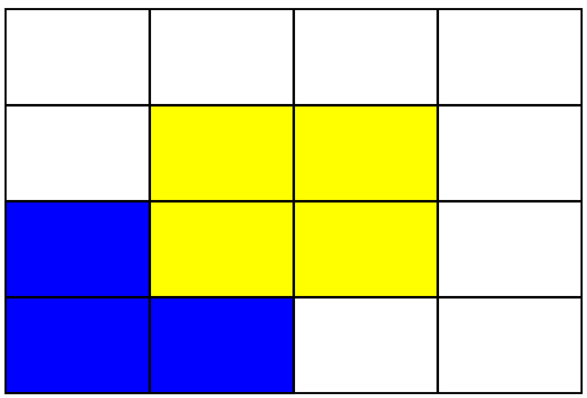
OpenGL - Image Space Approach

- Paint pixel with color of **closest** object

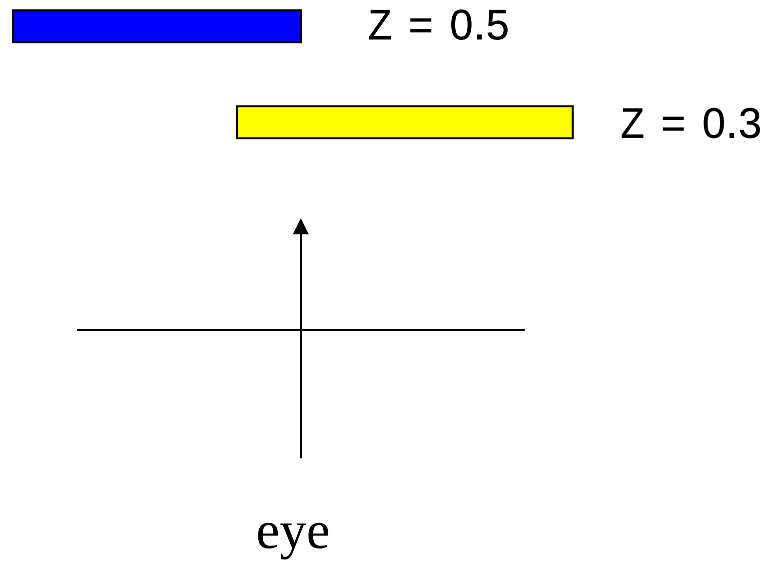
```
for (each pixel in image) {
    determine the object closest to the pixel
    draw the pixel using the object's color
}
```



Z buffer Illustration



Correct Final image



Top View

Z buffer Illustration

Step 1: Initialize the depth buffer

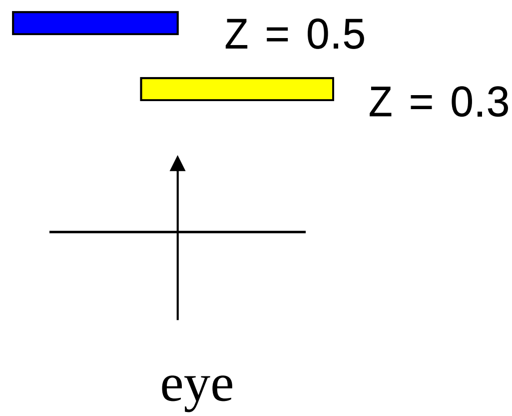
1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0

Largest possible z values is 1.0

Z buffer Illustration

Step 2: Draw blue polygon
(actually order does not affect final result)

1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0
0.5	0.5	1.0	1.0
0.5	0.5	1.0	1.0

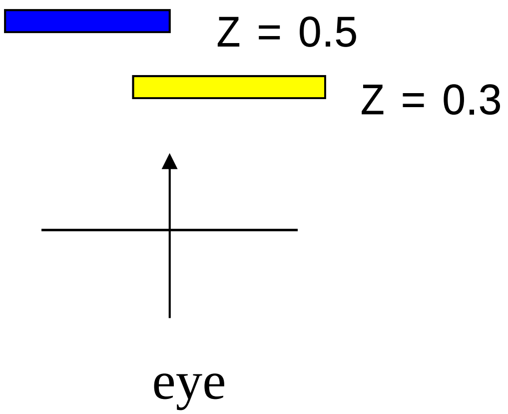


1. Determine group of pixels corresponding to blue polygon
2. Figure out z value of blue polygon for each covered pixel (0.5)
3. For each covered pixel, z = 0.5 is less than 1.0
    1. Smallest z so far = 0.5, color = blue

Z buffer Illustration

Step 3: Draw the yellow polygon

1.0	1.0	1.0	1.0
1.0	0.3	0.3	1.0
0.5	0.3	0.3	1.0
0.5	0.5	1.0	1.0



1. Determine group of pixels corresponding to yellow polygon
2. Figure out z value of yellow polygon for each covered pixel (0.3)
3. For each covered pixel, z = 0.3 becomes minimum, color = yellow

z-buffer drawback: wastes resources drawing and redrawing faces

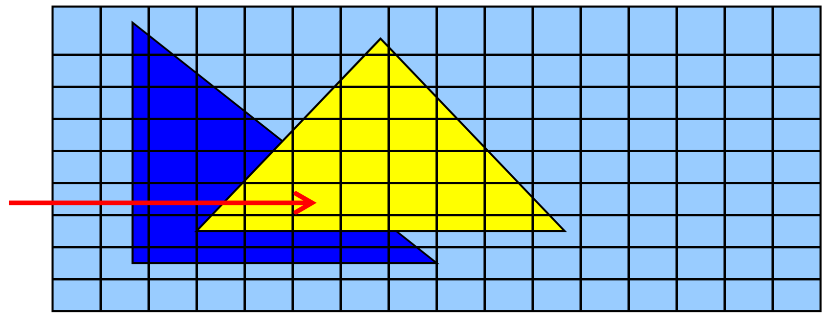
OpenGL HSR Commands

- 3 main commands to do HSR
- **glutInitDisplayMode(GLUT_DEPTH | GLUT_RGB)** instructs openGL to create depth buffer
- **glEnable(GL_DEPTH_TEST)** enables depth testing
- **glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT)** initializes depth buffer every time we draw a new picture

Z-buffer Algorithm

- Initialize every pixel's z value to 1.0
- rasterize every polygon
- For each pixel in polygon, find its z value (interpolate)
- Track smallest z value so far through each pixel
- As we rasterize polygon, for each pixel in polygon
  - If polygon's z through this pixel < current min z through pixel
  - Paint pixel with polygon's color

Find depth (z) of every polygon at each pixel



Z (depth) Buffer Algorithm

Depth of polygon being rasterized at pixel (x, y)

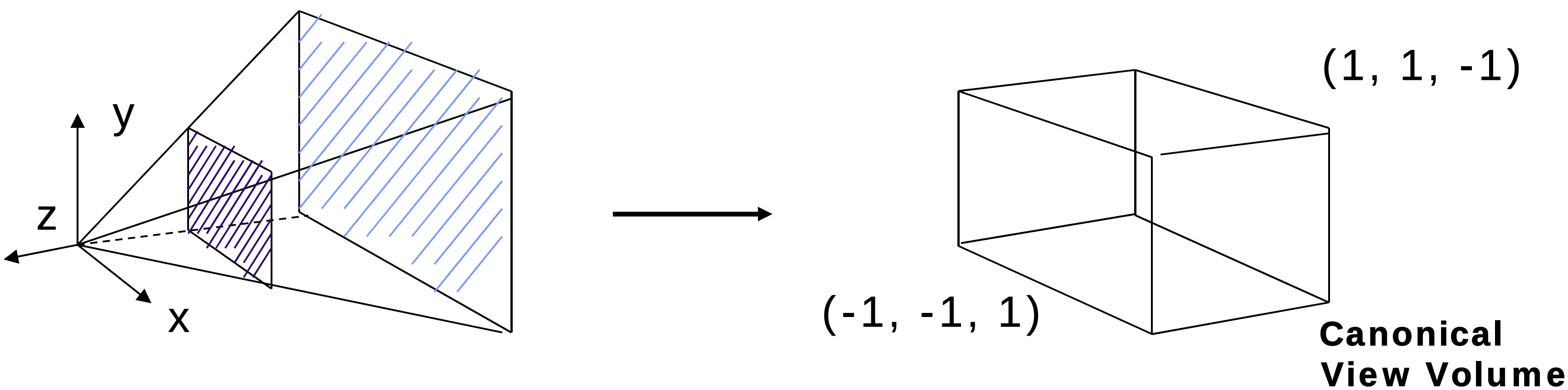
Largest depth seen so far Through pixel (x, y)

```
For  each polygon  {
   for each pixel (x,y) in polygon area  {
         if  (z_polygon_pixel(x,y) < depth_buffer(x,y) ) {
               depth_buffer(x,y) = z_polygon_pixel(x,y);
               color_buffer(x,y) = polygon color at (x,y)
          }
      }
  }
```

Note: know depths at vertices. Interpolate for interior z_polygon_pixel(x, y) depths

Perspective Transformation: Z-Buffer Depth Compression

- **Pseudodepth calculation:** Recall we chose parameters (a and b) to map z from range [near, far] to **pseudodepth** range[-1,1]



$$
\begin{pmatrix}
\frac{2N}{x\max - x\min} & 0 & \frac{right+left}{right-left} & 0 \\
0 & \frac{2N}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\
0 & 0 & \frac{-(F+N)}{F-N} & \frac{-2FN}{F-N} \\
0 & 0 & -1 & 0
\end{pmatrix}
\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}
$$

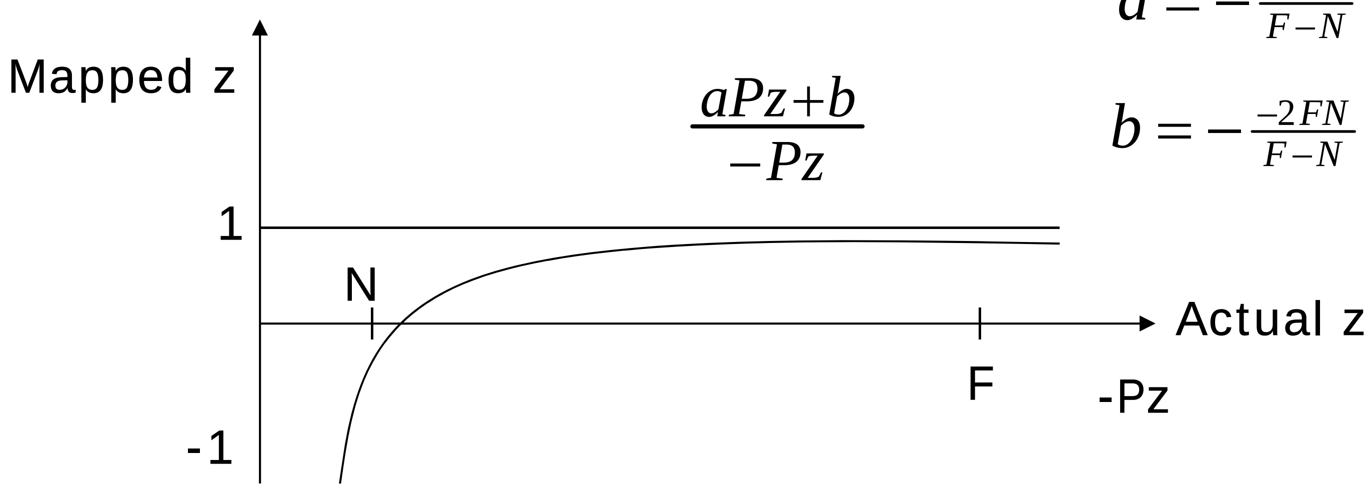
These values map z values of original view volume to [-1, 1] range

Z-Buffer Depth Compression

- This mapping is almost linear close to eye
- Non-linear further from eye, approaches asymptote
- Also limited number of bits
- Thus, two z values close to far plane may map to same pseudodepth: ***Errors!!***

$$a = -\frac{F+N}{F-N}$$

$$b = -\frac{-2FN}{F-N}$$



References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Chapter 9