

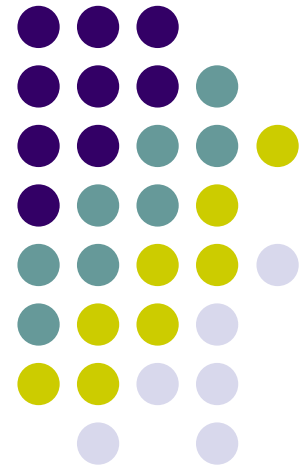
# Computer Graphics (CS 4731)

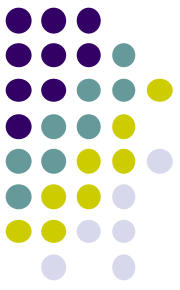
## Lecture 13: Viewing & Camera Control

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Prof Emmanuel Agu

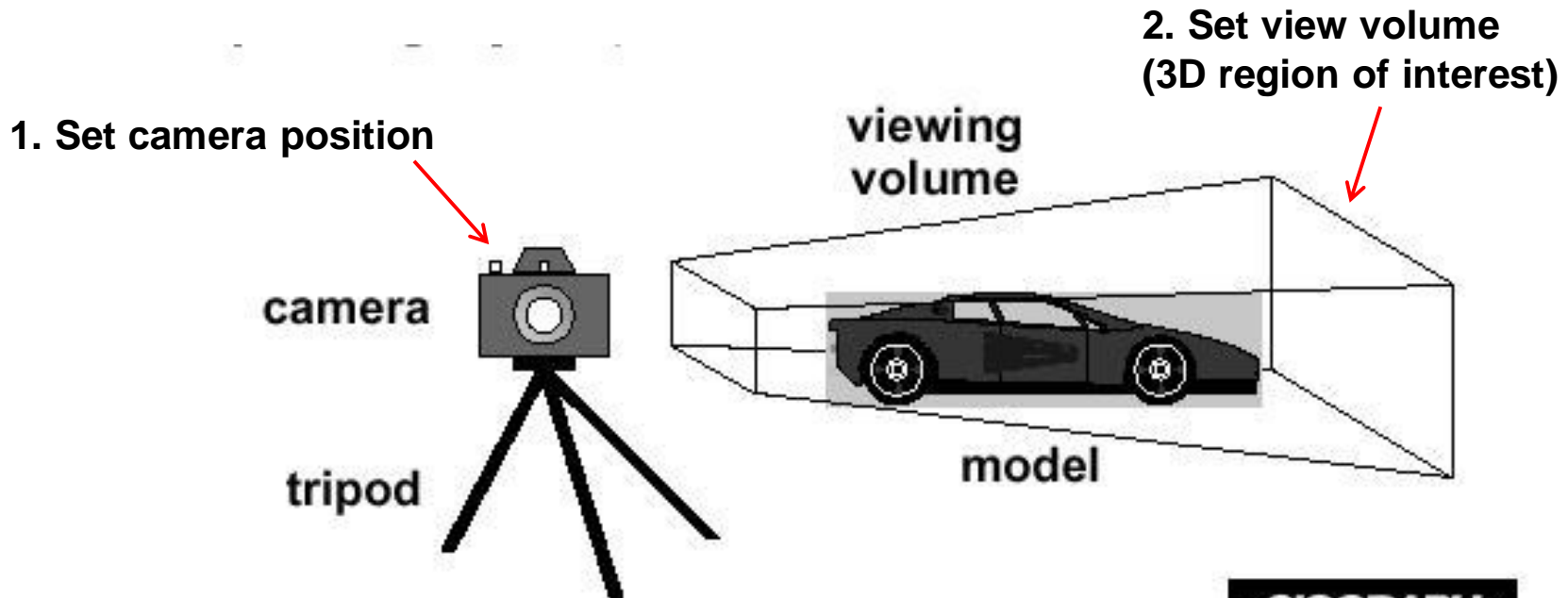
*Computer Science Dept.  
Worcester Polytechnic Institute (WPI)*



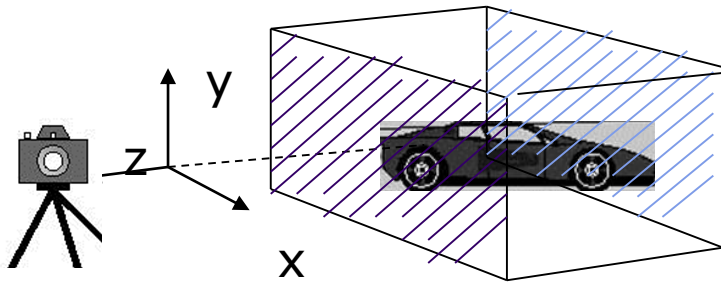
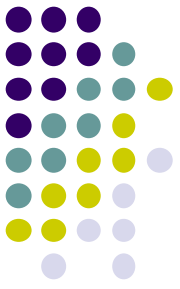


# 3D Viewing?

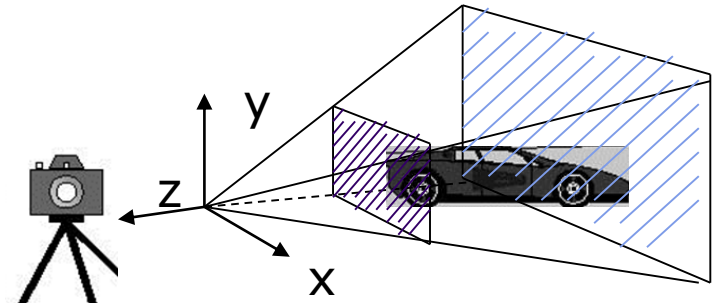
- Objects **inside** view volume drawn to viewport (screen)
- Objects outside view volume **clipped** (not drawn)!



# Different View Volume Shapes



**Orthogonal view volume**



**Perspective view volume**



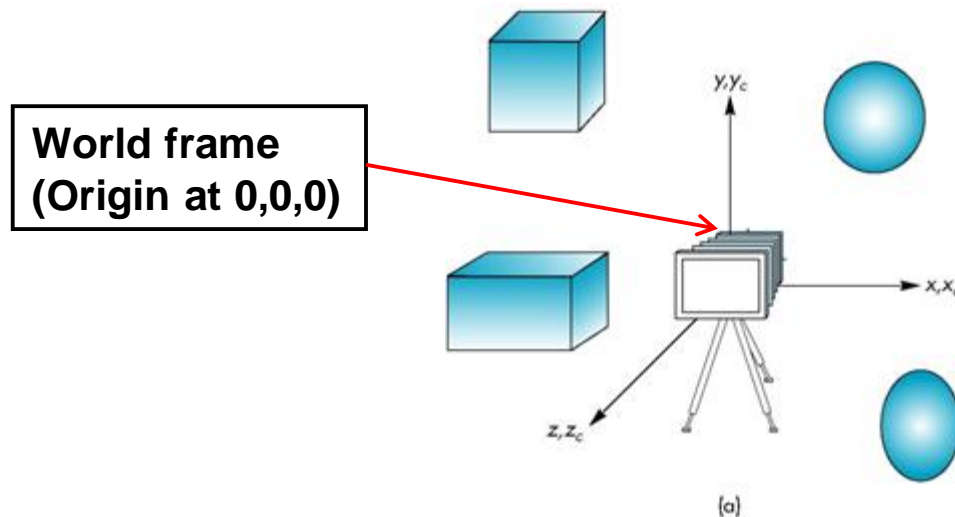
- Different view volume => different look
- **Foreshortening?** Near objects bigger
  - Perspective projection has **foreshortening**
  - Orthogonal projection: no foreshortening

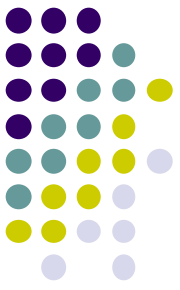




# The World Frame

- Objects/scene initially defined in **world frame**
- **World Frame origin** at  $(0,0,0)$
- Objects positioned, oriented (translate, scale, rotate transformations) applied to objects in **world frame**





# Camera Frame

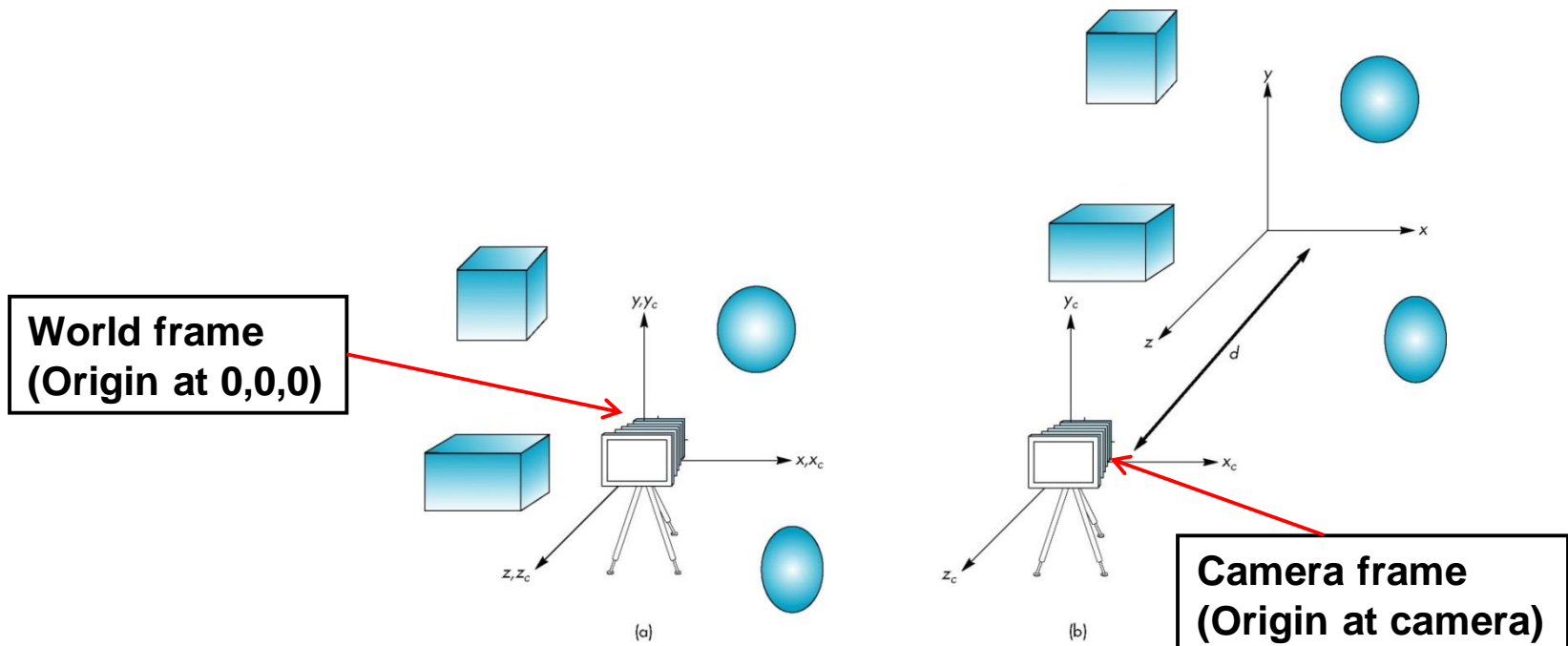
- More natural to describe object positions **relative to camera (eye)**
- Why?
  - Our view of the world
  - First person shooter games

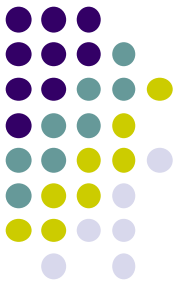




# Camera Frame

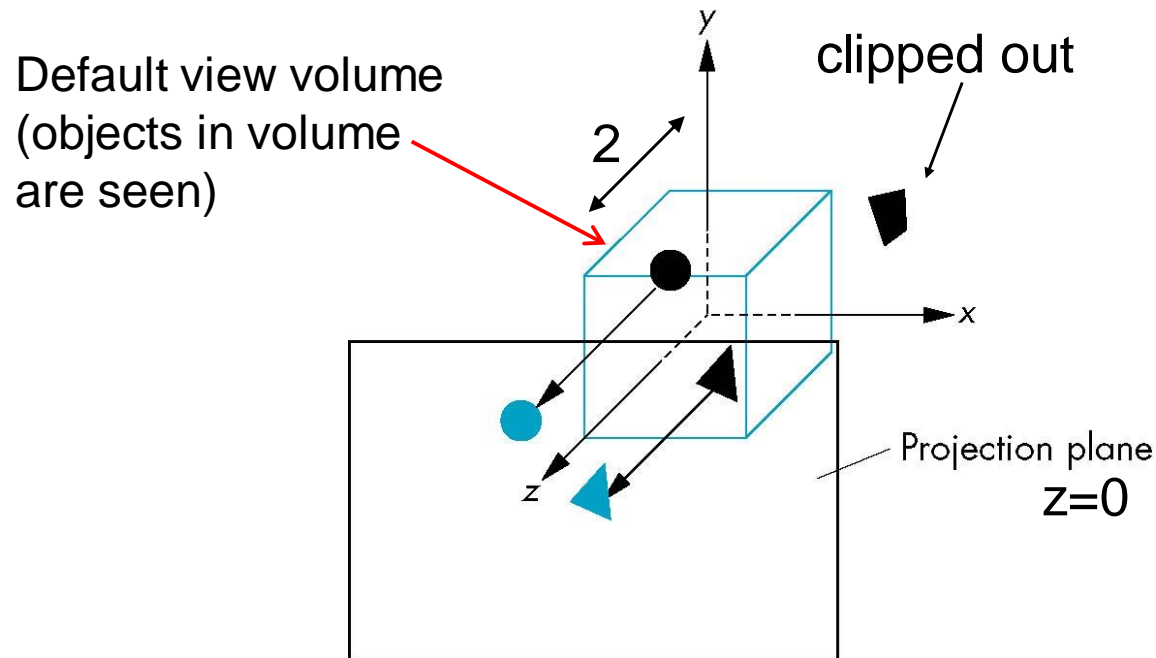
- **Viewing:** After user chooses camera (eye) position, represent objects in **camera frame** (origin at eye position)
- **Viewing transformation:** Converts object  $(x,y,z)$  positions in world frame to positions in camera frame





# Default OpenGL Camera

- Initially Camera at origin: object and camera frames same
- Points in negative z direction
- Default view volume is cube with sides of length 2



# Moving Camera Frame

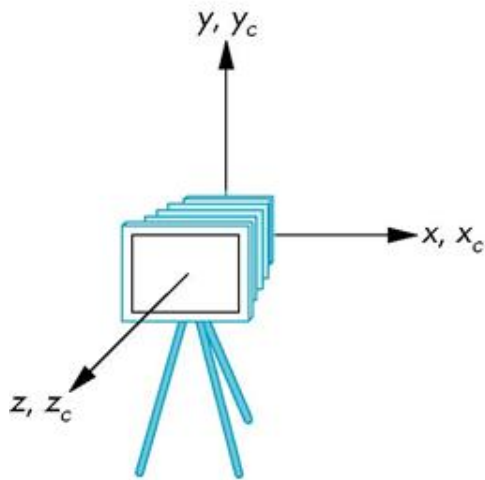


Same relative distance after  
Same result/look

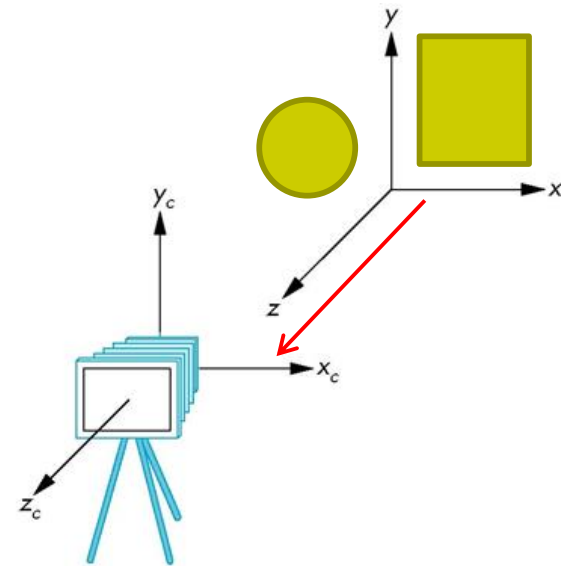
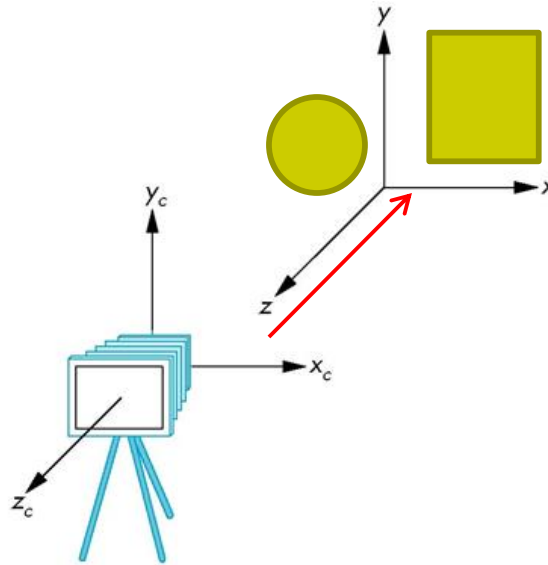
Translate **objects -5**  
away from camera

Translate **camera +5**  
away from objects

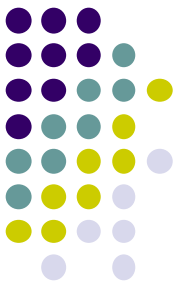
default frames



(a)





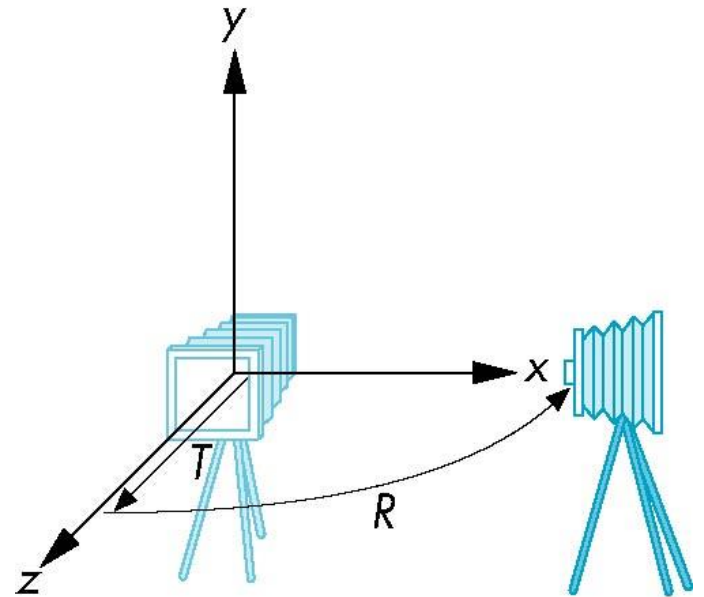


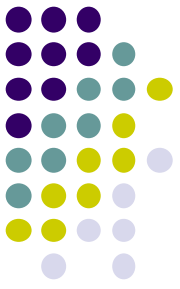
# Moving the Camera

- We can move camera using sequence of rotations and translations
- Example: side view
  - Rotate the camera
  - Move it away from origin
  - Model-view matrix  $C = TR$

```
// Using mat.h
```

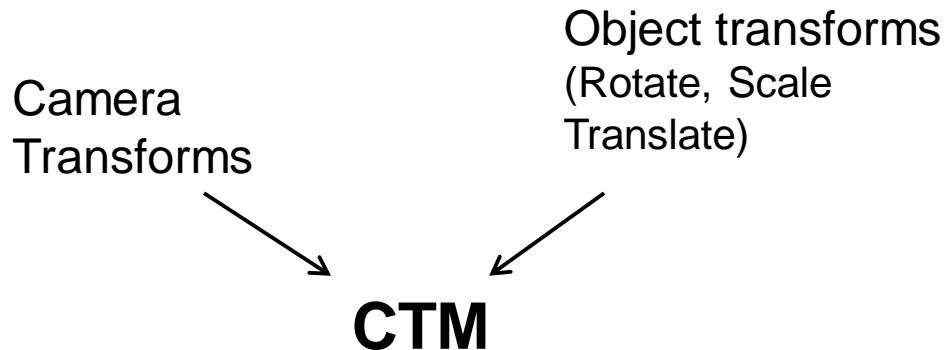
```
mat4 t = Translate (0.0, 0.0, -d);  
mat4 ry = RotateY(90.0);  
mat4 m = t*ry;
```





# Moving the Camera Frame

- Object distances **relative to camera** determined by the model-view matrix
  - Transforms (scale, translate, rotate) go into **modelview matrix**
  - Camera transforms also go in **modelview matrix (CTM)**





# The LookAt Function

- Previously, command **gluLookAt** to position camera
- **gluLookAt deprecated!!**
- Homegrown mat4 method LookAt() in mat.h
  - Sets camera position, transforms object distances to camera frame

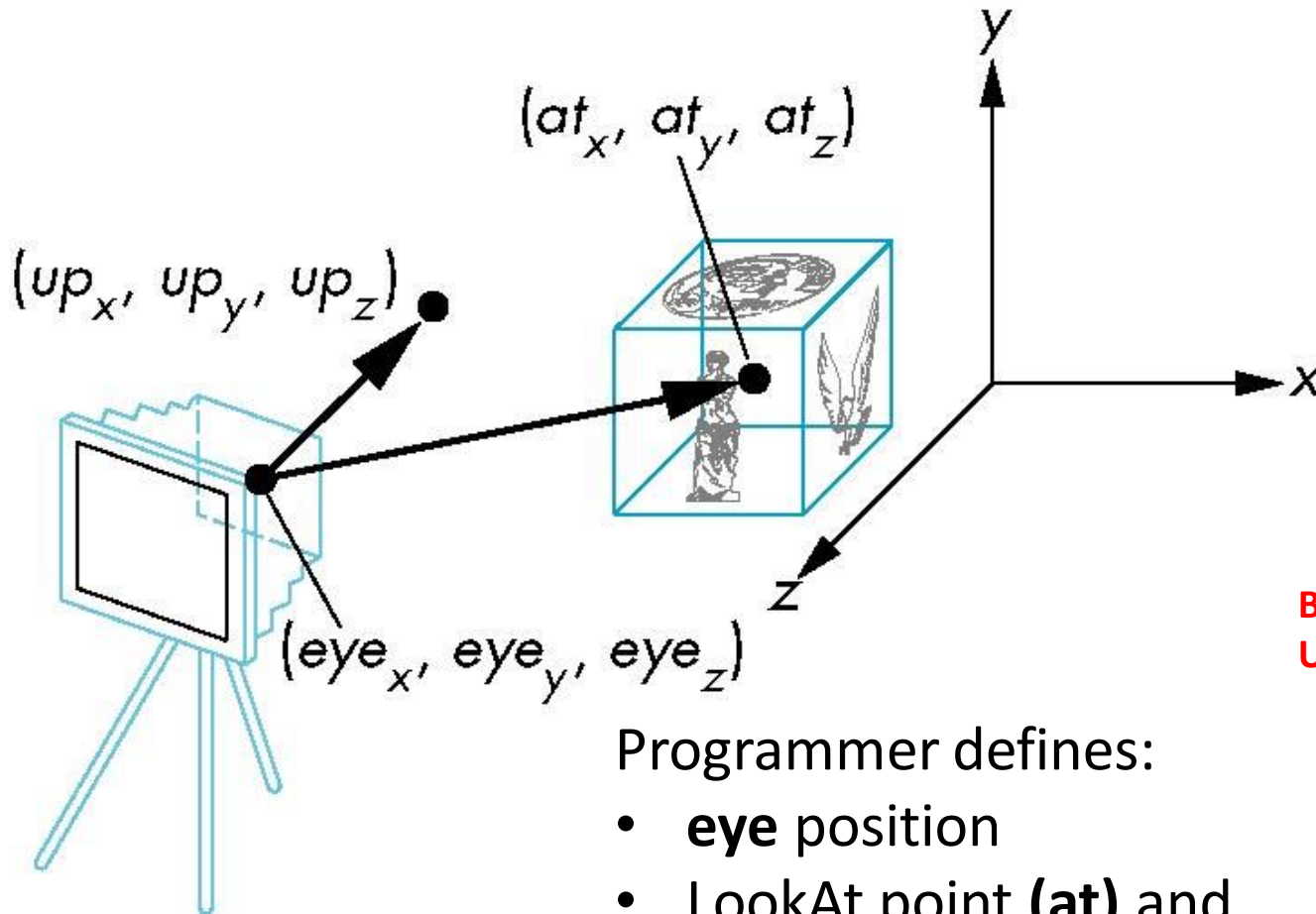
```
void display( ) {  
    .....  
  
    mat4 mv = LookAt(vec4 eye, vec4 at, vec4 up);  
    .....  
}
```

Builds 4x4 matrix for positioning, orienting Camera and puts it into variable **mv**



# The LookAt Function

LookAt(eye, at, up)

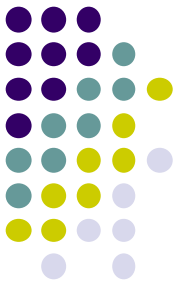


But Why do we set  
Up direction?

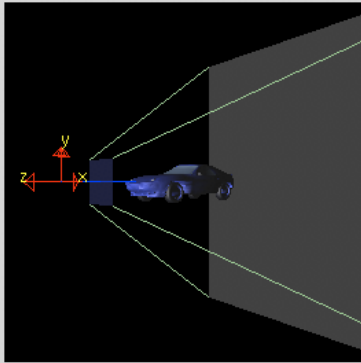
Programmer defines:

- **eye** position
- LookAt point (**at**) and
- **Up** vector (**Up** direction usually  $(0,1,0)$ )

# Nate Robbins LookAt Demo



World-space view



Screen-space view

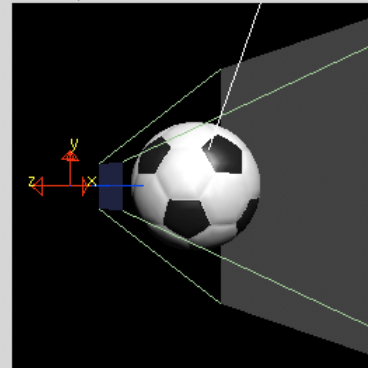


Command manipulation window

```
glTranslatef( 0.00 , 0.00 , 0.00 );  
glRotatef( 0.0 , 0.00 , 1.00 , 0.00 );  
glScalef( 1.00 , 1.00 , 1.00 );  
glBegin( ... );  
...
```

Click on the arguments and move the mouse to modify values.

World-space view



Screen-space view



Command manipulation window

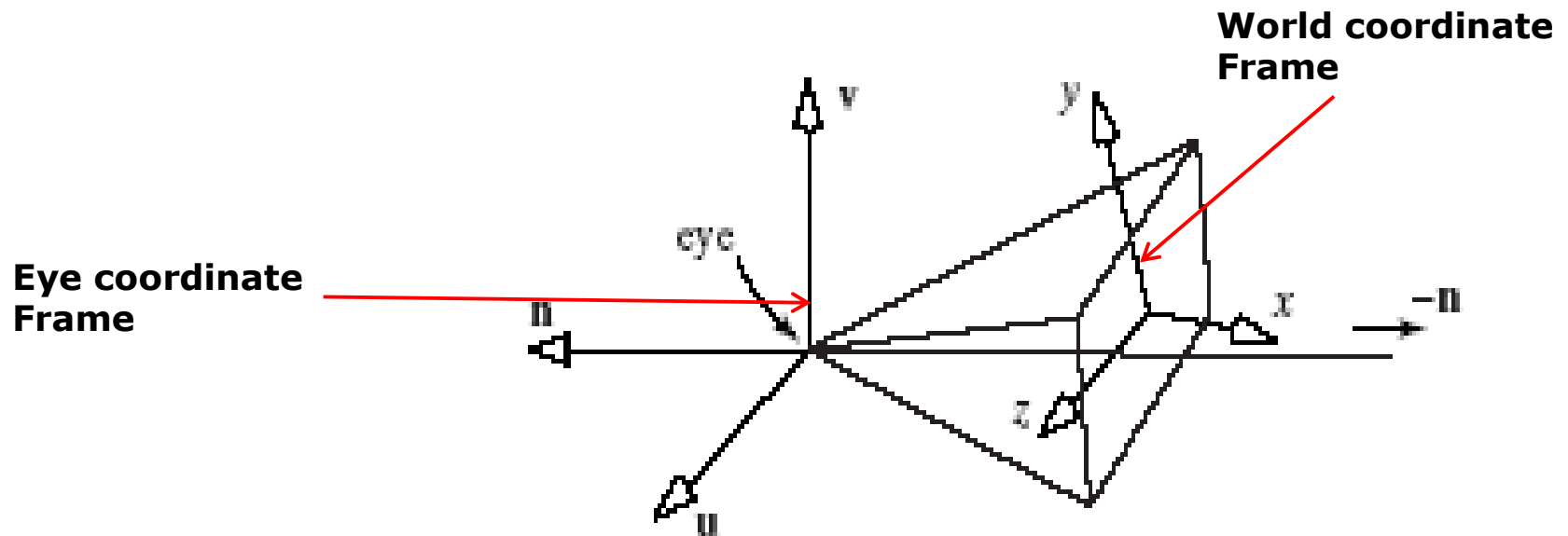
```
GLfloat pos[4] = { 1.50 , 1.00 , 1.00 , 0.00 };  
gluLookAt( 0.00 , 0.00 , 2.00 , <- eye  
          0.00 , 0.00 , 0.00 , <- center  
          0.00 , 1.00 , 0.00 ); <- up  
glLightfv(GL_LIGHT0, GL_POSITION, pos);
```

Click on the arguments and move the mouse to modify values.



# What does LookAt do?

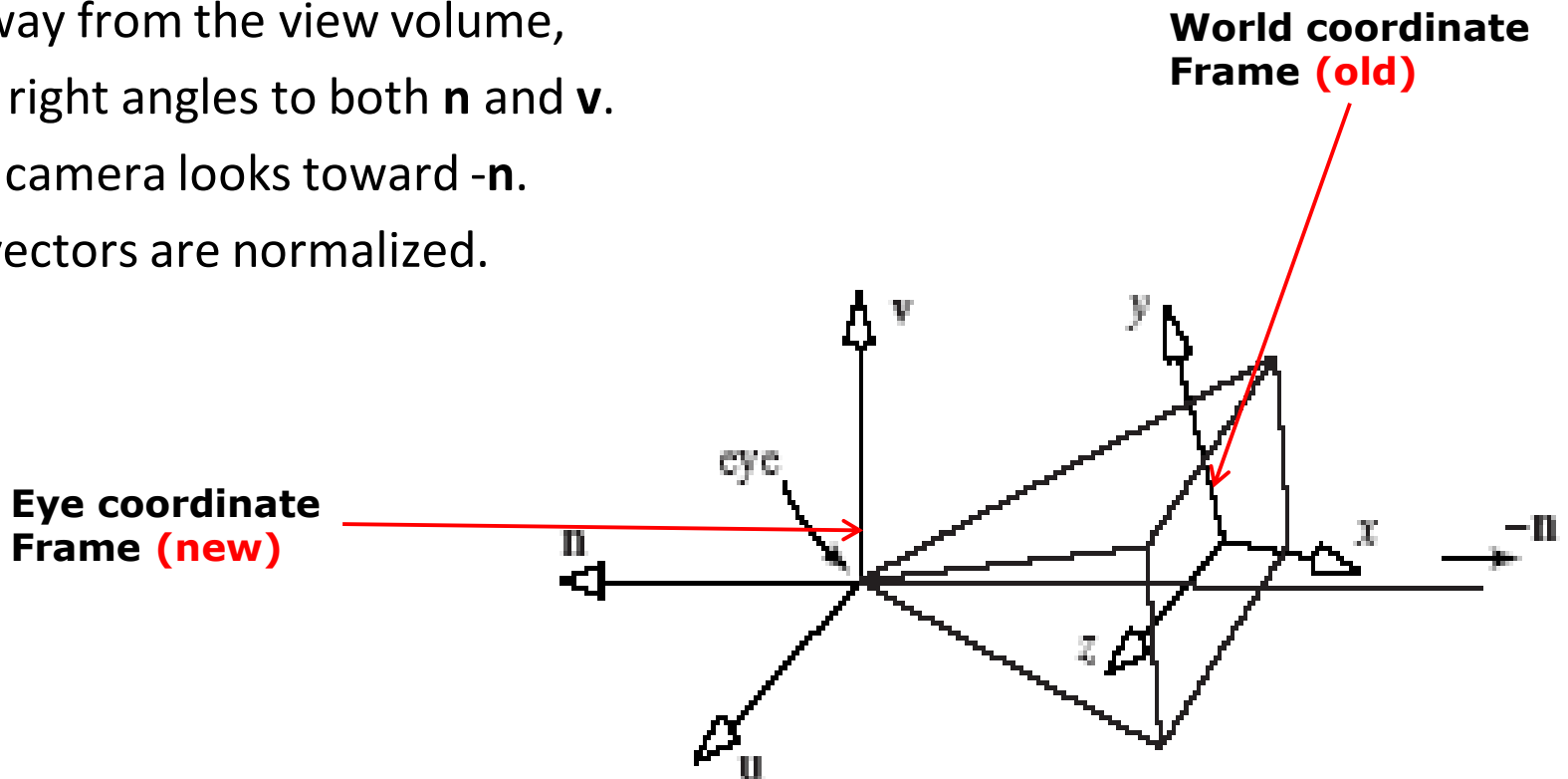
- Programmer defines eye, lookAt and Up
- **LookAt method:**
  - Forms new axes (u, v, n) at camera
  - Transform objects from world to eye camera frame



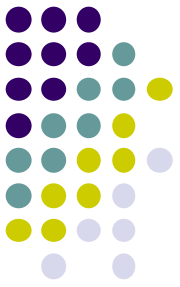
# Camera with Arbitrary Orientation and Position



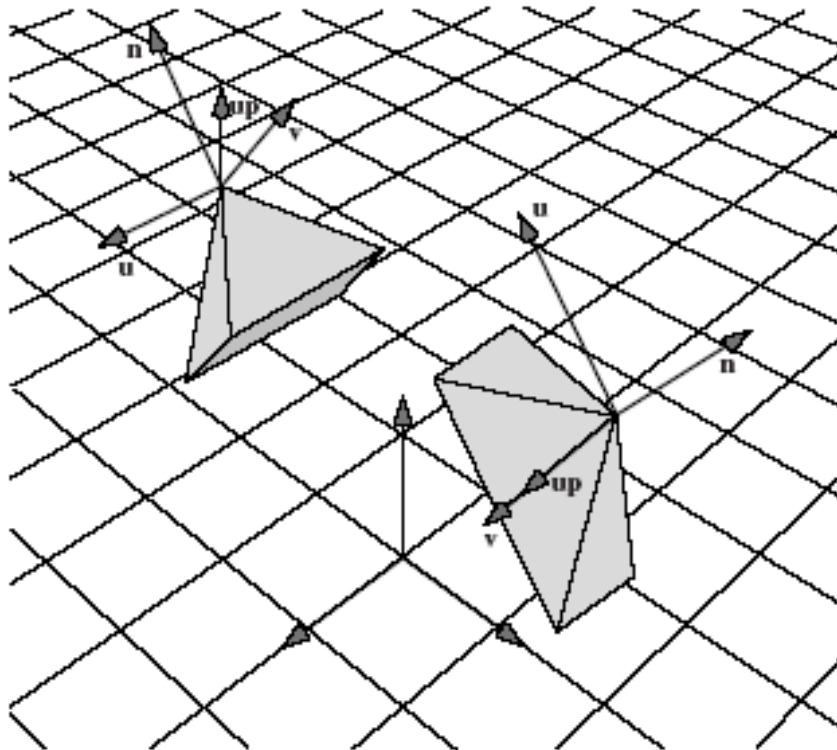
- Define new axes ( $u, v, n$ ) at eye
  - $v$  points vertically upward,
  - $n$  away from the view volume,
  - $u$  at right angles to both  $n$  and  $v$ .
  - The camera looks toward  $-n$ .
  - All vectors are normalized.



# LookAt: Effect of Changing Eye Position or LookAt Point



- Programmer sets **LookAt** (*eye*, *at*, *up*)
- If **eye**, **lookAt** point changes  $\Rightarrow$  **u,v,n** changes



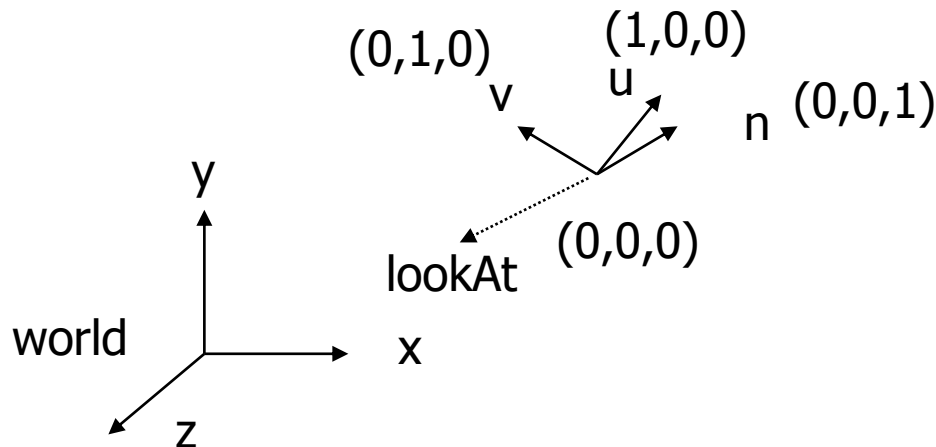


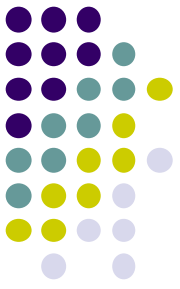


# Viewing Transformation Steps

1. Form camera  $(u,v,n)$  frame
2. Transform objects from world frame (Composes matrix for coordinate transformation)

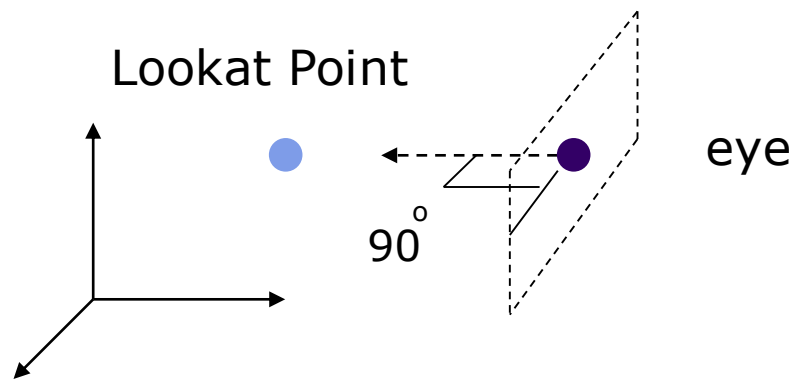
- Next, let's form camera  $(u,v,n)$  frame

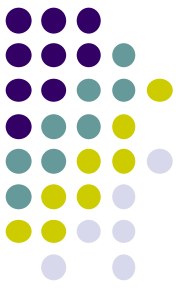




# Constructing U,V,N Camera Frame

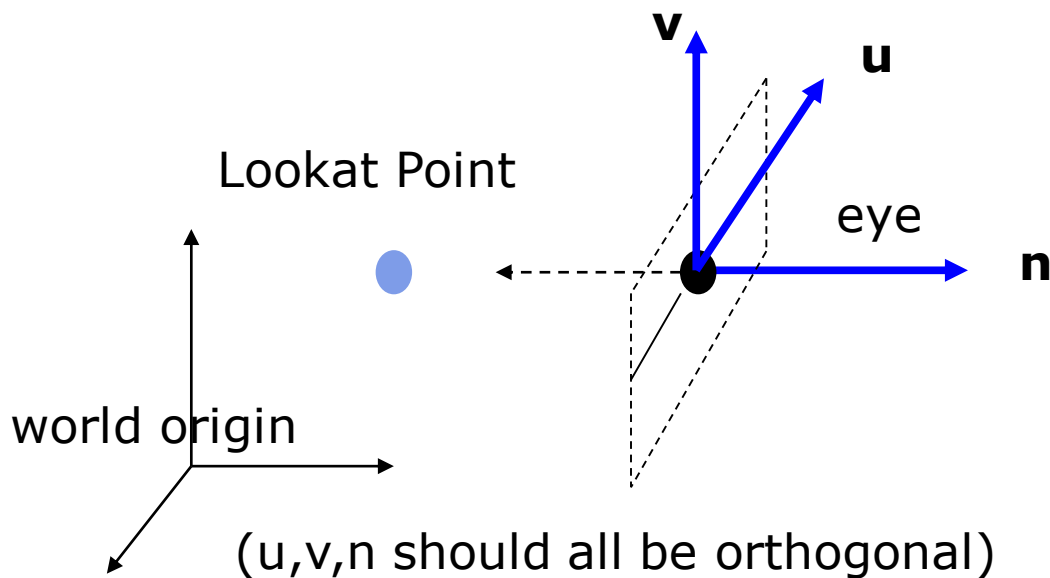
- Lookat arguments: **LookAt** (*eye*, *at*, *up*)
- **Known:** eye position, LookAt Point, up vector
- **Derive:** new origin and three basis (u,v,n) vectors





# Eye Coordinate Frame

- **New Origin: eye position** (that was easy)
- 3 basis vectors:
  - one is the normal vector (**n**) of the viewing plane,
  - other two (**u** and **v**) span the viewing plane



**n** is pointing away from the world because we use left hand coordinate system

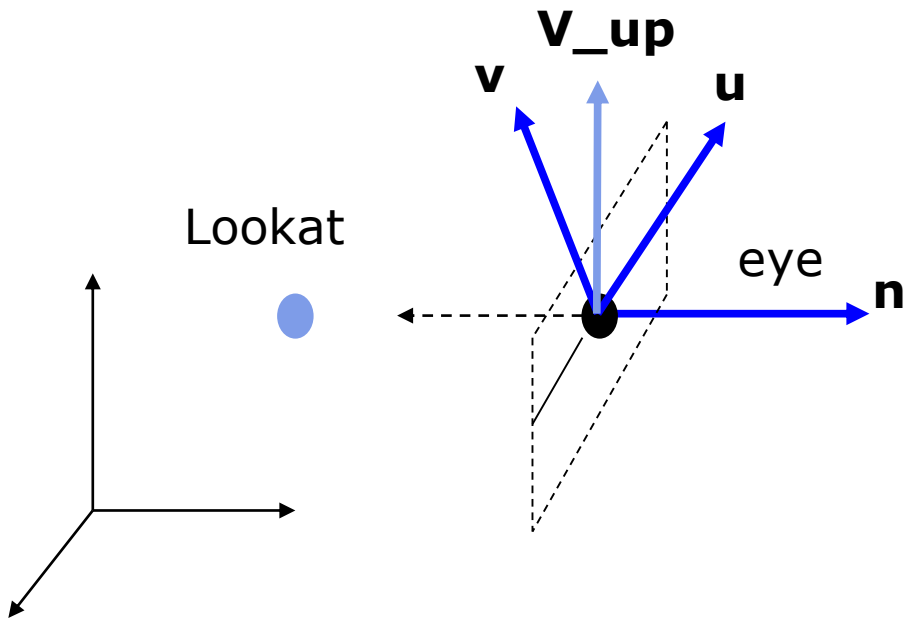
$$\mathbf{N} = \text{eye} - \text{Lookat Point}$$
$$\mathbf{n} = \mathbf{N} / |\mathbf{N}|$$

Remember **u,v,n** should be all unit vectors



# Eye Coordinate Frame

- How about  $u$  and  $v$ ?



- We can get  $u$  first -
  - $u$  is a vector that is perp to the plane spanned by  $N$  and view up vector ( $V_{up}$ )

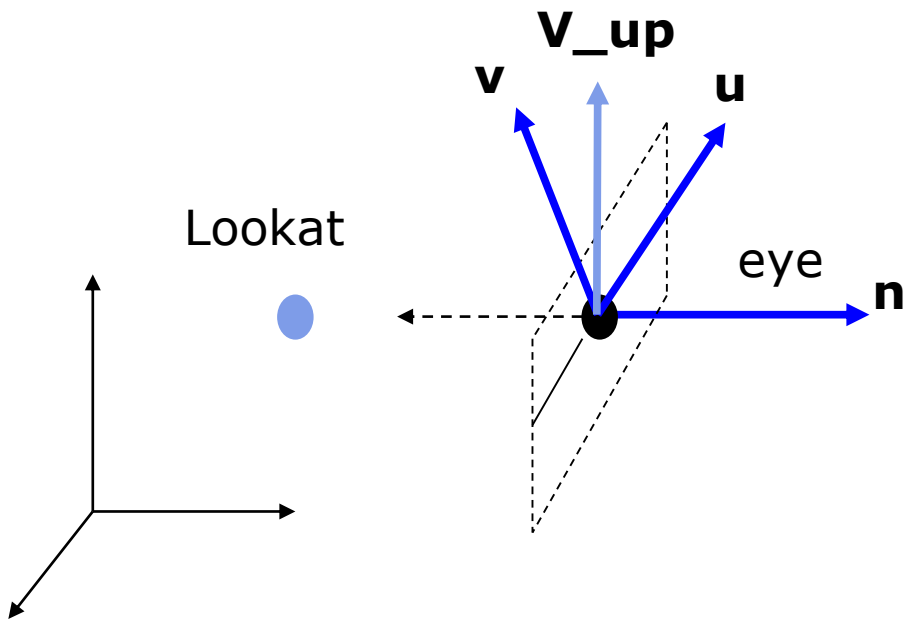
$$U = V_{up} \times n$$

$$u = U / |U|$$



# Eye Coordinate Frame

- How about  $v$ ?



Knowing  $n$  and  $u$ , getting  $v$  is easy

$$\mathbf{v} = \mathbf{n} \times \mathbf{u}$$

**$v$  is already normalized**



# Eye Coordinate Frame

- Put it all together

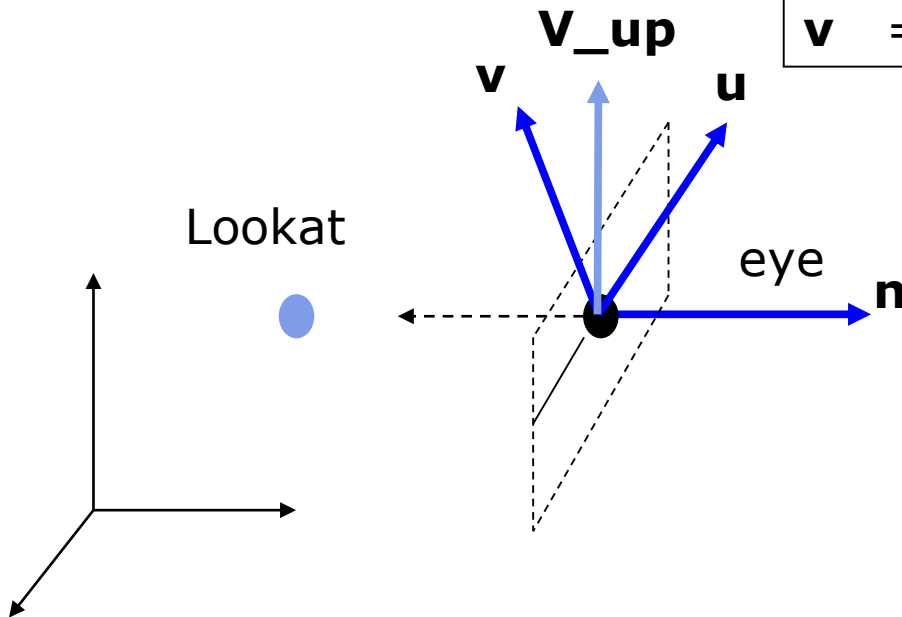
Eye space **origin: (Eye.x , Eye.y, Eye.z)**

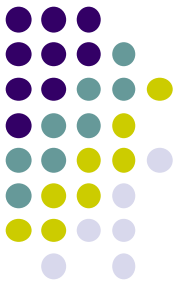
Basis vectors:

$$\mathbf{n} = (\text{eye} - \text{Lookat}) / |\text{eye} - \text{Lookat}|$$

$$\mathbf{u} = (\mathbf{V\_up} \times \mathbf{n}) / |\mathbf{V\_up} \times \mathbf{n}|$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u}$$

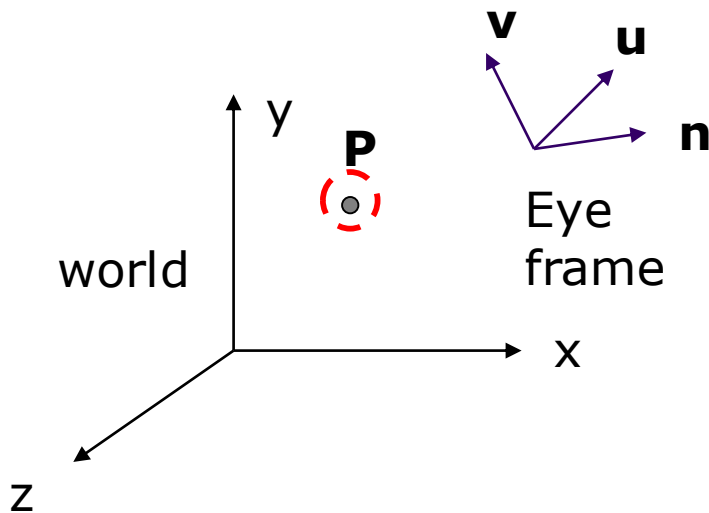




## Step 2: World to Eye Transformation

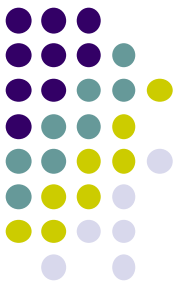
- Next, use  $u$ ,  $v$ ,  $n$  to compose LookAt matrix
- Transformation matrix ( $M_{w2e}$ ) ?

$$P' = M_{w2e} P$$



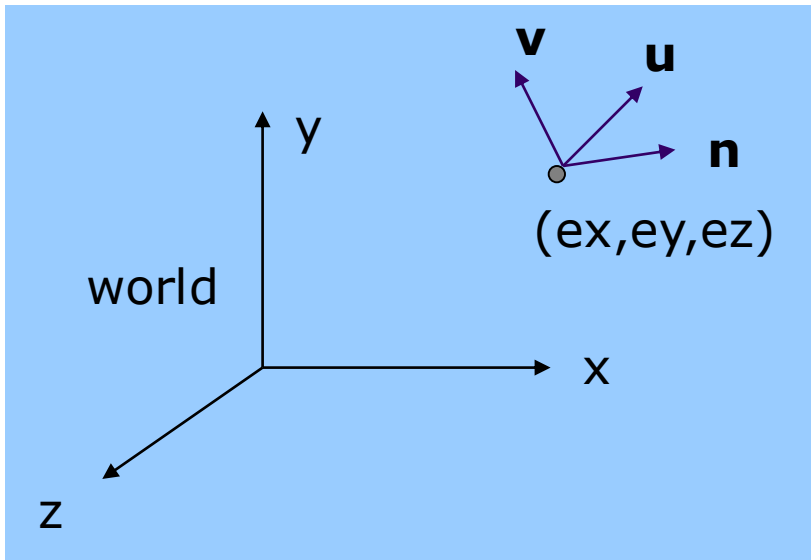
1. Come up with transformation sequence that lines up eye frame with world frame

2. Apply this transform sequence to point  $P$  in reverse order



# World to Eye Transformation

1. Rotate eye frame to “align” it with world frame
2. Translate  $(-ex, -ey, -ez)$  to align origin with eye



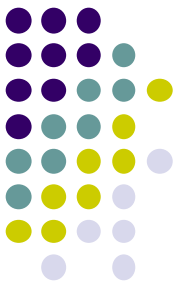
Rotation:

$$\begin{vmatrix} ux & uy & uz & 0 \\ vx & vy & vz & 0 \\ nx & ny & nz & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Translation:

$$\begin{vmatrix} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

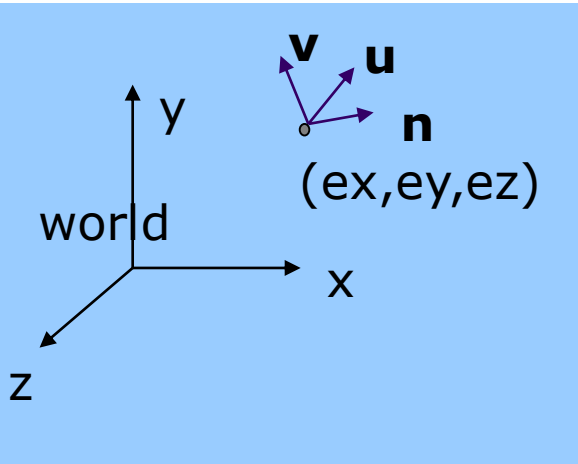




# World to Eye Transformation

- Transformation order: apply the transformation to the object in reverse order - translation first, and then rotate

$$M_{w2e} = \begin{array}{c|cc} \text{Rotation} & & \text{Translation} \\ \hline \begin{array}{cccc} ux & uy & uz & 0 \\ vx & vy & vz & 0 \\ nx & ny & nz & 0 \\ 0 & 0 & 0 & 1 \end{array} & & \begin{array}{cccc} 1 & 0 & 0 & -ex \\ 0 & 1 & 0 & -ey \\ 0 & 0 & 1 & -ez \\ 0 & 0 & 0 & 1 \end{array} \end{array}$$



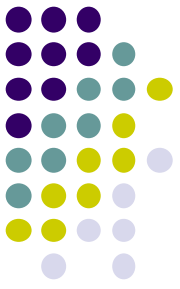
$$= \begin{array}{c|cccc} & ux & uy & uz & -\mathbf{e} \cdot \mathbf{u} \\ & vx & vy & vz & -\mathbf{e} \cdot \mathbf{v} \\ & nx & ny & nz & -\mathbf{e} \cdot \mathbf{n} \\ & 0 & 0 & 0 & 1 \end{array}$$

Multiplied together = lookAt transform

Note:  $\mathbf{e} \cdot \mathbf{u} = ex \cdot ux + ey \cdot uy + ez \cdot uz$

$\mathbf{e} \cdot \mathbf{v} = ex \cdot vx + ey \cdot vy + ez \cdot vz$

$\mathbf{e} \cdot \mathbf{n} = ex \cdot nx + ey \cdot ny + ez \cdot nz$



# lookAt Implementation (from mat.h)

Eye space **origin: (Eye.x , Eye.y, Eye.z)**

Basis vectors:

$$\begin{aligned} \mathbf{n} &= (\text{eye} - \text{Lookat}) / |\text{eye} - \text{Lookat}| \\ \mathbf{u} &= (\mathbf{V\_up} \times \mathbf{n}) / |\mathbf{V\_up} \times \mathbf{n}| \\ \mathbf{v} &= \mathbf{n} \times \mathbf{u} \end{aligned}$$

$$\begin{vmatrix} u_x & u_y & u_z & -\mathbf{e} \cdot \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e} \cdot \mathbf{v} \\ n_x & n_y & n_z & -\mathbf{e} \cdot \mathbf{n} \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

```
mat4 LookAt( const vec4& eye, const vec4& at, const vec4& up )
{
    vec4 n = normalize(eye - at);
    vec4 u = normalize(cross(up, n));
    vec4 v = normalize(cross(n, u));
    vec4 t = vec4(0.0, 0.0, 0.0, 1.0);
    mat4 c = mat4(u, v, n, t);
    return c * Translate( -eye );
}
```



# References

- Interactive Computer Graphics, Angel and Shreiner, Chapter 4
- Computer Graphics using OpenGL (3<sup>rd</sup> edition), Hill and Kelley