Computer Graphics (CS 4731)
Lecture 9: Implementing Transformations

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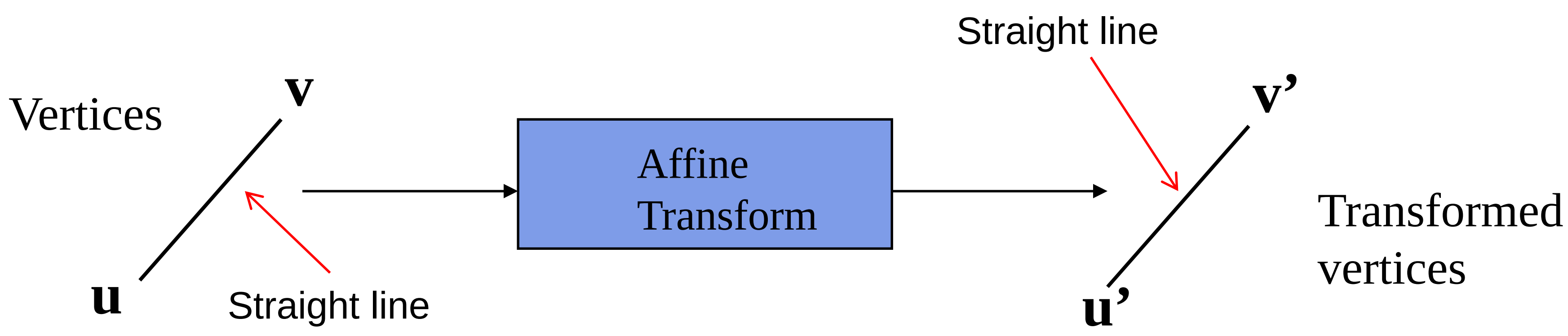
Objectives

- Learn how to implement transformations in OpenGL
  - Rotation
  - Translation
  - Scaling
- Introduce mat.h and vec.h transformations
  - Model-view
  - Projection

Affine Transformations

- Translate, Scale, Rotate, Shearing, are affine transforms
- **Rigid body transformations:** rotation, translation, scaling, shear
- **Line preserving:** important in graphics since we can
    1. Transform endpoints of line segments
    2. Draw line segment between the transformed endpoints



Previously: Transformations in OpenGL

- Pre 3.0 OpenGL had a set of transformation functions
  - glTranslate
  - glRotate( )
  - glScale( )
- Previously, OpenGL would
  - Receive transform commands (Translate, Rotate, Scale)
  - Multiply tranform matrices together and maintain transform matrix stack known as **modelview matrix**

Previously: Modelview Matrix Formed?

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

Specify transforms
In OpenGL Program

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix | glScale Matrix | glTranslate Matrix | Modelview Matrix

OpenGL implementations
(glScale, glTranslate, etc)
in Hardware (Graphics card)

OpenGL multiplies transforms together
To form modelview matrix
Applies final matrix to vertices of objects

Previously: OpenGL Matrices

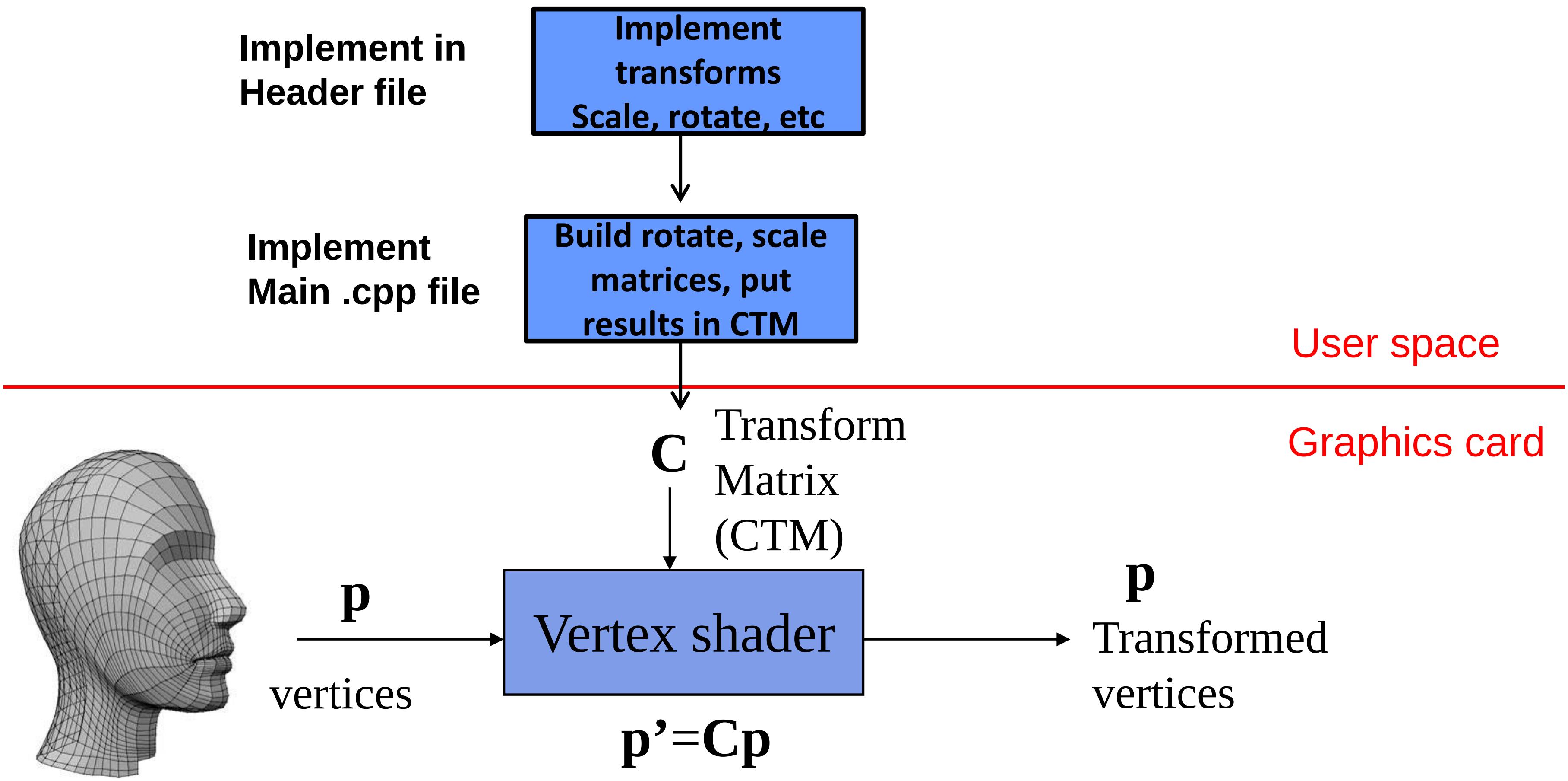
- OpenGL maintained 4 matrix stacks maintained as part of OpenGL state
  - Model-View (**GL_MODELVIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**)
  - Color(**GL_COLOR**)

Now: Transformations in OpenGL

- **From OpenGL 3.0:** No transform commands (scale, rotate, etc), matrices maintained by OpenGL!!
- glTranslate, glScale, glRotate, OpenGL modelview all deprecated!!
- If programmer needs transforms, matrices implement it!
- **Optional:** Programmer ***may*** now choose to maintain transform matrices **or NOT!**

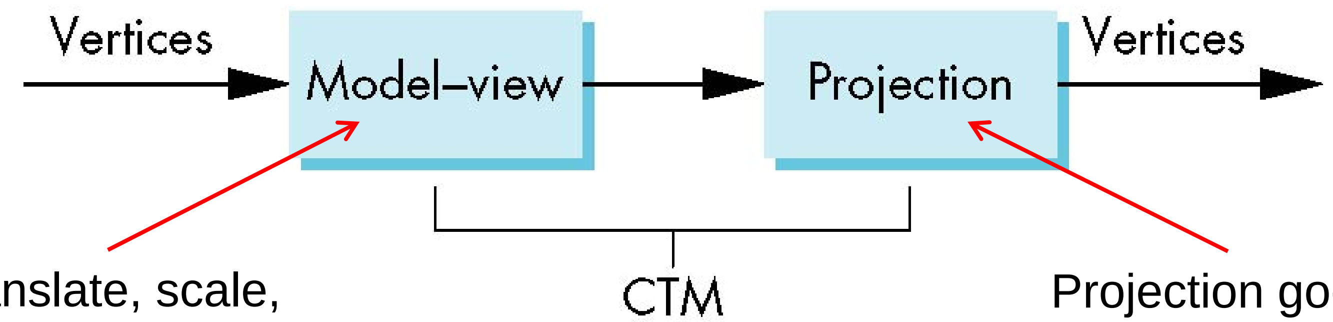
Current Transformation Matrix (CTM)

- Conceptually user can implement a 4 x 4 homogeneous coordinate matrix, the *current transformation matrix* (CTM)
- The **CTM** defined and updated in user program



CTM in OpenGL

- Previously, OpenGL had **model-view** and **projection matrix** in the pipeline that we can concatenate together to form **CTM**
- Essentially, emulate these two matrices using CTM



Translate, scale, rotate go here

Projection goes Here. More later

CTM Functionality

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

1. We need to implement our own transforms

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix — **glScale Matrix** — **glTranslate Matrix** — **Modelview Matrix**

2. Multiply our transforms together to form CTM matrix
3. Apply final matrix to vertices of objects

Implementing Transforms and CTM

- Where to implement transforms and CTM?
- We implement CTM in 3 parts
  1. mat.h (Header file)
  2. Application code (.cpp file)
  3. GLSL functions (vertex and fragment shader)

Implementing Transforms and CTM

- After including mat.h, we can declare mat4 type for CTM

```
class mat4 {
    vec4  _m[4];
    …….…
}
```

- **Transform functions**: Translate, Scale, RotateX (x-roll), etc. E.g.

```
mat4 Translate(const GLfloat x, const GLfloat y, const GLfloat z )
mat4 Scale( const GLfloat x, const GLfloat y, const GLfloat z )
```

- We just have to include mat.h **`(#include "mat.h")`**, use it

Implementing Transforms and CTM

- mat.h (Header files) implements
  - **Matrix Types: mat4** (4x4 matrix), **mat3** (3x3 matrix). E.g

```
mat4 ctm = Translate(3,6,4);
```

$$\text{CTM} \longleftarrow \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{Translation Matrix}$$

- **Note:** mat.h is home-grown (by text)
- Allows easy matrix creation manipulation
- **Uniformity:** Syntax of **mat.h** code resembles GLSL language used in shaders

CTM operations

- The CTM can be altered either by loading a new CTM or by postmutiplication

Load identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Load arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{C\ R}$
Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{C\ S}$

Example: Rotation, Translation, Scaling

Create an identity matrix:

```
mat4 m = Identity();
```

Form Translation and Scale matrices, multiply together

```
mat4 s = Scale( sx, sy, sz)
mat4 t = Transalate(dx, dy, dz);
m = m*s*t;
```

Example: Rotation about a Fixed Point

- We want $\mathbf{C} = \mathbf{T}\,\mathbf{R}\,\mathbf{T}^{-1}$
- Be careful with order. Do operations in following order

$$\mathbf{C} \leftarrow \mathbf{I}$$
$$\mathbf{C} \leftarrow \mathbf{CT}$$
$$\mathbf{C} \leftarrow \mathbf{CR}$$
$$\mathbf{C} \leftarrow \mathbf{CT}^{-1}$$

- Each operation corresponds to one function call in the program.
- **Note:** last operation specified is first executed

Example

- Rotation about z axis by 30 degrees about a fixed point (1.0, 2.0, 3.0)

```
mat 4 m = Identity();
m = Translate(1.0, 2.0, 3.0)*
  Rotate(30.0, 0.0, 0.0, 1.0)*
  Translate(-1.0, -2.0, -3.0);
```

- Remember last matrix specified in program (i.e. translate matrix in example) is first applied

Transformation matrices Formed?

- Converts all transforms (translate, scale, rotate) to 4x4 matrix
- We put 4x4 transform matrix into **CTM**
- Example

```
mat4 m = Identity();
```

mat4 type stores 4x4 matrix
Defined in mat.h

CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

```
mat4 m = Identity();
mat4 t = Translate(3,6,4);
m = m*t;
```

Identity Matrix | **Translation Matrix** | **CTM Matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

- Consider following code snipet

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
```

Identity Matrix **Scaling Matrix** **CTM Matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

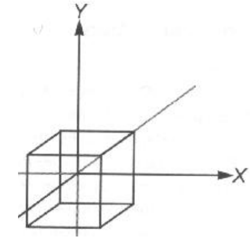
Transformation matrices Formed?

- What of translate, then scale, then ....
- Just multiply them together. Evaluated in ***reverse order***!! E.g:

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
```

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&1\end{pmatrix}\times\begin{pmatrix}1&0&0&3\\0&1&0&6\\0&0&1&4\\0&0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0&3\\0&2&0&12\\0&0&3&12\\0&0&0&1\end{pmatrix}$$

Identity Matrix **Scale Matrix** **Translate Matrix** **Final CTM Matrix**



How are Transform matrices Applied?

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

1. In application:
Load object vertices into points[ ] array -> VBO
Call glDrawArrays

CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. CTM built in application, passed to vertex shader

Application code

Object Vertices

CTM

Vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

Transformed vertex

3. In vertex shader: Each vertex of object (cube) is multiplied by CTM to get transformed vertex position

Passing CTM to Vertex Shader

- Build CTM (modelview) matrix in application program
- Pass matrix to shader

```
void display( ){
    .....
    mat4 m = Identity();
    mat4 s = Scale(1,2,3);
    mat4 t = Translate(3,6,4);
    m = m*s*t;

      // find location of matrix variable "model_view" in shader
      // then pass matrix to shader

      matrix_loc = glGetUniformLocation(program, "model_view");
      glUniformMatrix4fv(matrix_loc, 1, GL_TRUE, m);
        .....
}
```

Build CTM
in application

CTM matrix **m** in application
is same as **model_view** in shader

Implementation: Vertex Shader

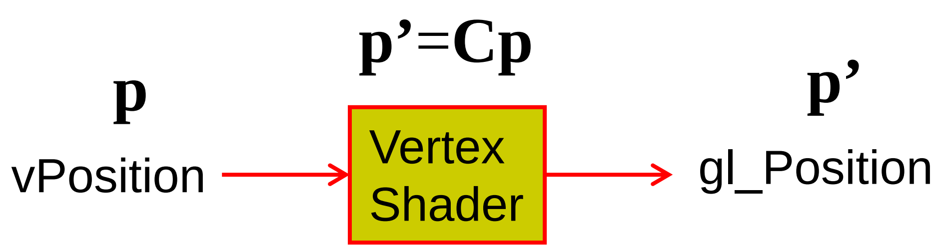
- On glDrawArrays( ), vertex shader invoked with different vPosition per shader
- E.g. If colorcube( ) generates 8 vertices, each vertex shader receives a vertex stored in vPosition
- Shader calculates modified vertex position, stored in gl_Position

```
in vec4 vPosition;
uniform mat4 model_view;

void main( )
{
   gl_Position = model_view*vPosition;
}
```

Transformed vertex **position**

Contains **CTM**

Original vertex **position**



Transformation matrices Formed?

- Example: Vertex (1, 1, 1) is one of 8 vertices of cube

In application

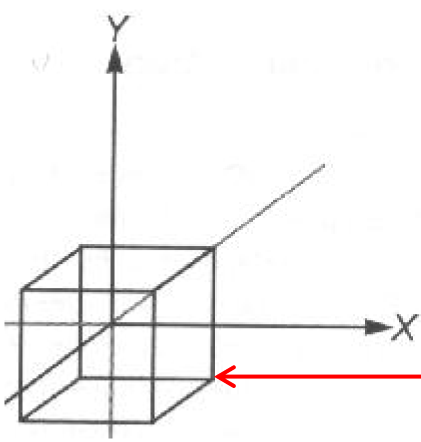
```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
colorcube( );
```

In vertex shader

$$\underset{\text{CTM (m)}}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} * \underset{\mathbf{p}}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}} = \underset{\mathbf{p'}}{\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}}$$

Original vertex (p)

Transformed vertex (p')



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

Transformation matrices Formed?

- **Another example:** Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```
mat4 m = Identity();
mat4 t = Translate(3,6,4);
m = m*t;
colorcube( );
```

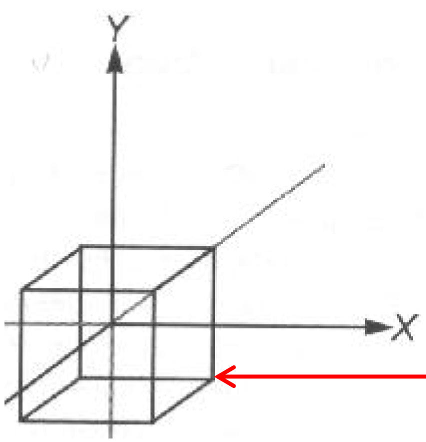
In vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 5 \\ 1 \end{pmatrix}$$

CTM Matrix

Original vertex

Transformed vertex



Each vertex of cube is multiplied by CTM matrix to get translated vertex

Transformation matrices Formed?

- **Another example:** Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

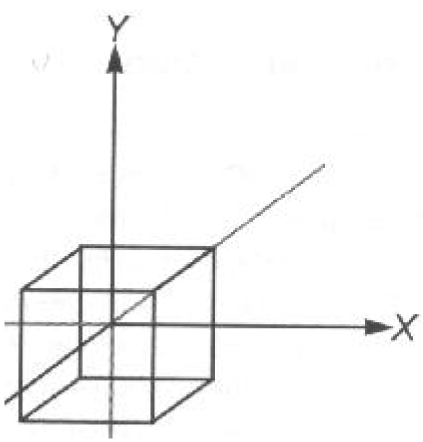
In vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

CTM Matrix

Original vertex

Transformed vertex



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

Arbitrary Matrices

- Can multiply by matrices from transformation commands (Translate, Rotate, Scale) into CTM
- Can also load arbitrary 4x4 matrices into CTM

Load into
CTM Matrix

$$\begin{pmatrix} 1 & 0 & 15 & 3 \\ 0 & 2 & 0 & 12 \\ 34 & 0 & 3 & 12 \\ 0 & 24 & 0 & 1 \end{pmatrix}$$

Matrix Stacks

- CTM is actually not just 1 matrix but a matrix **STACK**
  - Multiple matrices in stack, “current” matrix at top
  - Can save transformation matrices for use later (push, pop)
- E.g: Traversing hierarchical data structures (Ch. 8)
- Pre 3.1 OpenGL also maintained matrix stacks
- Right now just implement 1-level CTM
- Matrix stack later for hierarchical transforms

Reading Back State

- Can also access OpenGL variables (and other parts of the state) by *query* functions

```
glGetIntegerv
glGetFloatv
glGetBooleanv
glGetDoublev
glIsEnabled
```

- Example: to find out maximum number of texture units

```
glGetIntegerv(GL_MAX_TEXTURE_UNITS, &MaxTextureUnits);
```

Using Transformations

- **Example:** use idle function to rotate a cube and mouse function to change direction of rotation
- Start with program that draws cube as before
  - Centered at origin
  - Sides aligned with axes

main.c

```
void main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGB |
       GLUT_DEPTH);
    glutInitWindowSize(500, 500);
    glutCreateWindow("colorcube");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(display);
    glutIdleFunc(spinCube);
    glutMouseFunc(mouse);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```

Calls spinCube continuously
Whenever OpenGL program is idle

Idle and Mouse callbacks

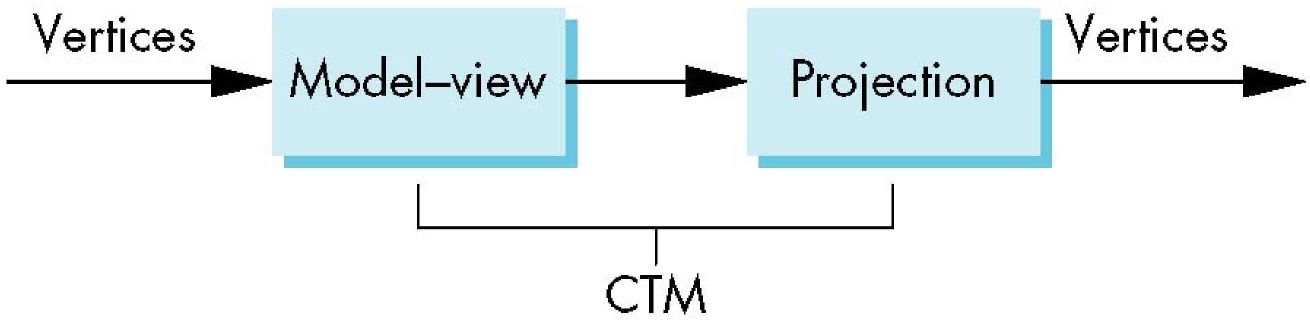
```
void spinCube()
{
   theta[axis] += 2.0;
   if( theta[axis] > 360.0 ) theta[axis] -= 360.0;
   glutPostRedisplay();
}

  void mouse(int button, int state, int x, int y)
  {
     if(button==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
             axis = 0;
     if(button==GLUT_MIDDLE_BUTTON && state == GLUT_DOWN)
             axis = 1;
     if(button==GLUT_RIGHT_BUTTON && state == GLUT_DOWN)
             axis = 2;
  }
```

Display callback

```
void display()
{
   glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
   ctm = RotateX(theta[0])*RotateY(theta[1])
                                             *RotateZ(theta[2]);
    glUniformMatrix4fv(matrix_loc,1,GL_TRUE,ctm);
    glDrawArrays(GL_TRIANGLES, 0, N);
   glutSwapBuffers();
}
```

- Alternatively, we can send rotation angle and axis to vertex shader,
- Let shader form CTM then do rotation
- Inefficient to apply vertex transform data in application (CPU) and send data to GPU to render

Using the Model-view Matrix



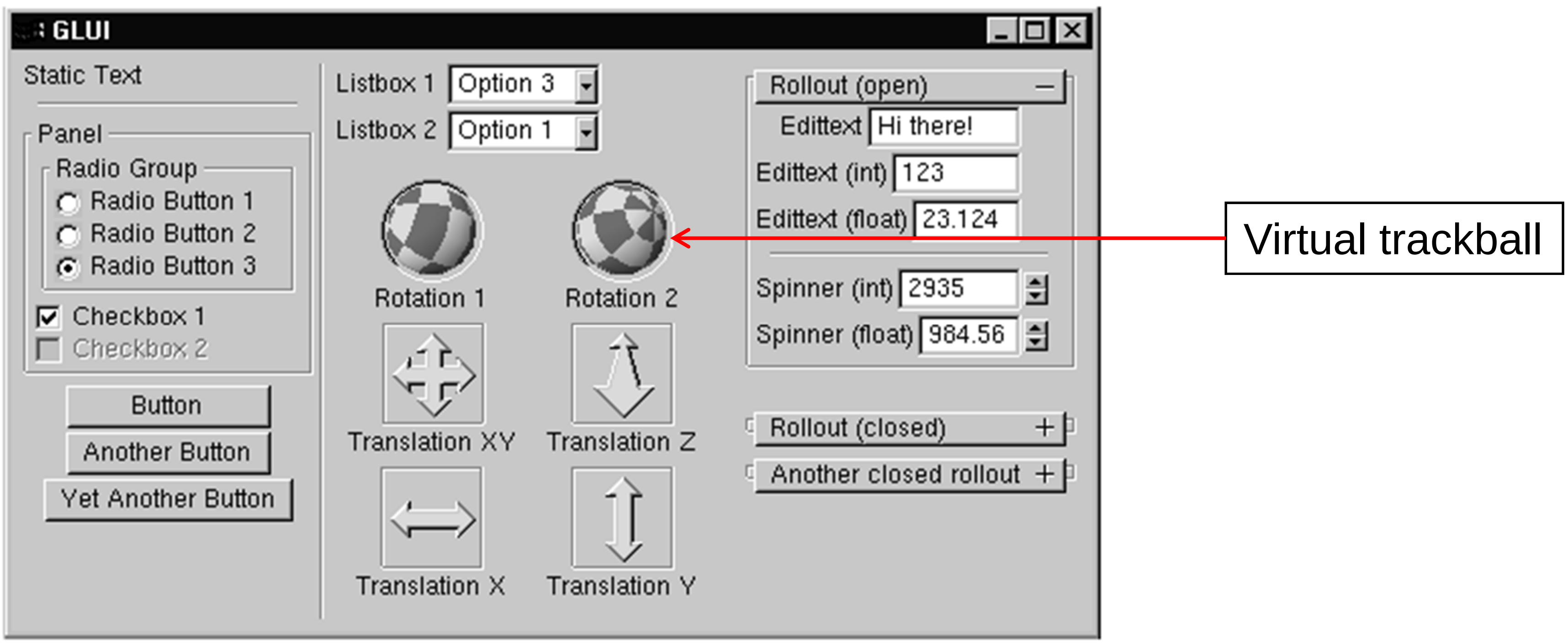
- In OpenGL the model-view matrix used to
  - Transform 3D models (translate, scale, rotate)
  - Position camera (using LookAt function) (next)
- The projection matrix used to define view volume and select a camera lens (later)
- Although these matrices no longer part of OpenGL, good to create them in our applications (as CTM)

3D? Interfaces

- Major interactive graphics problem: how to use 2D devices (e.g. mouse) to control 3D objects
- Some alternatives
  - Virtual trackball
  - 3D input devices such as the spaceball
  - Use areas of the screen
    - Distance from center controls angle, position, scale depending on mouse button depressed

GLUI

- User Interface Library by Paul Rademacher
- Provides sophisticated controls and menus
- Not used in this class/optional



References

- Angel and Shreiner, Chapter 3
- Hill and Kelley, appendix 4