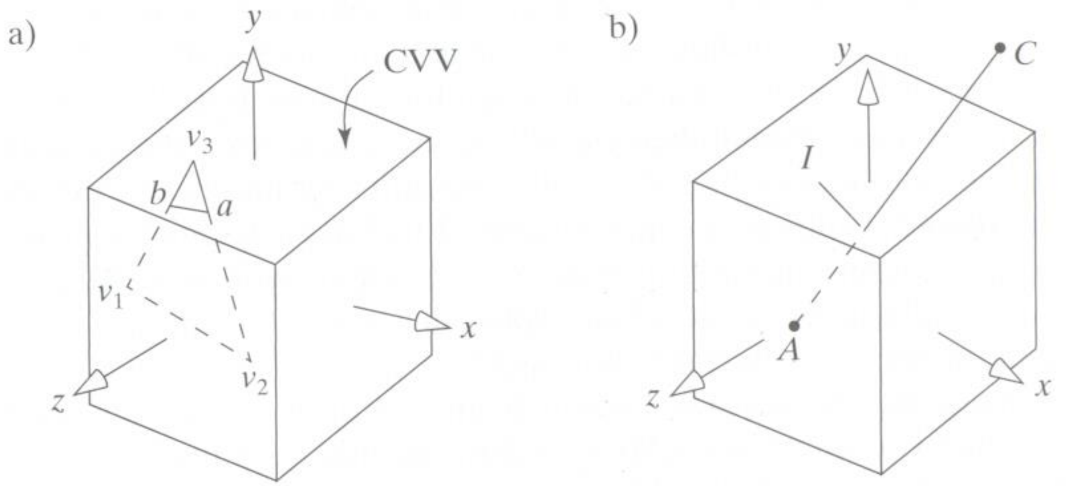
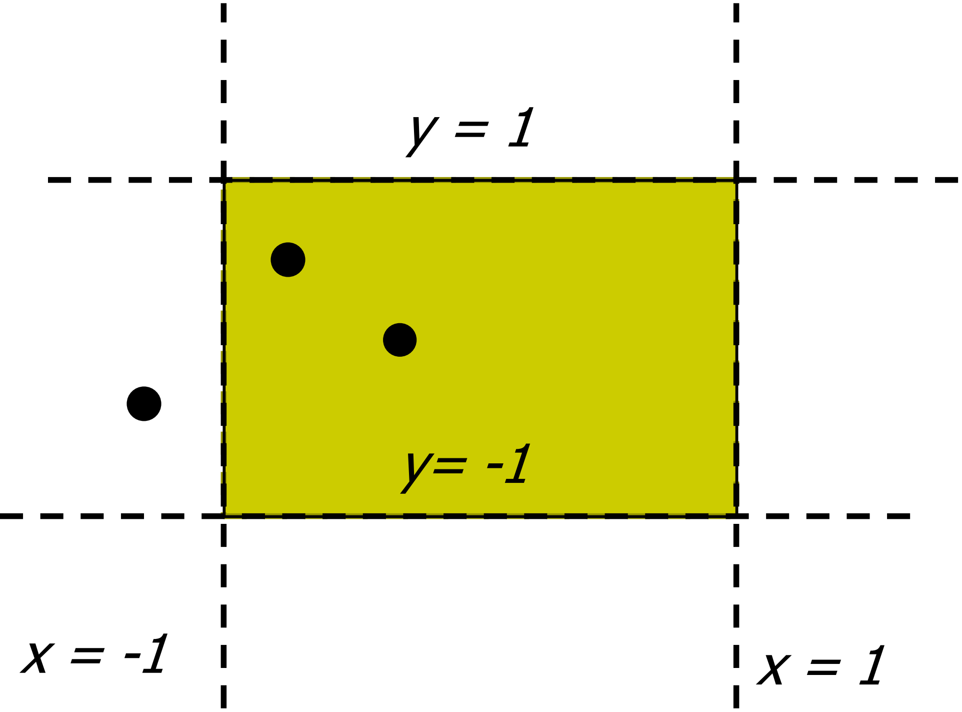
# Recall: Liang-Barsky 3D Clipping

- **Goal:** Clip object edge-by-edge against Canonical View volume (CVV)
- **Problem:**
  - 2 end-points of edge: **A = (Ax, Ay, Az, Aw)** and **C = (Cx, Cy, Cz, Cw)**
  - If edge intersects with CVV, compute intersection point **I =(Ix,Iy,Iz,Iw)**



# **Recall:** Determining if point is inside CVV



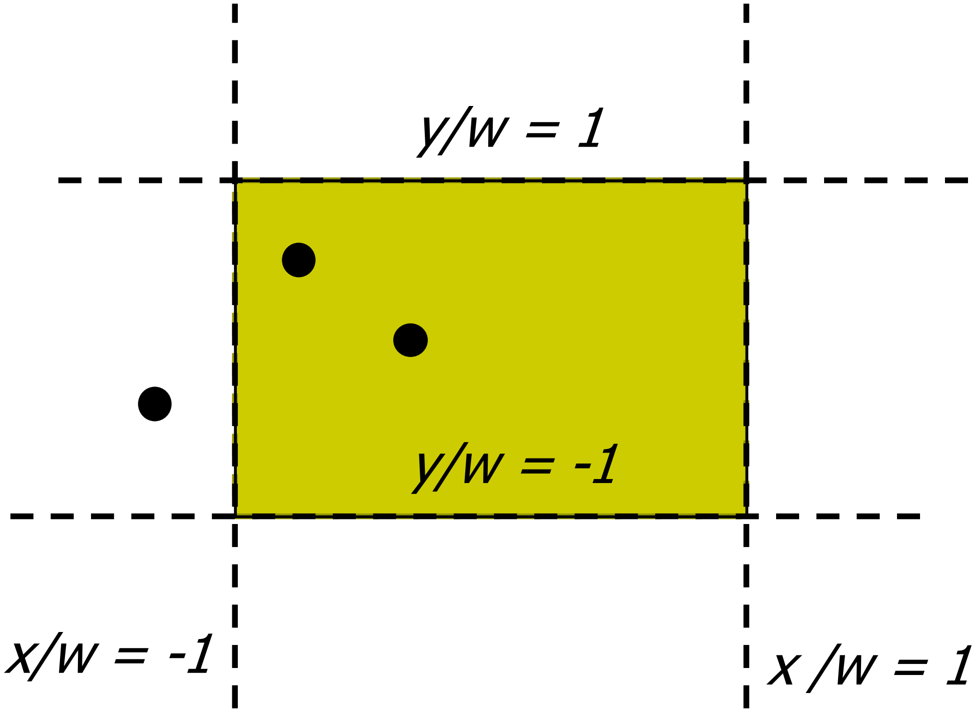
- **Problem:** Determine if point (x,y,z) is inside or outside CVV?

Point (x,y,z) is **inside CVV if**
(-1 <= x <= 1)
**and** (-1 <= y <= 1)
**and** (-1 <= z <= 1)

else point **is outside CVV**

- **CVV == 6 infinite planes (x=-1,1; y=-1,1; z=-1,1)**

# Recall: Determining if point is inside CVV



- If point specified as (x,y,z,w)

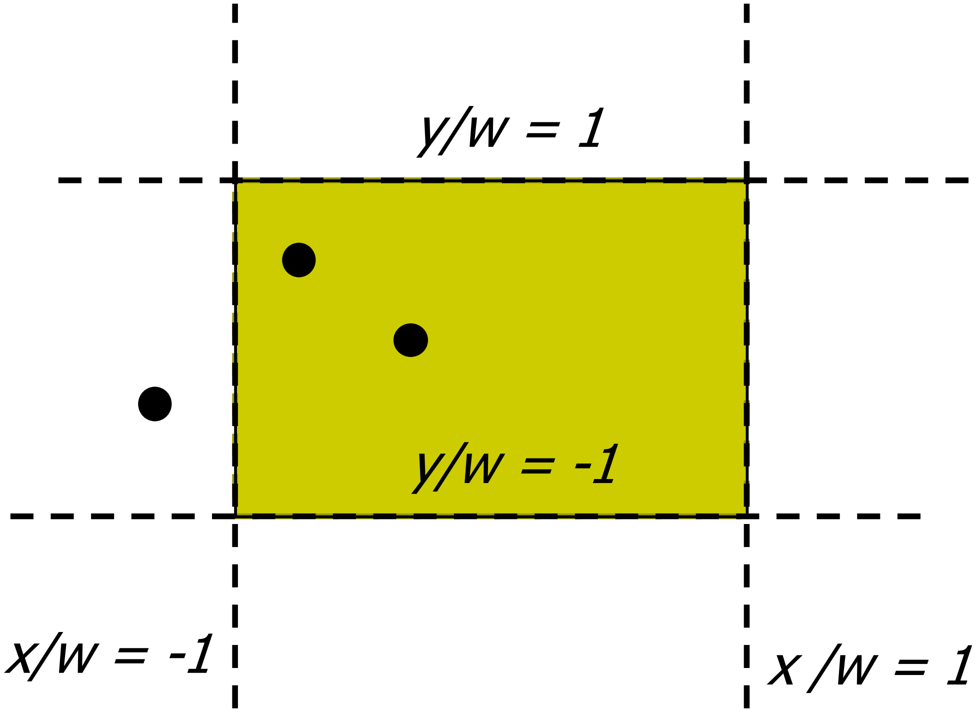
\- **Test (x/w, y/w , z/w)!**

Point (x/w, y/w, z/w) is inside CVV

if (-1 <= **x/w** <= 1)
**and** (-1 <= **y/w** <= 1)
**and** (-1 <= **z/w** <= 1)

else point is outside CVV

# Recall: Modify Inside/Outside Tests Slightly



Our test: $(-1 < \mathbf{x/w} < 1)$

Point (x,y,z,w) inside plane x = 1 if

$x/w < 1$

=> $\mathbf{w - x > 0}$

Point (x,y,z,w) inside plane x = -1 if

$-1 < x/w$

=> $\mathbf{w + x > 0}$

# Recall: Numerical Example: Inside/Outside CVV Test

- Point (x,y,z,w) is
  - inside plane x=-1 **if w+x > 0**
  - inside plane x=1 **if w – x > 0**

- Example Point (0.5, 0.2, 0.7) inside planes (x = -1,1) because - 1 <= 0.5 <= 1
- If w = 10, (0.5, 0.2, 0.7) = (5, 2, 7, 10)
- Can either **divide by w** then test: – 1 <= 5/10 <= 1 **OR**

To test if inside x = - 1, **w + x** = 10 + 5 = 15 > 0

To test if inside x = 1, **w - x** = 10 - 5 = 5 > 0

# Recall: 3D Clipping

- Do same for y, z to form boundary coordinates for 6 planes as:

| **Boundary coordinate (BC)** | **Homogenous coordinate** | **Clip plane** | **Example (5,2,7,10)** |
|---|---|---|---|
| BC0 | w+x | x=-1 | 15 |
| BC1 | w-x | x=1 | 5 |
| BC2 | w+y | y=-1 | 12 |
| BC3 | w-y | y=1 | 8 |
| BC4 | w+z | z=-1 | 17 |
| BC5 | w-z | z=1 | 3 |

- **Consider line that goes from point A to C**
  - **Trivial accept:** 12 BCs (6 for pt. A, 6 for pt. C) > 0
  - **Trivial reject:** Both endpoints outside (-ve) for same plane

# Edges as Parametric Equations

- Implicit form $F(x, y) = 0$

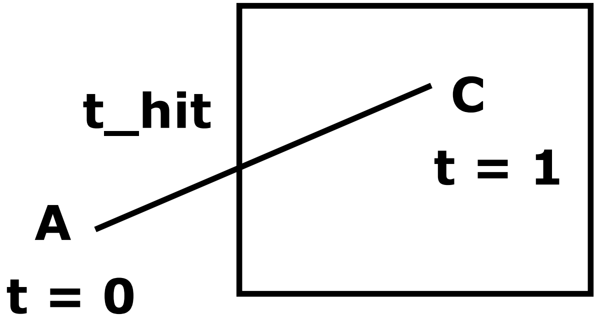
- Parametric forms:
  - points specified based on single parameter value
  - Typical parameter: time *t*

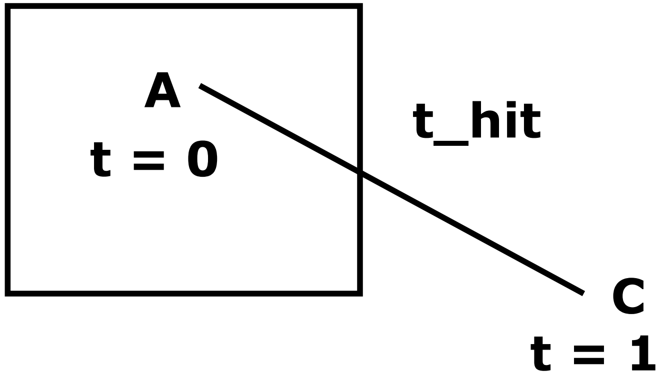
$$P(t) = P_0 + (P_1 - P_0) * t \qquad 0 \le t \le 1$$

- Represent each edge parametrically as A + (C – A)t
  - at time t=0, point at A
  - at time t=1, point at C

# Inside/outside?

- Test A, C against 6 walls **(x=-1,1; y=-1,1; z=-1,1)**
- There is an intersection if BCs have opposite signs. i.e. if either
  - A is outside (< 0), C is inside ( > 0) or
  - A inside (> 0) , C outside (< 0)
- Edge intersects with plane at some t_hit between [0,1]





# Calculating hit time (t_hit)

- How to calculate t_hit?
- Represent an edge t as:

$$Edge(t) = ((Ax + (Cx - Ax)t, (Ay + (Cy - Ay)t, (Az + (Cz - Az)t, (Aw + (Cw - Aw)t)$$

- E.g. If x = 1,

$$\frac{Ax + (Cx - Ax)t}{Aw + (Cw - Aw)t} = 1$$

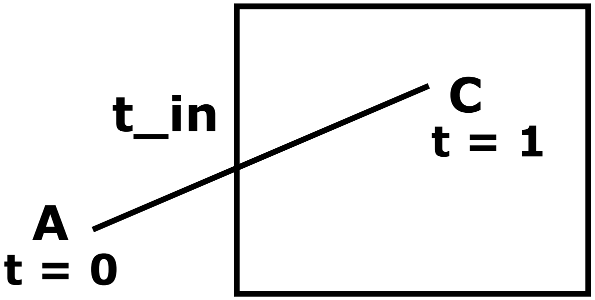
- Solving for t above,

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

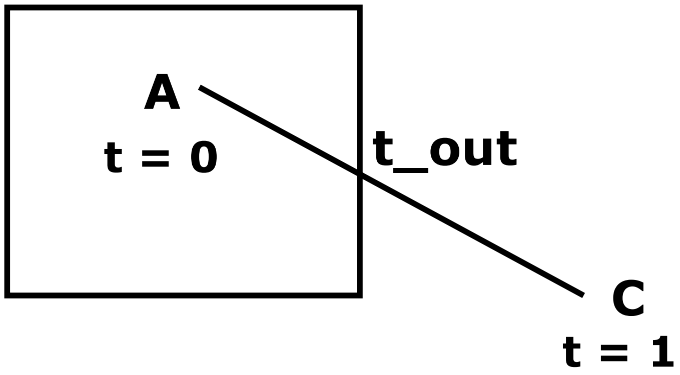
# Inside/outside?

- t_hit can be **“entering (t_in) ”** or **”leaving (t_out)”**
- Define: **“entering”** if A outside, C inside
  - Why? As t goes [0-1], edge goes from outside (at A) to inside (at C)
- Define **“leaving”** if A inside, C outside
  - Why? As t goes [0-1], edge goes from inside (at A) to outside (at C)

**Entering**

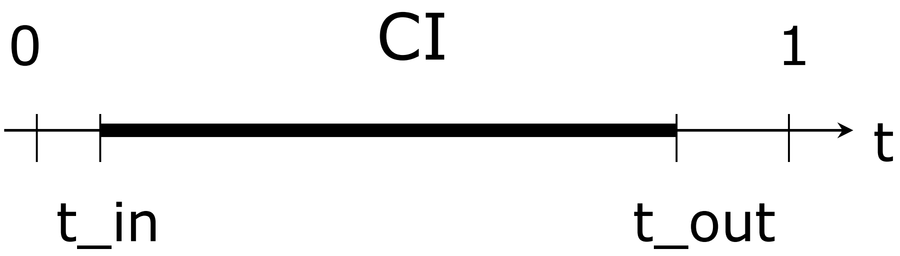


**Leaving**



# Candidate Interval

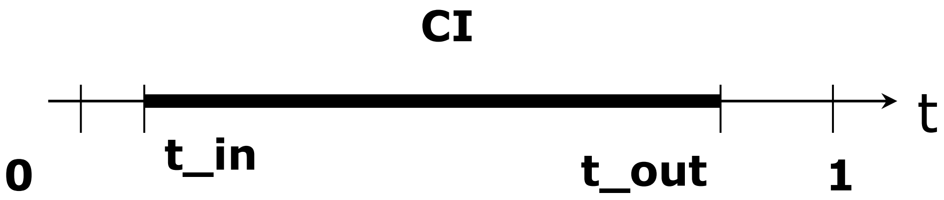
- Candidate Interval (CI): time interval during which edge might still be inside CVV. i.e. CI = t_in to t_out
- Initialize CI to [0,1]
- For each of 6 planes, calculate **t_in** or **t_out**, shrink CI



- Conversely: values of t outside CI = edge is outside CVV

# Shortening Candidate Interval

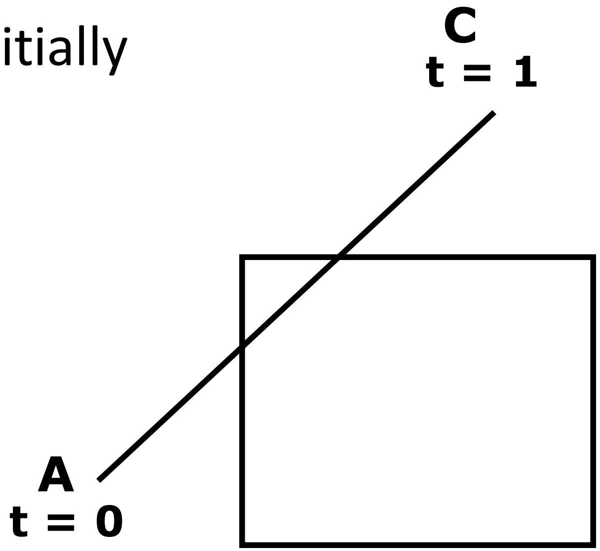
- **Algorithm:**
  - Test for trivial accept/reject (stop if either occurs)
  - Set CI to [0,1]
  - For each of 6 planes:
    - Find hit time t_hit
    - If t_in,   new t_in = max(t_in,t_hit)
    - If t_out,  new t_out = min(t_out, t_hit)
    - If t_in > t_out => exit (no valid intersections)



**Note:** seeking smallest valid CI without t_in crossing t_out

# Example: Chop step by Step against 6 planes

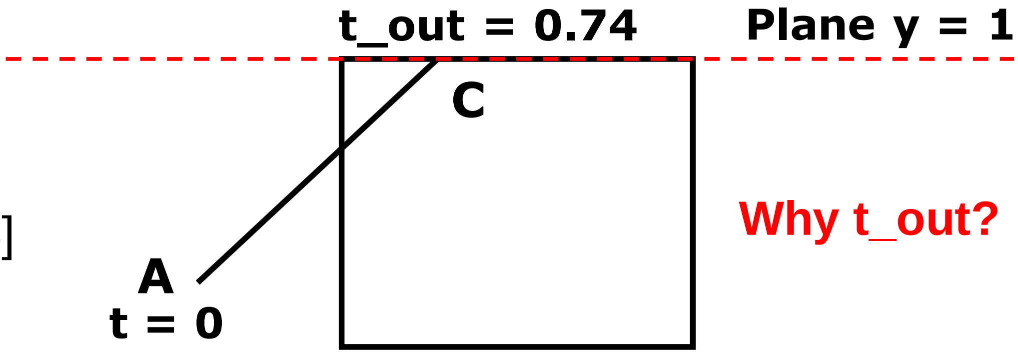
- Initially



t_in = 0,     t_out = 1
Candidate Interval (CI) = [0 to 1]

- Chop against each of 6 planes

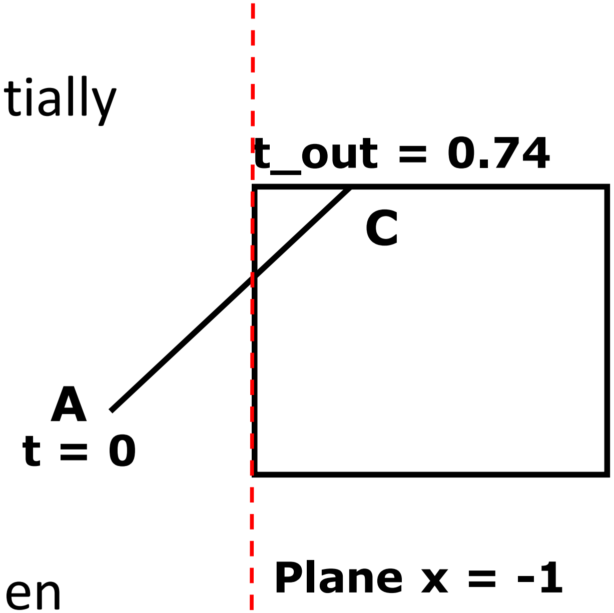
t_in = 0,     t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]



Why t_out?

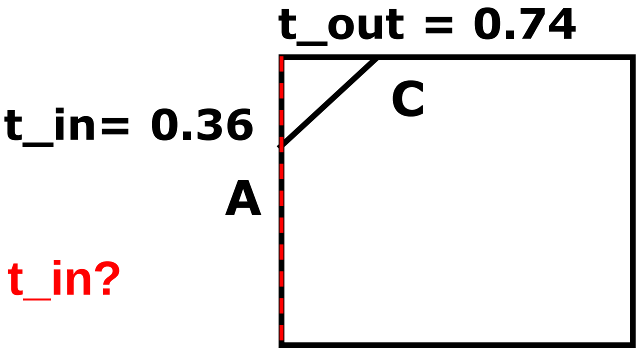
# Example: Chop step by Step against 6 planes

- Initially



t_in = 0, t_out = 0.74
Candidate Interval (CI) = [0 to 0.74]

- Then



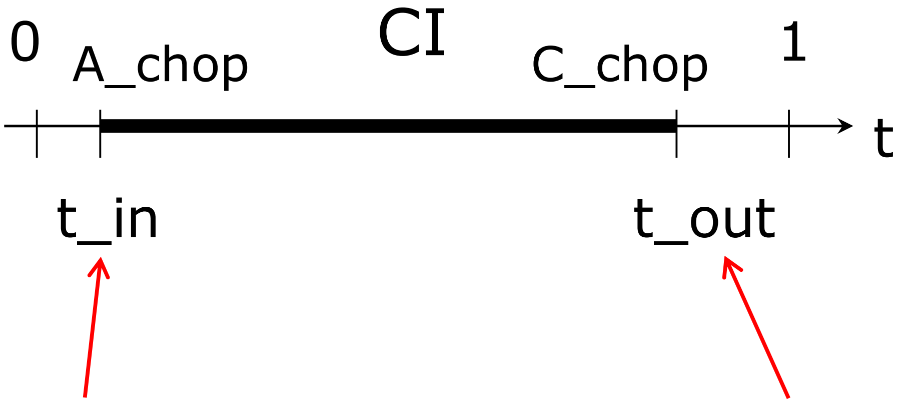
t_in = 0.36, t_out = 0.74
Candidate Interval (CI) CI = [0.36 to 0.74]

**Why t_in?**

# Calculate choppped A and C

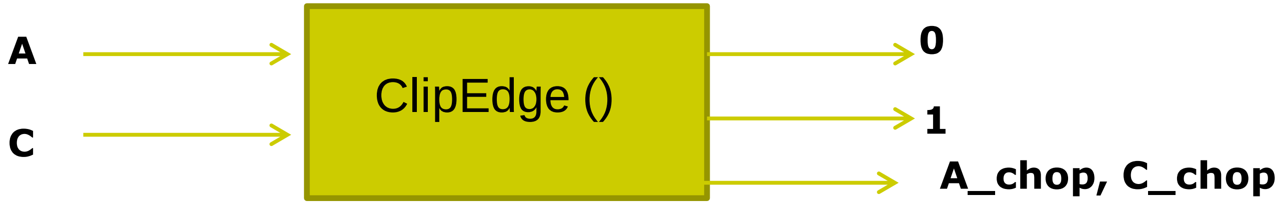
- If valid t_in, t_out, calculate adjusted edge endpoints A, C as

- A_chop = A + t_in ( C – A)  (calculate for Ax,Ay, Az)
- C_chop = A + t_out ( C – A) (calculate for Cx,Cy,Cz)



# 3D Clipping Implementation

- Function clipEdge( )
- Input: two points A and C (in homogenous coordinates)
- Output:
  - 0, if AC lies **completely outside** CVV
  - 1, **completely inside** CVV
  - Returns clipped A and C otherwise
- Calculate 6 BCs (w-x, w+x, etc) for A, 6 for C



# Store BCs as Outcodes

- Use outcodes to track in/out
  - Number walls x = +1, -1; y = +1, -1, and z = +1, -1 as 0 to 5
  - Bit *i* of A’s **outcode = 1** if A is outside ith wall
  - 1 otherwise
- **Example:** outcode for point outside walls 1, 2, 5

| **Wall no.** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **OutCode** | 0 | 1 | 1 | 0 | 0 | 1 |

# Trivial Accept/Reject using Outcodes

- **Trivial accept:** inside (not outside) any walls

| **Wall no.** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **A Outcode** | 0 | 0 | 0 | 0 | 0 | 0 |
| **C OutCode** | 0 | 0 | 0 | 0 | 0 | 0 |

**Logical bitwise test: A | C == 0**

- **Trivial reject:** point outside **same** wall. Example Both A and C outside wall 1

| **Wall no.** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **A Outcode** | 0 | 1 | 0 | 0 | 1 | 0 |
| **C OutCode** | 0 | 1 | 1 | 0 | 0 | 0 |

**Logical bitwise test: A & C != 0**

# 3D Clipping Implementation

- Compute BCs for A,C store as outcodes
- Test A, C outcodes for trivial accept, trivial reject
- If not trivial accept/reject, for each wall:
  - Compute tHit
  - Update t_in, t_out
  - If t_in > t_out, early exit

# 3D Clipping Pseudocode

```
int clipEdge(Point4& A, Point4& C)
{
    double tIn = 0.0, tOut = 1.0, tHit;
    double aBC[6], cBC[6];
    int aOutcode = 0, cOutcode = 0;

    .....find BCs for A and C
    .....form outcodes for A and C

    if((aOutCode & cOutcode) != 0) // trivial reject
        return 0;
    if((aOutCode | cOutcode) == 0) // trivial accept
        return 1;
```

# 3D Clipping Pseudocode

```
for(i=0;i<6;i++)  // clip against each plane
{
    if(cBC[i] < 0)  // C is outside wall i (exit so tOut)
    {
        tHit = aBC[i]/(aBC[i] – cBC[I]);     // calculate tHit
        tOut = MIN(tOut, tHit);
    }
    else if(aBC[i] < 0)  // A is outside wall I (enters so tIn)
    {
        tHit = aBC[i]/(aBC[i] – cBC[i]);     // calculate tHit
        tIn = MAX(tIn, tHit);
    }
    if(tIn > tOut) return 0; // CI is empty: early out
}
```

$$t = \frac{Aw - Ax}{(Aw - Ax) - (Cw - Cx)}$$

# 3D Clipping Pseudocode

```
Point4 tmp;  // stores homogeneous coordinates
If(aOutcode != 0) // A is outside: tIn has changed. Calculate A_chop
{
    tmp.x = A.x + tIn * (C.x – A.x);
    // do same for y, z, and w components
}
If(cOutcode != 0) // C is outside: tOut has changed. Calculate C_chop
{
    C.x = A.x + tOut * (C.x – A.x);
    // do same for y, z and w components
}
A = tmp;
Return 1; // some of the edges lie inside CVV
}
```

# Polygon Clipping

- Not as simple as line segment clipping
    - Clipping a line segment yields at most one line segment
    - Clipping a concave polygon can yield multiple polygons

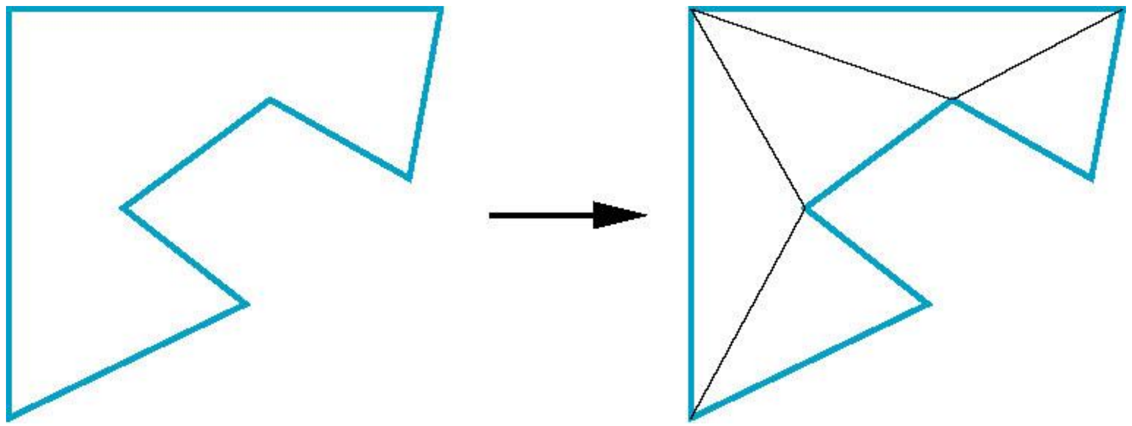
- Clipping a convex polygon can yield at most one other polygon

# Clipping Polygons

- Need more sophisticated algorithms to handle polygons:
  - ***Sutherland-Hodgman:*** clip any given polygon against a convex clip polygon (or window)
  - ***Weiler-Atherton:*** Both clipped polygon and clip polygon (or window) can be concave

# Tessellation and Convexity

- One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)
- Also makes fill easier



# Computer Graphics (CS 4731)
# Lecture 21: Viewport Transformation & Hidden Surface Removal

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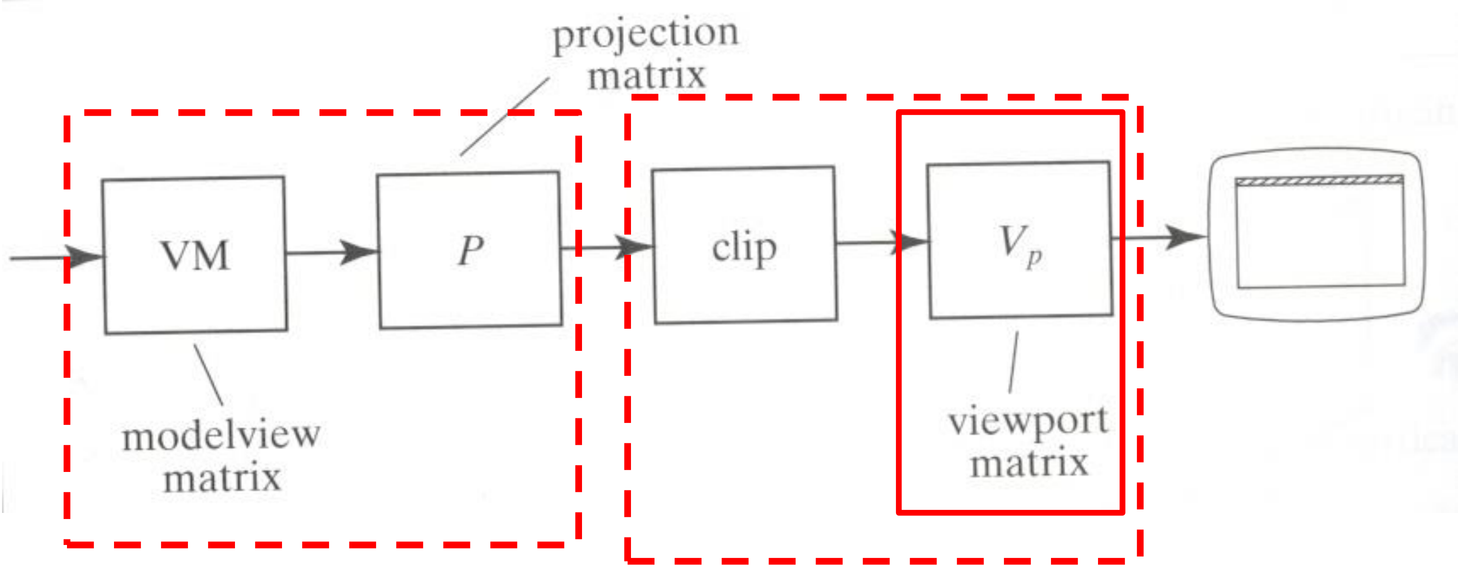
# Viewport Transformation

- After clipping, do viewport transformation

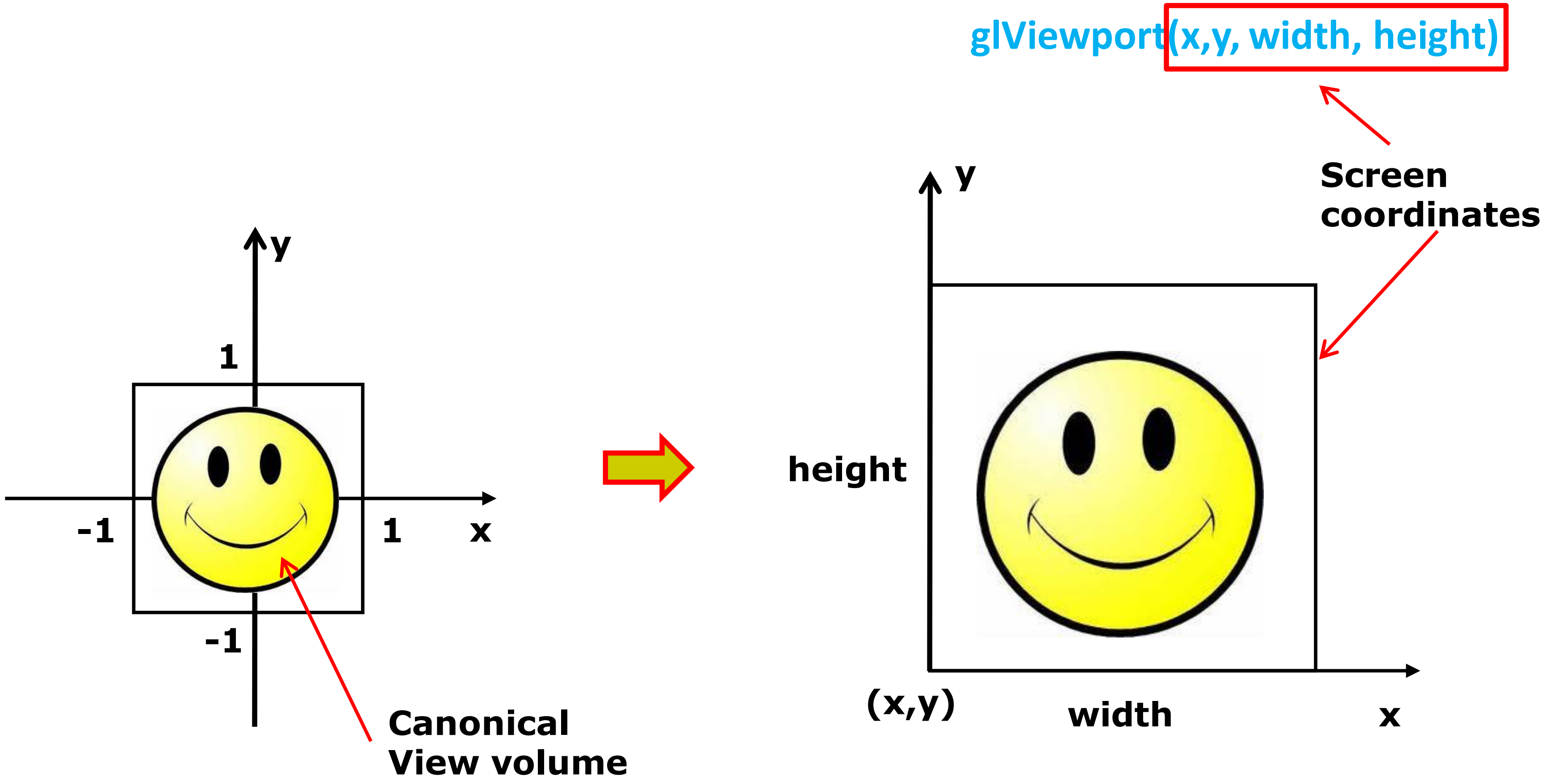


**User implements in Vertex shader**

**Manufacturer implements In hardware**

# Viewport Transformation

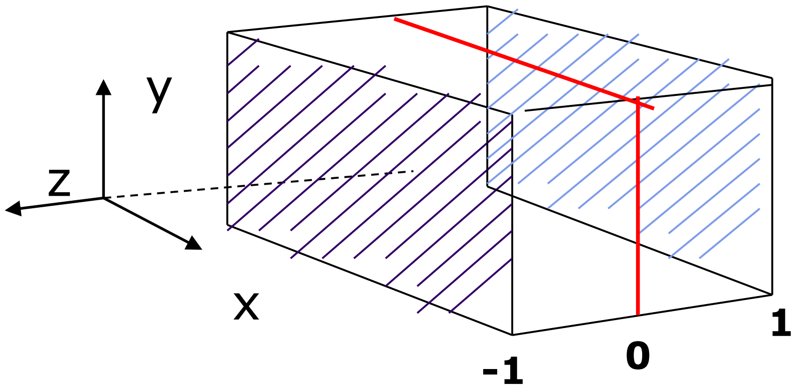
- Maps **CVV (x, y)** -> **screen (x, y)** coordinates



`glViewport(x,y, width, height)`

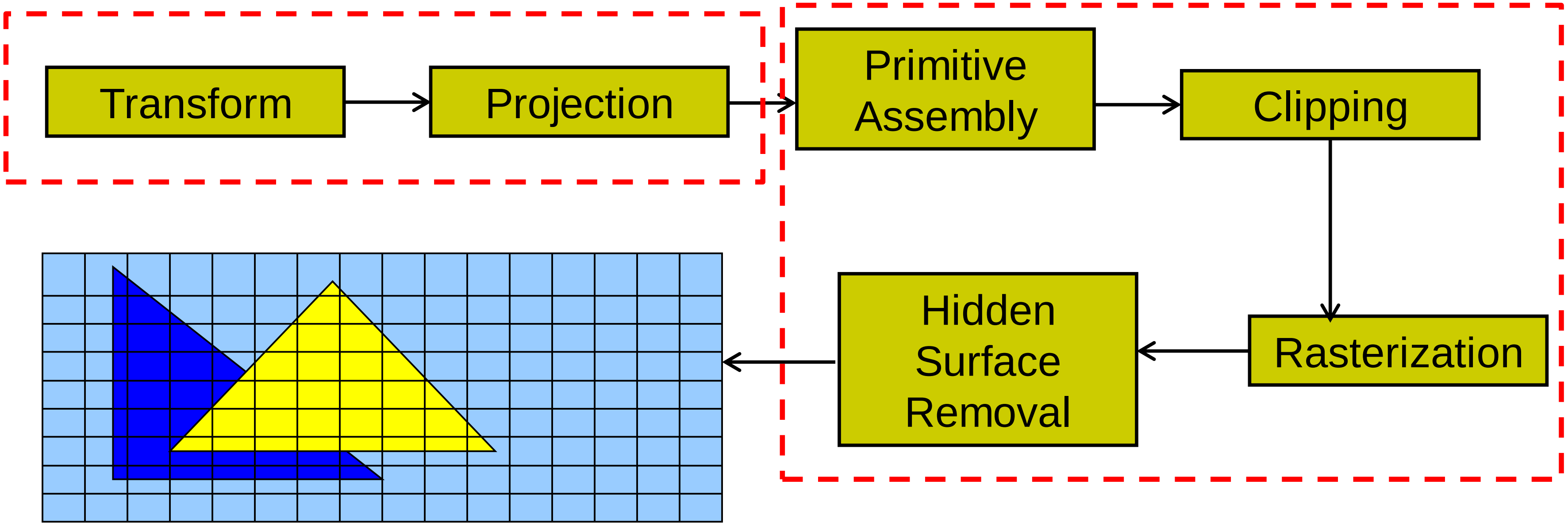
# Viewport Transformation: What of z?

- Also maps CVV z (pseudo-depth) from [-1,1] to [0,1]
- [0,1] pseudo-depth stored in depth buffer,
  - Used for Depth testing (Hidden Surface Removal)



# Recall: OpenGL Stages

- After projection, several stages before objects drawn to screen
- These stages are **NOT** programmable



Vertex shader: programmable

In hardware: **NOT** programmable

Transform

Projection

Primitive Assembly

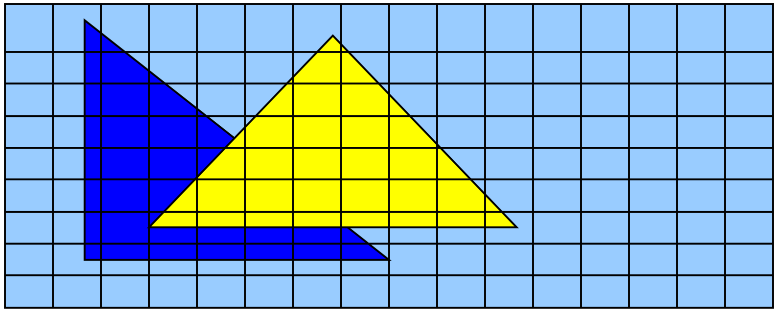
Clipping

Rasterization

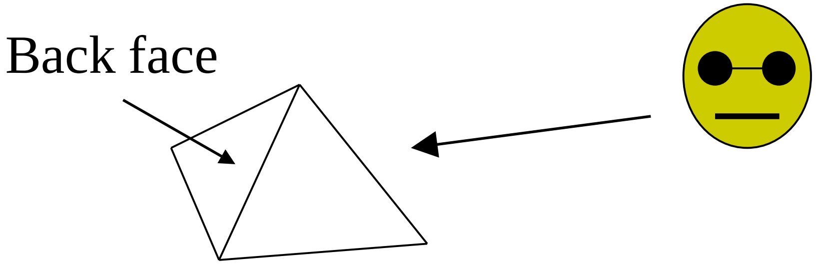
Hidden Surface Removal

# Hidden surface Removal

- Drawing polygonal faces on screen consumes CPU cycles
- User cannot see every surface in scene
- To save time, draw only surfaces we see
- Surfaces we cannot see and elimination methods?



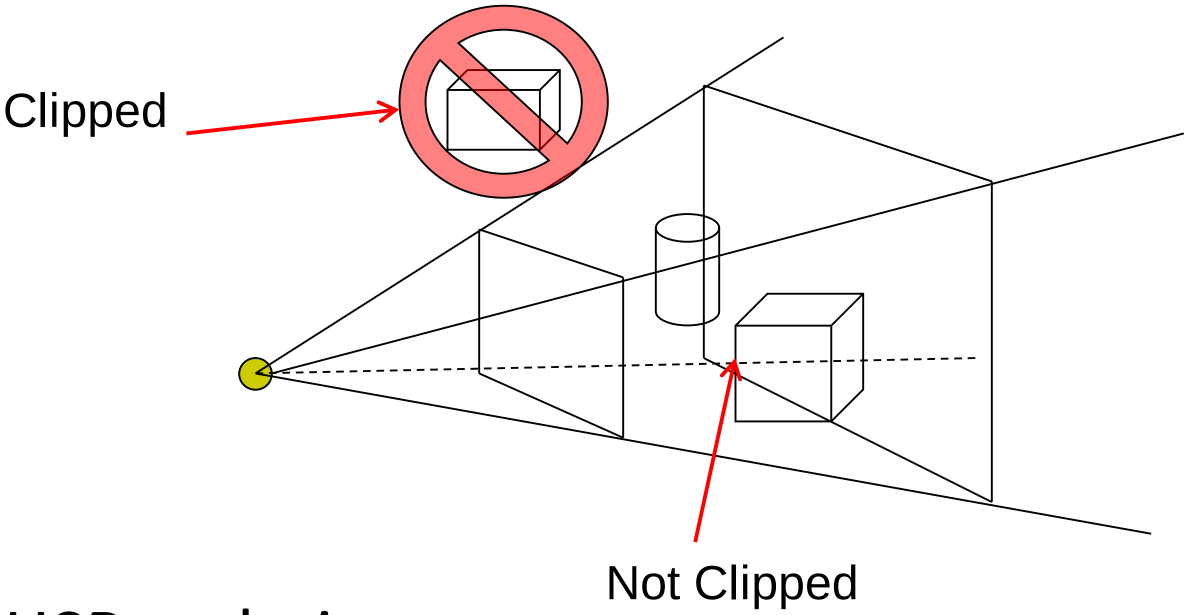
**1. Occluded surfaces:** hidden surface removal (visibility)



**2. Back faces:** back face culling

# Hidden surface Removal

- Surfaces we cannot see and elimination methods:
  - **3. Faces outside view volume:** viewing frustrum culling



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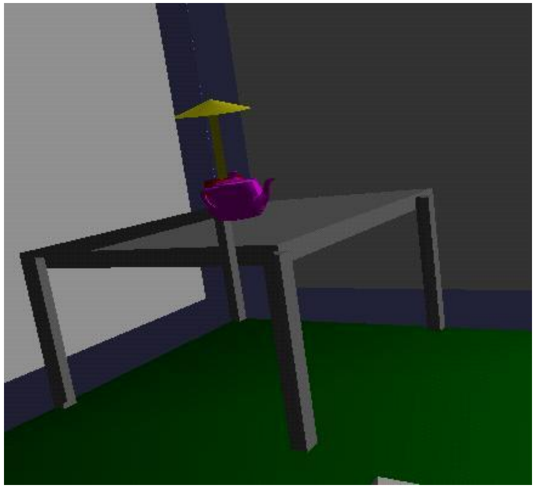
Classes of HSR techniques:

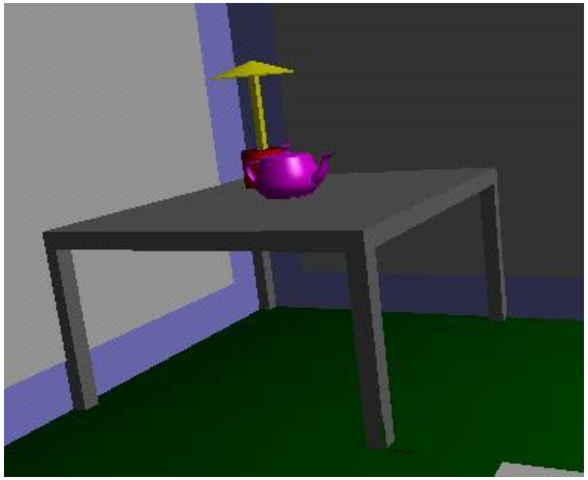
  - **Object space techniques:** applied before rasterization
  - **Image space techniques:** applied after vertices have been rasterized

# Visibility (hidden surface removal)

- Overlapping opaque polygons
- **Correct visibility?** Draw only the closest polygon
  - (remove the other hidden surfaces)



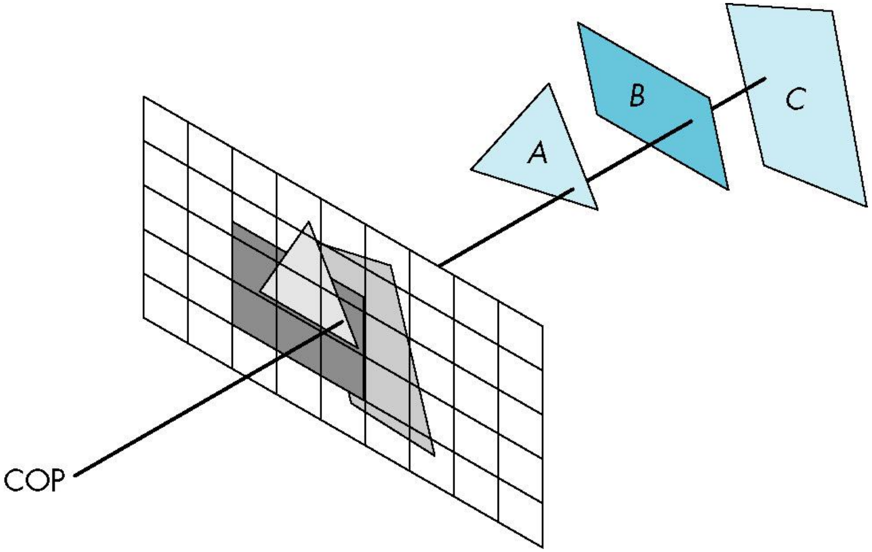
**wrong visibility**



**Correct visibility**

# Image Space Approach

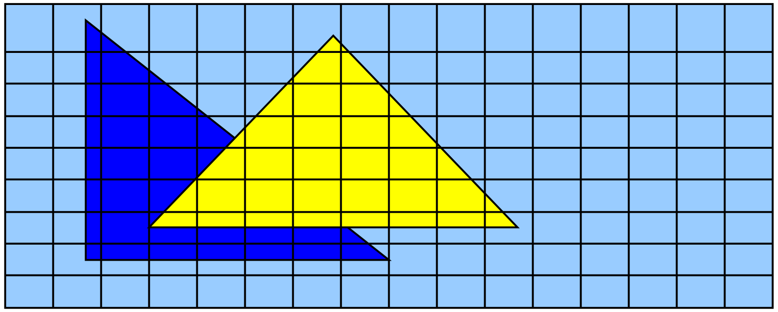
- Start from pixel, work backwards into the scene
- Through each pixel, (nm for an n x m frame buffer) find closest of k polygons
- Complexity O(nmk)
- Examples:
  - Ray tracing
  - z-buffer : OpenGL



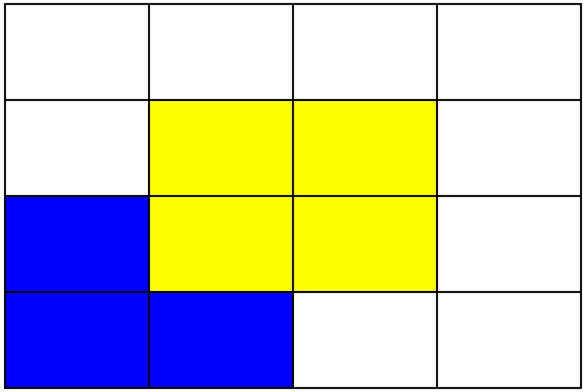
# OpenGL - Image Space Approach

- Paint pixel with color of **closest** object

```
for (each pixel in image) {
    determine the object closest to the pixel
    draw the pixel using the object's color
}
```



# Z buffer Illustration



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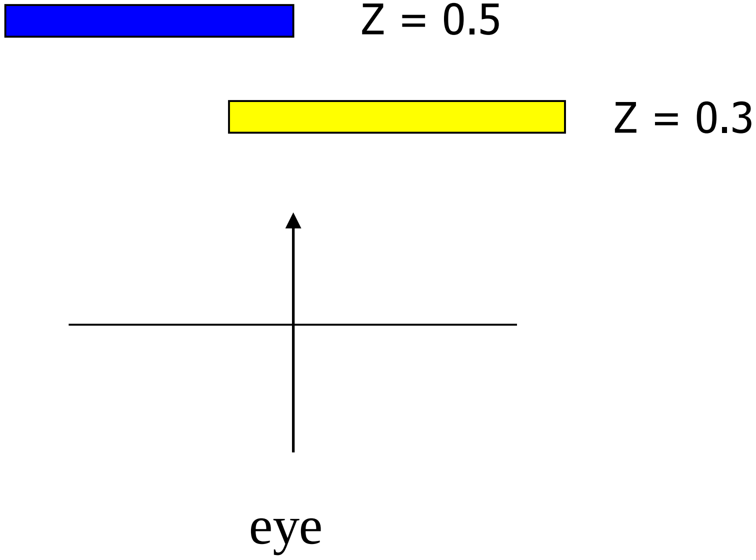
Correct Final image



Top View

# Z buffer Illustration

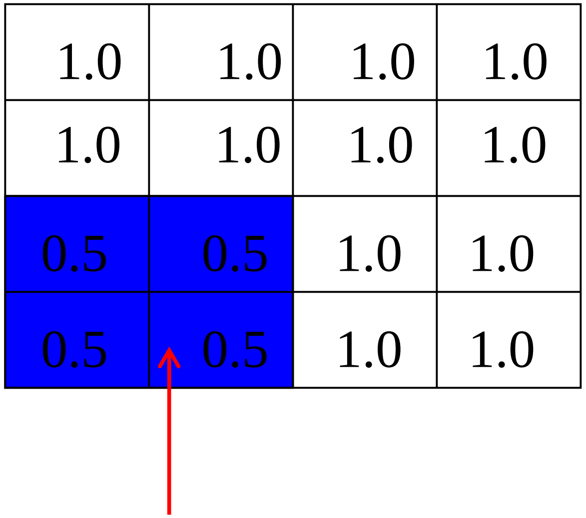
**Step 1:** Initialize the depth buffer

| | | | |
|---|---|---|---|
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |

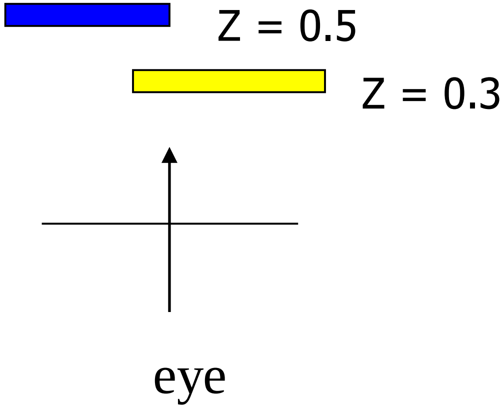
Largest possible
z values is 1.0

# Z buffer Illustration

**Step 2:** Draw blue polygon
(order does not affect final result)



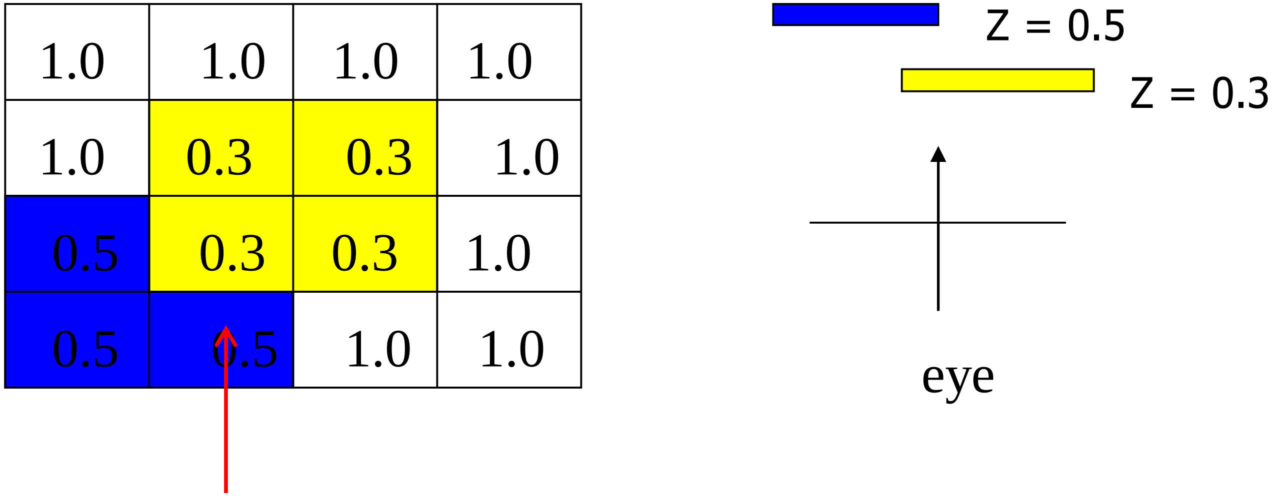
| | | | |
|---|---|---|---|
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 0.5 | 0.5 | 1.0 | 1.0 |
| 0.5 | 0.5 | 1.0 | 1.0 |



1. Determine group of pixels corresponding to blue polygon
2. Figure out z value of blue polygon for each covered pixel (0.5)
3. For each covered pixel, compare polygon z to current depth buffer z
    1. z = 0.5 is less than 1.0 so smallest z so far = 0.5, color = blue

# Z buffer Illustration

**Step 3:** Draw the yellow polygon



| | | | |
|---|---|---|---|
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 0.3 | 0.3 | 1.0 |
| 0.5 | 0.3 | 0.3 | 1.0 |
| 0.5 | 0.5 | 1.0 | 1.0 |

1. Determine group of pixels corresponding to yellow polygon
2. Figure out z value of yellow polygon for each covered pixel (0.3)
3. For each covered pixel, z = 0.3 becomes minimum, color = yellow

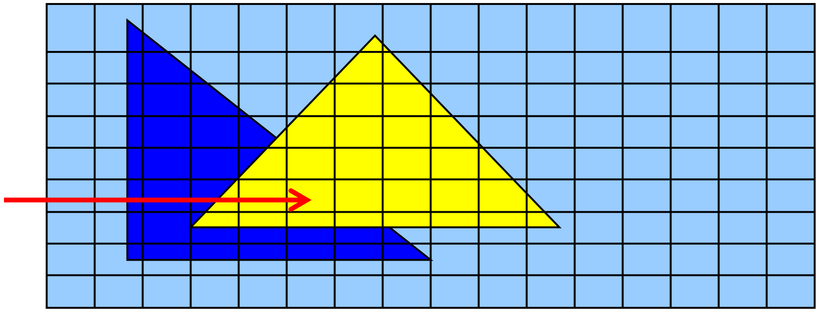
**z-buffer drawback:** wastes resources drawing and redrawing faces

# OpenGL HSR Commands

- 3 main commands to do HSR
- **`glutInitDisplayMode(GLUT_DEPTH | GLUT_RGB)`** instructs openGL to create depth buffer
- **`glEnable(GL_DEPTH_TEST)`** enables depth testing
- **`glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT)`** initializes depth buffer every time we draw a new picture

# Z-buffer Algorithm

- Initialize every pixel's z value to 1.0
- rasterize every polygon
- For each pixel in polygon, find its z value (interpolate)
- Track smallest z value so far through each pixel
- As we rasterize polygon, for each pixel in polygon
  - If polygon's z through this pixel < current min z through pixel
  - Paint pixel with polygon's color



**Find depth (z) of every polygon at each pixel**

# Z (depth) Buffer Algorithm

Depth of polygon being
rasterized at pixel (x, y)

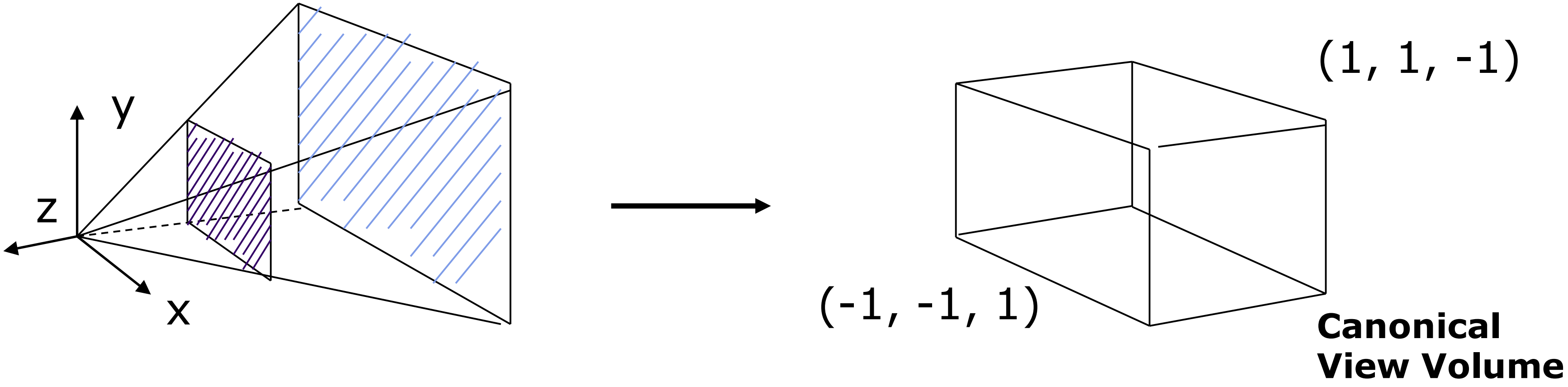
Largest depth seen so far
Through pixel (x, y)

```
For  each polygon  {
   for each pixel (x,y) in polygon area  {
        if  (z_polygon_pixel(x,y) < depth_buffer(x,y) ) {
             depth_buffer(x,y) = z_polygon_pixel(x,y);
             color_buffer(x,y) = polygon color at (x,y)
        }
   }
}
```

**Note: know depths at vertices. Interpolate for interior z_polygon_pixel(x, y) depths**

# Perspective Transformation Issue: Z-Buffer Depth Compression

- **Pseudodepth calculation:** Recall we chose parameters (a and b) to map z from range [near, far] to **pseudodepth** range[-1,1]

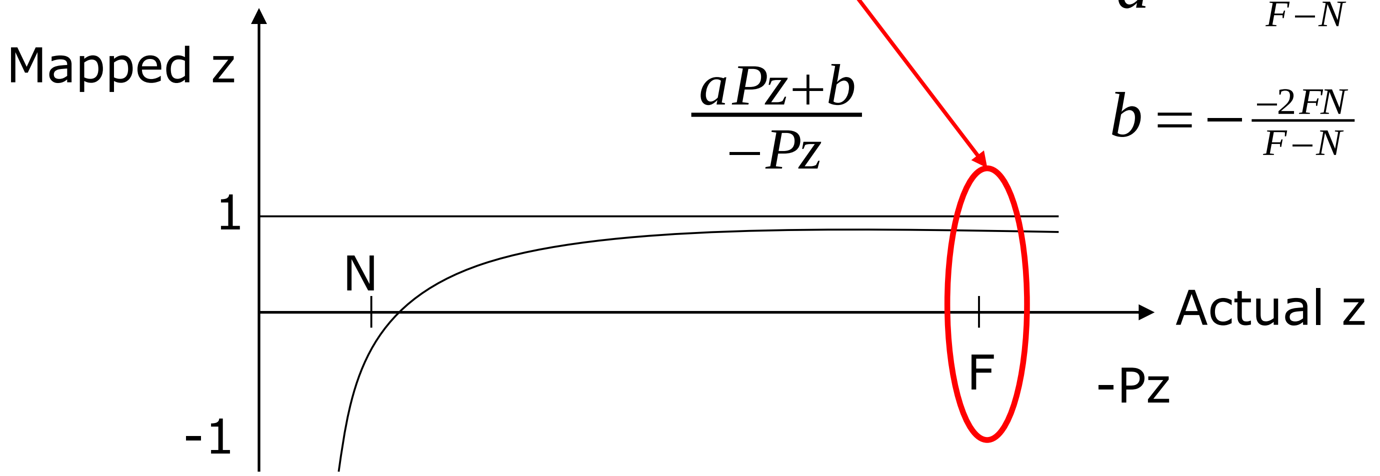


$$\begin{pmatrix} \frac{2N}{x\max - x\min} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2N}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & \boxed{\frac{-(F+N)}{F-N}} & \boxed{\frac{-2FN}{F-N}} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

**These values map z values of original view volume to [-1, 1] range**

# Z-Buffer Depth Compression

- This mapping is almost linear close to eye
- Non-linear further from eye, approaches asymptote
- Also limited number of bits
- Thus, two z values close to far plane may map to same pseudodepth: ***Errors!!***



$$\frac{aPz+b}{-Pz}$$

$$a = -\frac{F+N}{F-N}$$

$$b = -\frac{-2FN}{F-N}$$

# References

- Angel and Shreiner, Interactive Computer Graphics, 6th edition
- Hill and Kelley, Computer Graphics using OpenGL, 3rd edition, Chapter 9