CS/ECE 545 NAME:

Digital Image Processing Fall Semester 2011

Examination #2

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO!

Write your name at the top of this page now.

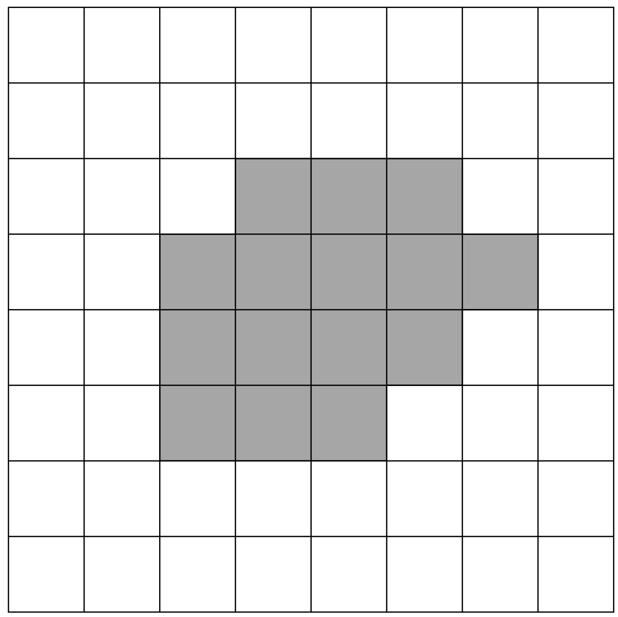
This examination is OPEN BOOK and OPEN NOTES.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work. If you do not understand a question, ask. It will be to your advantage to read the entire examination before beginning to work.

The examination will end exactly 1.5 hours after it begins. Good luck.

Problem 1:	/35
Problem 2:	/40
Problem 3:	/25
Total:	/100

PROBLEM 1 (35 Points)
Part A (15 Points)
Suppose that binary image A is given by



Indicate on the figure the erosion of A by structuring element

$$B_1 = \{(-1,-1),(-1,1),(1,-1),(1,1)\}$$

by marking with an `X` all locations in the erosion. Note that the center of the structuring element is not itself a member of the structuring element.

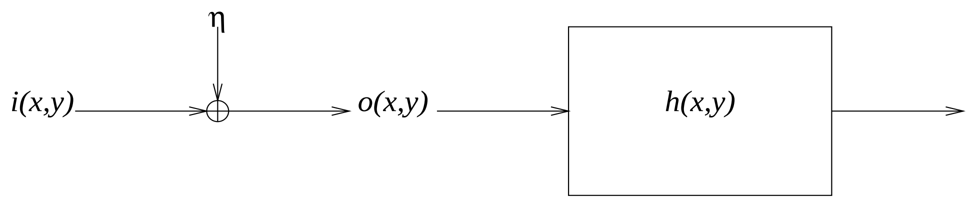
Part B (10 Points)
The same result as in Part A may be obtained by separating B_1 into two *smaller* structuring elements, B_2 and B_3, and eroding by first B_2, and then B_3. That is, $A \ominus B_1 = A \ominus B_2 \ominus B_3$. Find B_2 and B_3. Note that which is which does not matter.

Part C (10 Points)
Structuring element B_4 is formed by adding the center pixel to B_1. Can

$$B_4 = \{(-1,-1),(-1,1),(0,0),(1,-1),(1,1)\}$$

be separated into two *smaller* structuring elements? Why or why not? (Saying that B_4 can be separated into $\{(0,0)\}$ and B_4 does not count!)

PROBLEM 2 (40 Points)
Noise $\eta(x, y)$ is added to image $i(x, y)$ to produce output $o(x, y)$. It is desired to filter o with an optimal (Wiener) filter $h(x, y)$ to produce the best possible estimate of i as shown in the block diagram.

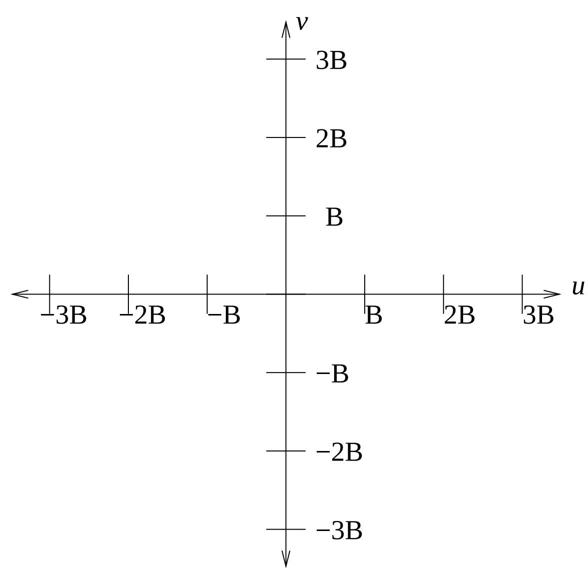


Let the signal be band-limited to frequencies from B to $2B$. That is, the power spectrum of i is

$$I(u, v) = \begin{cases} 0 & \text{if } |u| < B \text{ and } |v| < B; \\ 1 & \text{if } (|u| \geq B \text{ or } |v| \geq B) \text{ and } (|u| < 2B \text{ and } |v| < 2B); \\ 0 & \text{if } |u| \geq 2B \text{ or } |v| \geq 2B. \end{cases}$$

The noise has power spectrum that is 1 for all u, v.

Part A (5 Points)
Graph $I(u, v)$.



Part B (20 Points)
What is the frequency response $\hat{H}(u, v)$ of the optimal filter $\hat{h}$?

Part C (15 Points)
What is the impulse response of the optimal filter $\hat{h}$? Hint: Consider $I(u, v)$ to be the difference of 2 boxes (rectangles). What does that tell you about $\hat{H}(u, v)$ and, in turn, about $\hat{h}(x, y)$?

PROBLEM 3 (25 Points)
The Discrete Cosine Transform is often used in image processing applications, such as image compression, because it uses only real quantities. The 1-D Discrete Cosine Transform is given by

$$C_f(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{(2x+1)u\pi}{2N}\right], \qquad \text{where } \alpha(u) = \begin{cases} 1/\sqrt{N} & u = 0, \\ 2/\sqrt{N} & u = 1, 2, \ldots, N-1. \end{cases}$$

Let $g(x)$ be the reverse of $f(x)$, i.e., $g(x) = f(N-1-x)$. Show that the DCT of g obeys

$$C_g(u) = \begin{cases} C_f(u) & u \text{ even}, \\ -C_f(u) & u \text{ odd}. \end{cases}$$

Hint: There is nothing particularly difficult about this problem if you are careful with the algebra.