Computer Graphics (CS 543)
Lecture 4 (Part 2): Implementing Transformations

Prof Emmanuel Agu

Computer Science Dept.

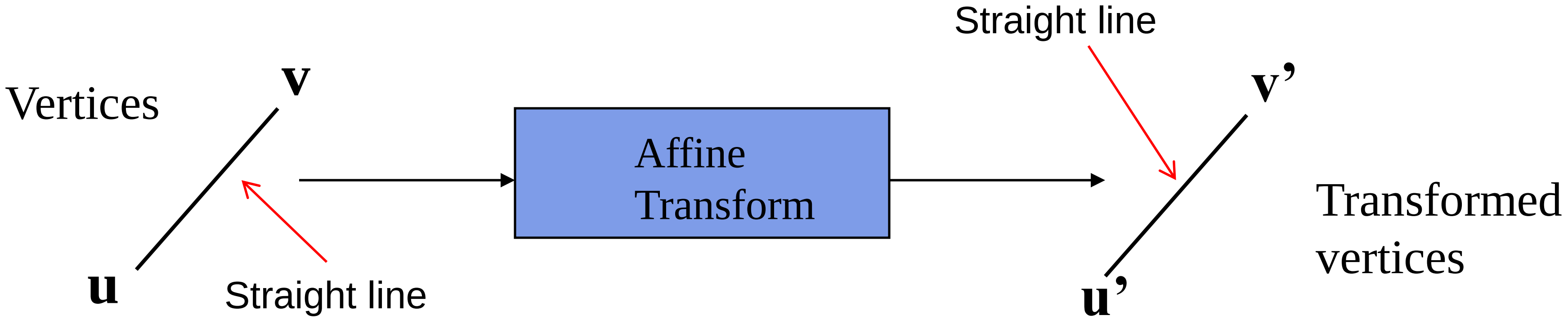
Worcester Polytechnic Institute (WPI)

Objectives

- Learn how to implement transformations in OpenGL
  - Rotation
  - Translation
  - Scaling
- Introduce mat.h and vec.h header files for transformations
  - Model-view
  - Projection

Affine Transformations

- Translate, Scale, Rotate, Shearing, are affine transforms
- **Rigid body transformations:** rotation, translation, scaling, shear
- **Line preserving:** important in graphics since we can
    1. Transform endpoints of line segments
    2. Draw line segment between the transformed endpoints



Previously: Transformations in OpenGL

- Pre 3.0 OpenGL had a set of transformation functions
  - glTranslate
  - glRotate( )
  - glScale( )
- Previously, OpenGL would
  - Receive transform commands (glTranslate, glRotate, glScale)
  - Multiply tranform matrices together and maintain transform matrix stack known as **modelview matrix**

Previously: Modelview Matrix Formed?

```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity();
glScale(1,2,3);
glTranslate(3,6,4);
```

Specify transforms
In OpenGL Program (.cpp file)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix **glScale Matrix** **glTranslate Matrix** **Modelview Matrix**

OpenGL implementations
(glScale, glTranslate, etc)
in Hardware (Graphics card)

OpenGL multiplies transforms together
To form modelview matrix
Applies final matrix to vertices of objects

Previously: OpenGL Matrices

- OpenGL maintained 4 matrix stacks maintained as part of OpenGL state
  - Model-View (**`GL_MODELVIEW`**)
  - Projection (**`GL_PROJECTION`**)
  - Texture (**`GL_TEXTURE`**)
  - Color(**`GL_COLOR`**)

Now: Transformations in OpenGL

- **From OpenGL 3.0:** No transform commands (scale, rotate, etc), matrices maintained by OpenGL!!
- glTranslate, glScale, glRotate, OpenGL modelview matrix all deprecated!!
- If programmer needs transforms, matrices implement it!
- **Optional:** Programmer ***may*** now choose to maintain transform matrices **or NOT!**

Current Transformation Matrix (CTM)

- Conceptually user can implement a 4 x 4 homogeneous coordinate matrix, the *Current Transformation Matrix* (CTM)
- The **CTM** defined and updated in user program

Implement in .h Header file

Implement transforms Scale, rotate, etc

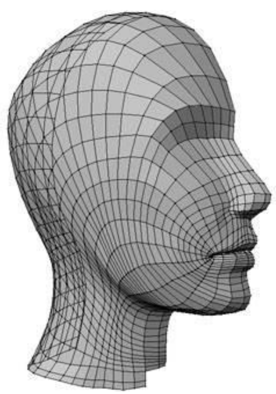
Implement Main .cpp file

Build rotate, scale matrices, put results in CTM

User space

Graphics card

C Transform Matrix (CTM)



p vertices

Vertex shader

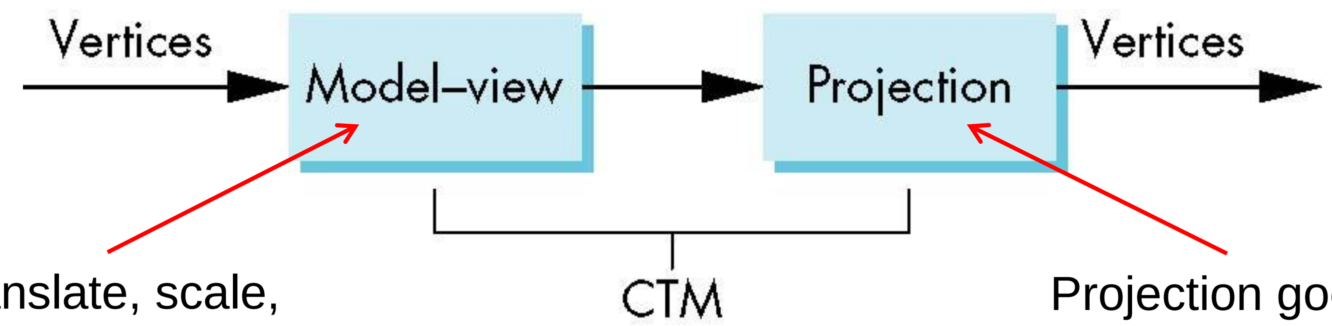
p Transformed vertices

p’=Cp

Homegrown CTM Matrices

- CTM = modelview + projection
  - Model-View (**GL_MODELVIEW**)
  - Projection (**GL_PROJECTION**)
  - Texture (**GL_TEXTURE**)
  - Color(**GL_COLOR**)

CTM



Translate, scale, rotate go here

Projection goes Here. More later

CTM Functionality

```
LoadIdentity();
Scale(1,2,3);
Translate(3,6,4);
```

1. **We need to implement our own transforms (in header file)**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix **Scale Matrix** **Translate Matrix** **CTM Matrix**

2. Multiply our transforms together to form CTM matrix
3. Apply final matrix to vertices of objects

Implementing Transforms and CTM

- Where to implement transforms and CTM?
- We implement CTM in 3 parts
  1. mat.h (Header file)
     - Implementations of translate( ) , scale( ), etc
  2. Application code (.cpp file)
     - Multiply together translate( ) , scale( ) = final CTM matrix
  3. GLSL functions (vertex and fragment shader)
     - Apply final CTM matrix to vertices

Implementing Transforms and CTM

- We just have to include mat.h **(#include "mat.h")**, use it
- **Uniformity: mat.h** syntax resembles GLSL language in shaders
- **Matrix Types: mat4** (4x4 matrix), **mat3** (3x3 matrix).

```
class mat4 {
    vec4  _m[4];
    …….….
}
```

- Can declare CTM as mat4 type

$$\textbf{CTM} \longleftarrow \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Translation Matrix

```
mat4 ctm = Translate(3,6,4);
```

- **mat.h also has transform functions**: Translate, Scale, Rotate, etc.

```
mat4 Translate(const GLfloat x, const GLfloat y, const GLfloat z )
mat4 Scale( const GLfloat x, const GLfloat y, const GLfloat z )
```

CTM operations

- The CTM can be altered either by loading a new CTM or by postmutiplication

Load identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Load arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{C}\,\mathbf{R}$
Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{C}\,\mathbf{S}$

Example: Creating Identity Matrix

- All transforms (translate, scale, rotate) converted to 4x4 matrix
- We put 4x4 transform matrix into **CTM**
- Example: Create an identity matrix

```
mat4 m = Identity();
```

mat4 type stores 4x4 matrix
Defined in mat.h

CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

```
mat4 m = Identity();
mat4 t = Translate(3,6,4);
m = m*t;
```

Identity Matrix **Translation Matrix** **CTM Matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

- Consider following code snipet

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
```

Identity Matrix **Scaling Matrix** **CTM Matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformation matrices Formed?

- What of translate, then scale, then ….
- Just multiply them together. Evaluated in ***reverse order***!! E.g:

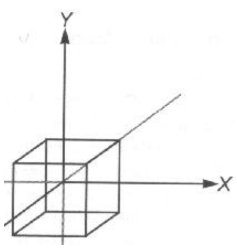
```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
```

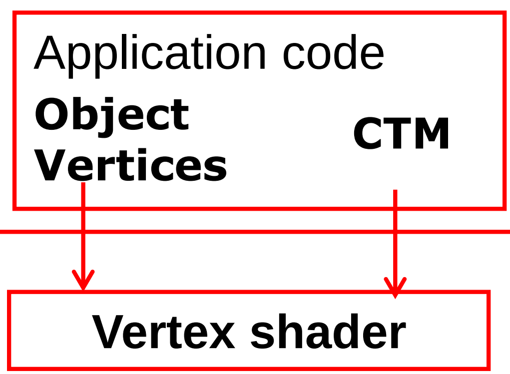
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\textbf{Identity Matrix}} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\textbf{Scale Matrix}} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\textbf{Translate Matrix}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\textbf{Final CTM Matrix}}$$

How are Transform matrices Applied?

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

1. In application:
Load object vertices into points[ ] array -> VBO
Call glDrawArrays





CTM Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. CTM built in application, passed to vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

Transformed vertex

3. In vertex shader: Each vertex of object (cube) is multiplied by CTM to get transformed vertex position

```
gl_Position = model_view*vPosition;
```

Passing CTM to Vertex Shader

- Build CTM (modelview) matrix in application program
- Pass matrix to shader

```
void display( ){
    .....
    mat4 m = Identity();
    mat4 s = Scale(1,2,3);
    mat4 t = Translate(3,6,4);
    m = m*s*t;

    // find location of matrix variable "model_view" in shader
    // then pass matrix to shader

    matrix_loc = glGetUniformLocation(program, "model_view");
    glUniformMatrix4fv(matrix_loc, 1, GL_TRUE, m);
      .....
}
```

Build CTM
in application

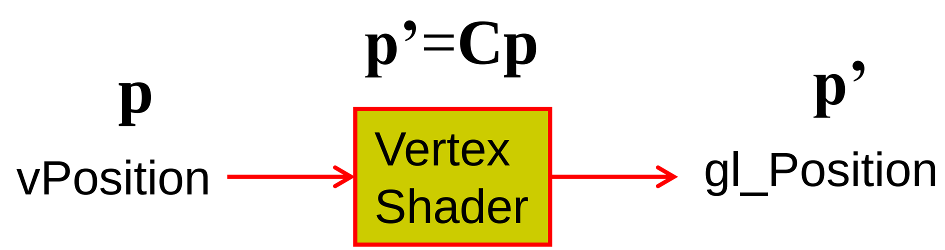
CTM matrix **m** in application
is same as **model_view** in shader

Implementation: Vertex Shader

- On glDrawArrays( ), vertex shader invoked with different vPosition per shader
- E.g. If colorcube( ) generates 8 vertices, each vertex shader receives a vertex stored in vPosition
- Shader calculates modified vertex position, stored in gl_Position

```
in vec4 vPosition;
uniform mat4 model_view;

void main( )
{
    gl_Position = model_view*vPosition;
}
```



Transformed vertex **position**

Contains **CTM**

Original vertex **position**

What Really Happens to Vertex Position Attributes?

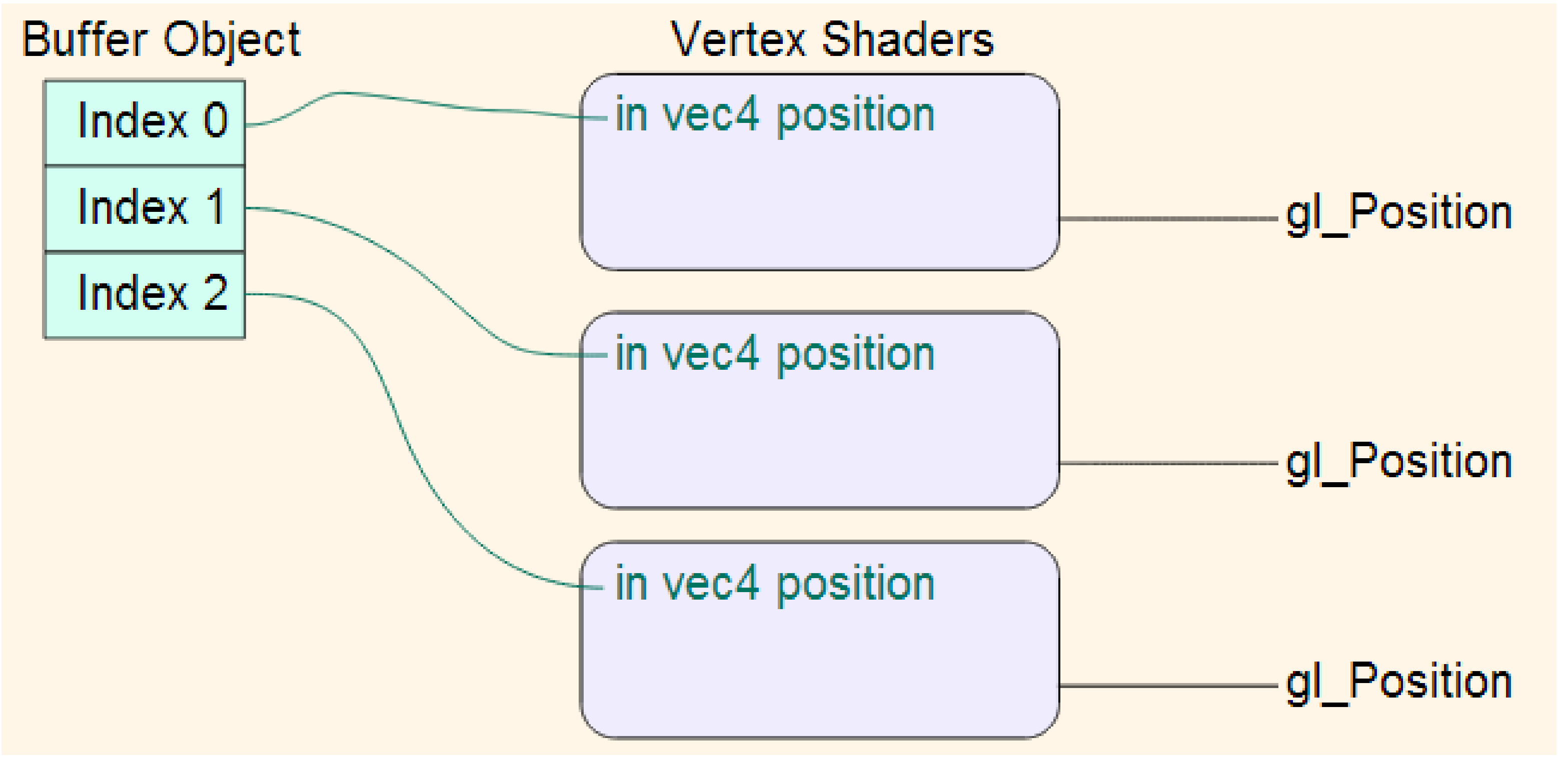


Image credit: Arcsynthesis tutorials

What About Multiple Vertex Attributes?

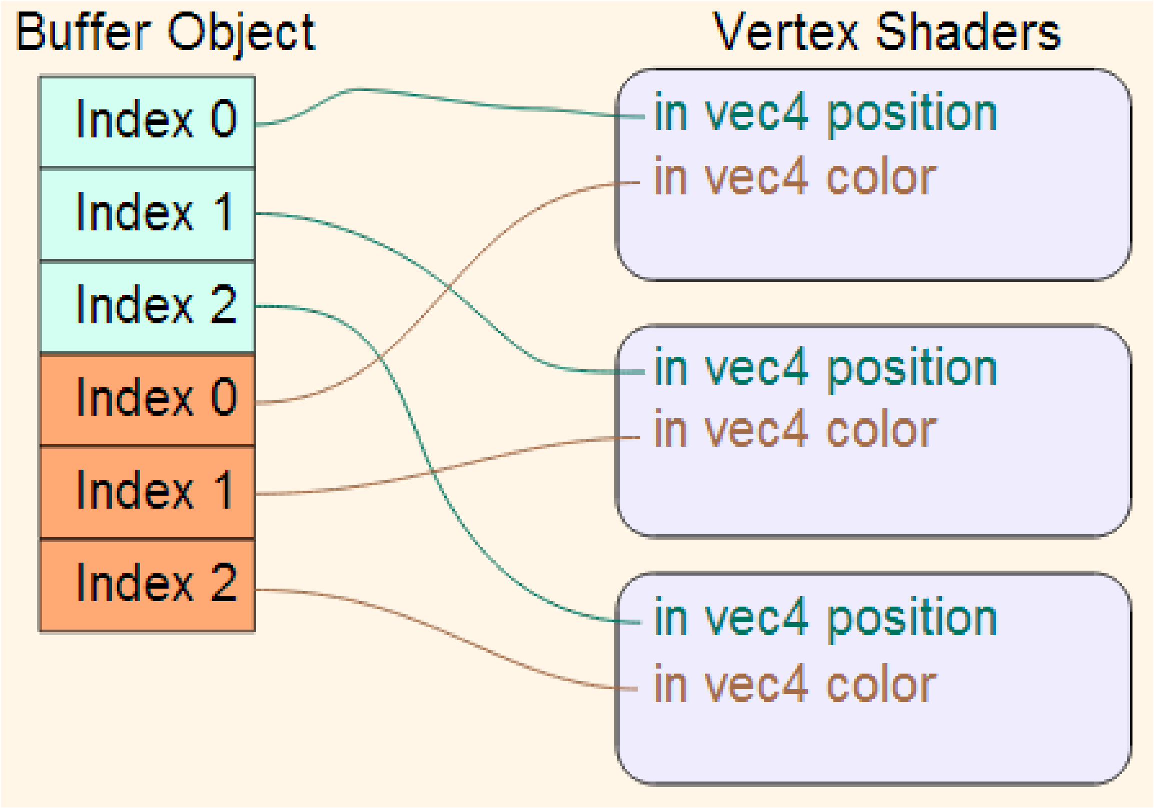


Image credit: Arcsynthesis tutorials

Transformation matrices Formed?

- Example: Vertex (1, 1, 1) is one of 8 vertices of cube

In application

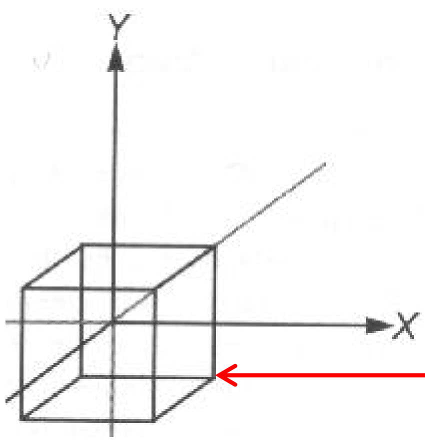
```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
m = m*s;
colorcube( );
```

In vertex shader

$$\begin{array}{ccccc} \textbf{CTM (m)} & & \mathbf{p} & & \mathbf{p'} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & * & \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} & = & \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} \end{array}$$

Original vertex

Transformed vertex



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

Transformation matrices Formed?

- **Another example:** Vertex (1, 1, 1) is one of 8 vertices of cube

In application

```
mat4 m = Identity();
mat4 s = Scale(1,2,3);
mat4 t = Translate(3,6,4);
m = m*s*t;
colorcube( );
```

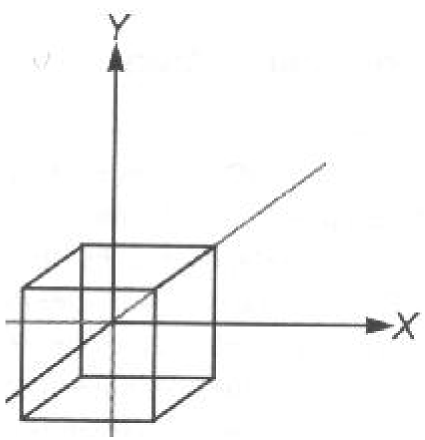
In vertex shader

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 12 \\ 0 & 0 & 3 & 12 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \\ 15 \\ 1 \end{pmatrix}$$

CTM Matrix

Original vertex

Transformed vertex



Each vertex of cube is multiplied by modelview matrix to get scaled vertex position

Arbitrary Matrices

- Can multiply by matrices from transformation commands (Translate, Rotate, Scale) into CTM
- Can also load arbitrary 4x4 matrices into CTM

Load into
CTM Matrix $\Leftarrow$

$$\begin{pmatrix} 1 & 0 & 15 & 3 \\ 0 & 2 & 0 & 12 \\ 34 & 0 & 3 & 12 \\ 0 & 24 & 0 & 1 \end{pmatrix}$$

Example: Rotation about a Fixed Point

- We want $\mathbf{C} = \mathbf{T}\,\mathbf{R}\,\mathbf{T}^{-1}$
- Be careful with order. Do operations in following order

$$\mathbf{C} \leftarrow \mathbf{I}$$
$$\mathbf{C} \leftarrow \mathbf{CT}$$
$$\mathbf{C} \leftarrow \mathbf{CR}$$
$$\mathbf{C} \leftarrow \mathbf{CT}^{-1}$$

- Each operation corresponds to one function call in the program.
- **Note:** last operation specified is first executed

Matrix Stacks

- CTM is actually not just 1 matrix but a matrix **STACK**
  - Multiple matrices in stack, "current" matrix at top
  - Can save transformation matrices for use later (push, pop)
- E.g: Traversing hierarchical data structures (Ch. 8)
- Pre 3.1 OpenGL also maintained matrix stacks
- Right now just implement 1-level CTM
- Matrix stack later for hierarchical transforms

Reading Back State

- Can also access OpenGL variables (and other parts of the state) by *query* functions

```
glGetIntegerv
glGetFloatv
glGetBooleanv
glGetDoublev
glIsEnabled
```

- Example: to find out max. number texture units on GPU

```
glGetIntegerv(GL_MAX_TEXTURE_UNITS, &MaxTextureUnits);
```

Using Transformations

- **Example:** use idle function to rotate a cube and mouse function to change direction of rotation
- Start with program that draws cube as before
  - Centered at origin
  - Sides aligned with axes

Recall: main.c

```
void main(int argc, char **argv)
{
    glutInit(&argc, argv);
    glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGB |
       GLUT_DEPTH);
    glutInitWindowSize(500, 500);
    glutCreateWindow("colorcube");
    glutReshapeFunc(myReshape);
    glutDisplayFunc(display);
    glutIdleFunc(spinCube);
    glutMouseFunc(mouse);
    glEnable(GL_DEPTH_TEST);
    glutMainLoop();
}
```

Calls spinCube continuously
Whenever OpenGL program is idle

Recall: Idle and Mouse callbacks

```
void spinCube()
{
  theta[axis] += 2.0;
  if( theta[axis] > 360.0 ) theta[axis] -= 360.0;
  glutPostRedisplay();
}

void mouse(int button, int state, int x, int y)
{
   if(button==GLUT_LEFT_BUTTON && state == GLUT_DOWN)
           axis = 0;
   if(button==GLUT_MIDDLE_BUTTON && state == GLUT_DOWN)
           axis = 1;
   if(button==GLUT_RIGHT_BUTTON && state == GLUT_DOWN)
           axis = 2;
}
```

Display callback

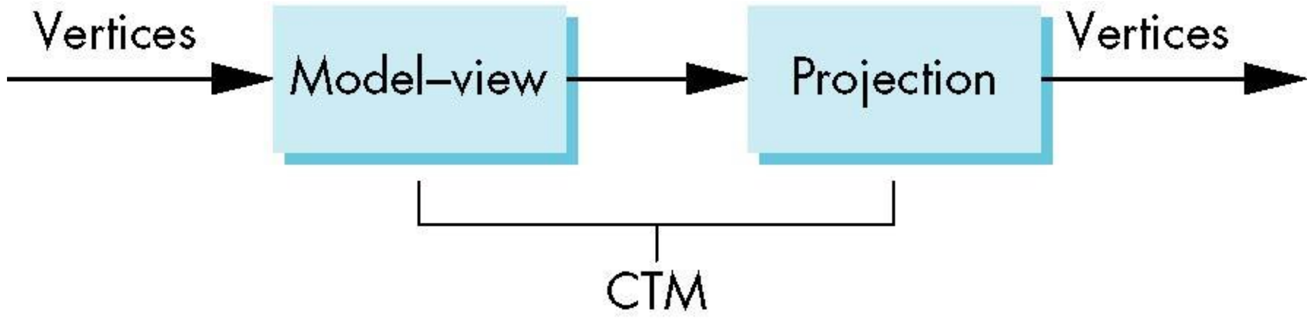
```
void display()
{
   glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
   ctm = RotateX(theta[0])*RotateY(theta[1])
                                  *RotateZ(theta[2]);
    glUniformMatrix4fv(matrix_loc,1,GL_TRUE,ctm);
     glDrawArrays(GL_TRIANGLES, 0, N);
   glutSwapBuffers();
}
```

Update CTM Matrix

Pass CTM to vertex shader

- Alternatively, we can
  - send rotation angle + axis to vertex shader,
  - Let shader form CTM then do rotation
- Inefficient: if mesh has 10,000 vertices each one forms CTM, redundant!!!!

Using the Model-view Matrix



- In OpenGL the model-view matrix used to
  - Transform 3D models (translate, scale, rotate)
  - Position camera (using LookAt function) (next)
- The projection matrix used to define view volume and select a camera lens (later)
- Although these matrices no longer part of OpenGL, good to create them in our applications (as CTM)

References

- Angel and Shreiner, Interactive Computer Graphics (6^{th} edition), Chapter 3