Digital Image Processing (CS/ECE 545)
Lecture 4: Filters (Part 2)
& Edges and Contours

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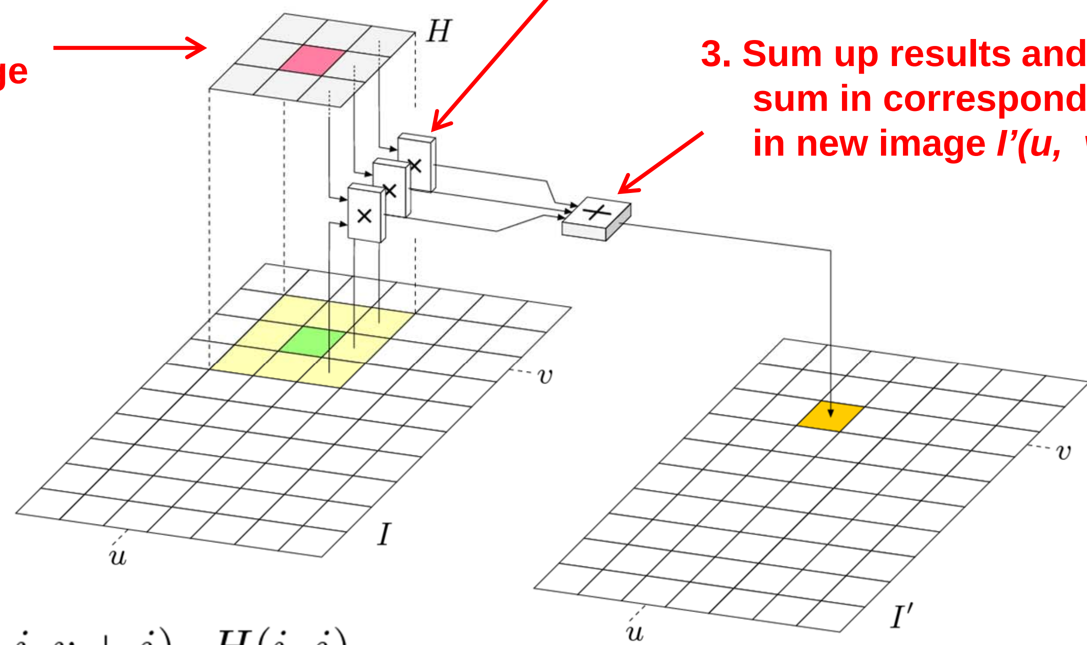
Recall: Applying Linear Filters: Convolution

For each image position *I(u,v)*:

1. Move filter matrix *H* over image such that *H(0,0)* coincides with current image position *(u,v)*

2. Multiply all filter coefficients *H(i,j)* with corresponding pixel *I(u + i, v + j)*

3. Sum up results and store sum in corresponding position in new image *I'(u, v)*



Stated formally:

$$I'(u,v) \leftarrow \sum_{(i,j)\in R_H} I(u+i,v+j)\cdot H(i,j)$$

R_H is set of all pixels Covered by filter.
For 3x3 filter, this is:

$$I'(u,v) \leftarrow \sum_{i=-1}^{i=1} \sum_{j=-1}^{j=1} I(u+i,v+j)\cdot H(i,j)$$

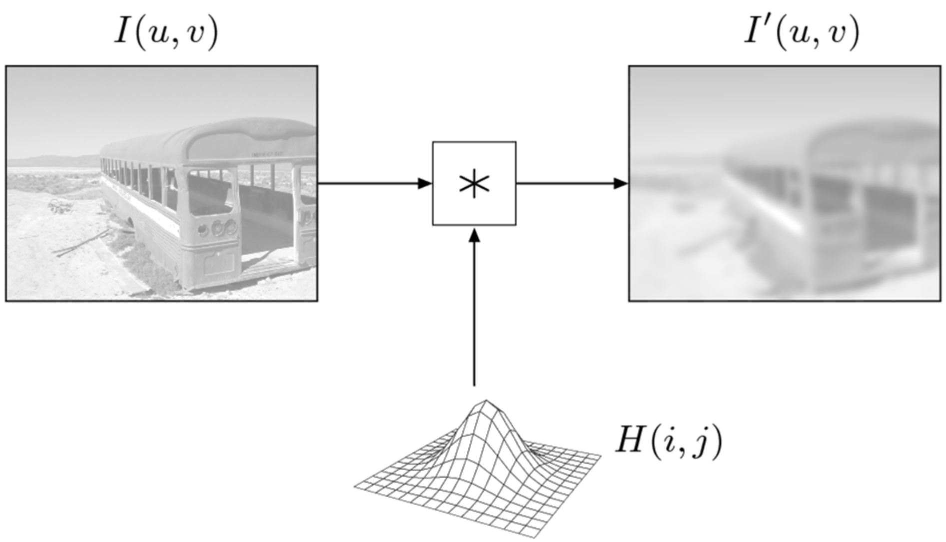
Recall: Mathematical Properties of Convolution

- Applying a filter as described called ***linear convolution***
- For discrete 2D signal, convolution defined as:

$$I'(u,v) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} I(u-i, v-j) \cdot H(i,j)$$

Formal definition:
Sum to ± ∞

$$I' = I * H$$



Recall: Properties of Convolution

- Commutativity

Same result if we convolve image with filter or vice versa

$$I * H = H * I$$

- Linearity

If image multiplied by scalar Result multiplied by same scalar

$$(s \cdot I) * H = I * (s \cdot H) = s \cdot (I * H)$$

$$(I_1 + I_2) * H = (I_1 * H) + (I_2 * H)$$

If 2 images added and convolve result with a kernel *H*, Same result if we each image is convolved individually + added

(notice)

$$(b + I) * H \neq b + (I * H)$$

- Associativity

$$A * (B * C) = (A * B) * C$$

Order of filter application irrelevant Any order, same result

Properties of Convolution

- Separability

$$H = H_1 * H_2 * \ldots * H_n$$

$$\begin{aligned} I * H &= I * (H_1 * H_2 * \ldots * H_n) \\ &= \big(\ldots((I * H_1) * H_2) * \ldots * H_n\big) \end{aligned}$$

- If a kernel H can be separated into multiple smaller kernels

Applying smaller kernels H_1 H_2 … H_N H one by one computationally cheaper than apply 1 large kernel H

$$H = H_1 * H_2 * \ldots * H_n$$

Computationally
More expensive

Computationally
Cheaper

Separability in *x* and *y*

- Sometimes we can separate a kernel into **"vertical"** and **"horizontal"** components
- Consider the kernels

$$H_x = [1\ 1\ 1\ 1\ 1], \quad \text{and} \quad H_y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Then

$$H = H_x * H_y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Complexity of *x/y* Separable Kernels

- What is the number of operations for 3 x 5 kernel *H*

Ans: 15*wh*

$$H = H_x * H_y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- What is the number of operations for H_x followed by H_y?

Ans: 3*wh* + 5*wh* = 8*wh*

$$H_x = [1\ 1\ 1\ 1\ 1], \quad \text{and} \quad H_y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Complexity of *x/y* Separable Kernels

- What is the number of operations for 3 x 5 kernel H

Ans: 15wh

- What is the number of operations for H_x followed by H_y?

Ans: $3wh + 5wh = 8wh$

- What about M x M kernel?

O(M^2) – no separability (M^2wh operations, grows quadratically!)

O(M^2) – with separability ($2Mwh$ operations, grows linearly!)

Gaussian Kernel

- 1D

$$g_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

- 2D

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

Separability of 2D Gaussian

- 2D gaussian is just product of 1D gaussians:

$$
\begin{aligned}
G_\sigma(x,y) &= \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \\
&= \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)\cdot\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{y^2}{2\sigma^2}\right) \\
&= g_\sigma(x)\cdot g_\sigma(y)
\end{aligned}
$$

Separable!

Separability of 2D Gaussian

- Consequently, convolution with a gaussian is separable

$$I * G = I * G_x * G_y$$

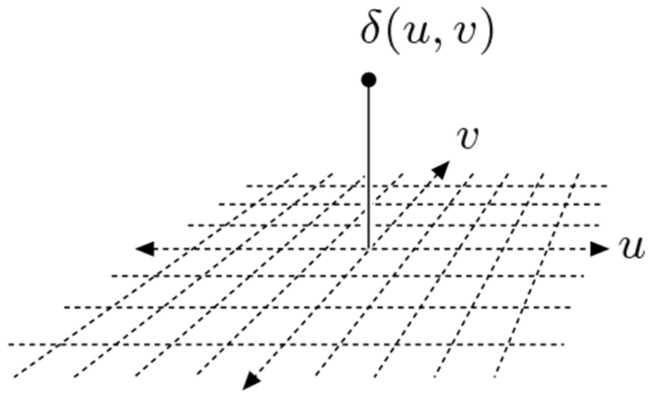
- Where *G* is the 2D discrete gaussian kernel;
- G_x is "horizontal" and G_y is "vertical" 1D discrete Gaussian kernels

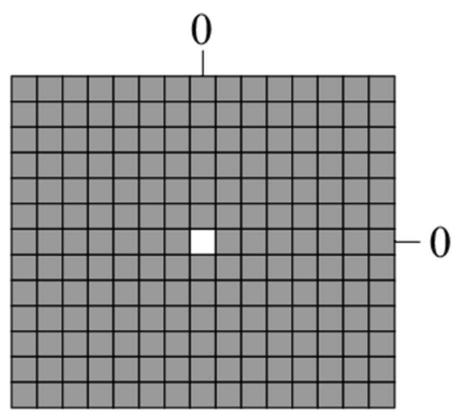
Impulse (or Dirac) Function

- In discrete 2D case, impulse function defined as:

$$\delta(u,v) = \begin{cases} 1 & \text{for } u = v = 0 \\ 0 & \text{otherwise.} \end{cases}$$

- Impulse function on image?
  - A white pixel at origin, on black background

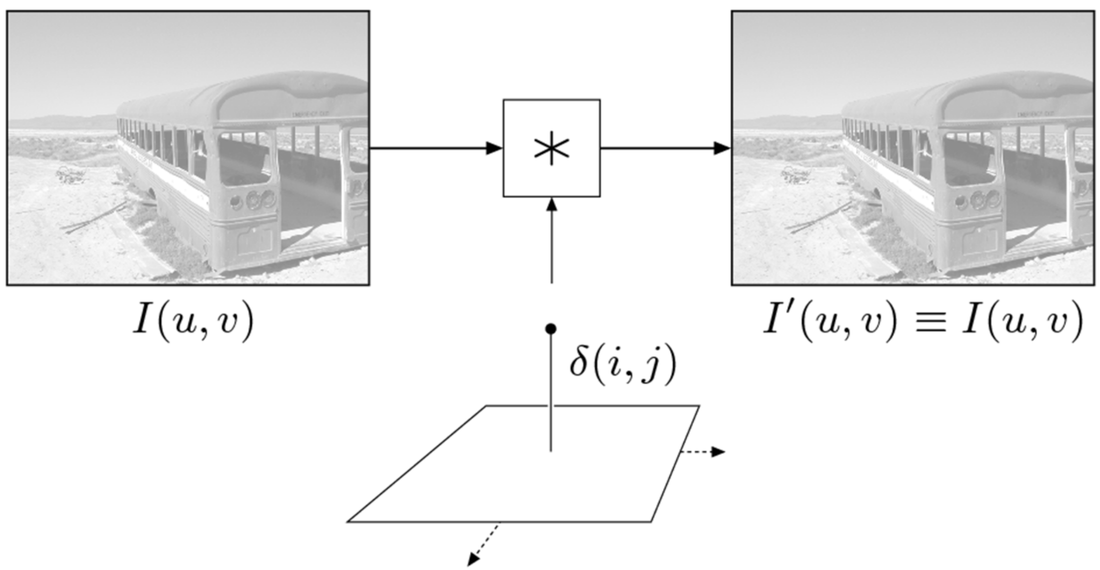




Impulse (or Dirac) Function

- Impulse function neutral under convolution (no effect)
- Convolving an image using impulse function as filter = image

$$I * \delta = I$$

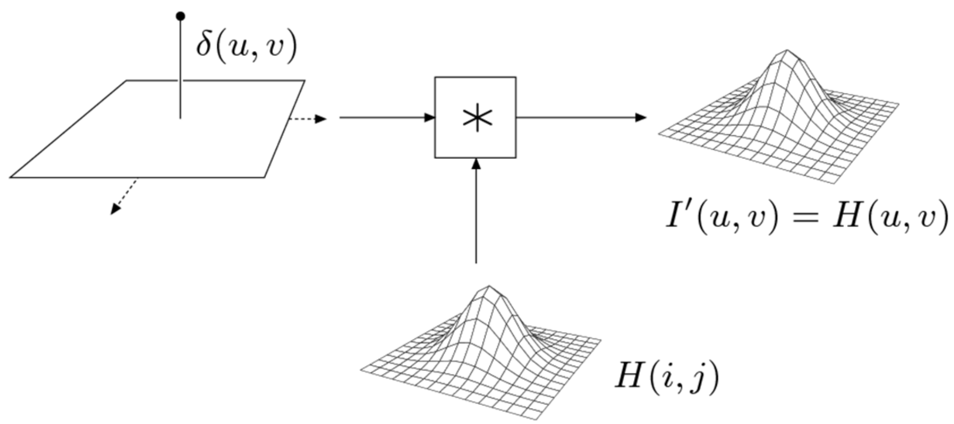


Impulse (or Dirac) Function

- Reverse case? Apply filter *H* to impulse function
- Using fact that convolution is commutative

$$H * \delta = \delta * H = H$$

- Result is the filter *H*



Noise

- While taking picture (during capture), noise may occur
- Noise? Errors, degradations in pixel values
- Examples of causes:
  - Focus blurring
  - Blurring due to camera motion
- Additive model for noise: $H * I + \textcolor{red}{Noise}$
- Removing noise called **Image Restoration**
- Image restoration can be done in:
  - Spatial domain, or
  - Frequency domain

Types of Noise

- **Type of noise determines best types of filters for removing it!!**
- **Salt and pepper noise:** Randomly scattered black + white pixels
- Also called **impulse noise, shot noise or binary noise**
- Caused by sudden sharp disturbance

(a) Original image

(b) With added salt & pepper noise

Courtesy Allasdair McAndrews

Types of Noise

- **Gaussian Noise:** idealized form of white noise *added to* image, normally distributed $I + Noise$
- **Speckle Noise:** pixel values *multiplied* by random noise

$$I(1 + Noise)$$

(a) Gaussian noise

(b) Speckle noise

Courtesy Allasdair McAndrews

Types of Noise

- **Periodic Noise:** caused by disturbances of a periodic nature

- Salt and pepper, gaussian and speckle noise can be cleaned using spatial filters

- Periodic noise can be cleaned using frequency domain filtering (later)

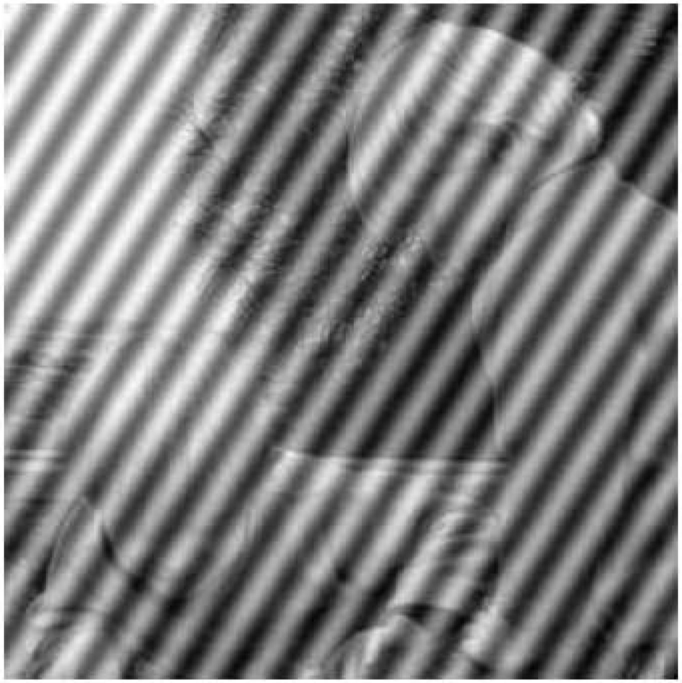
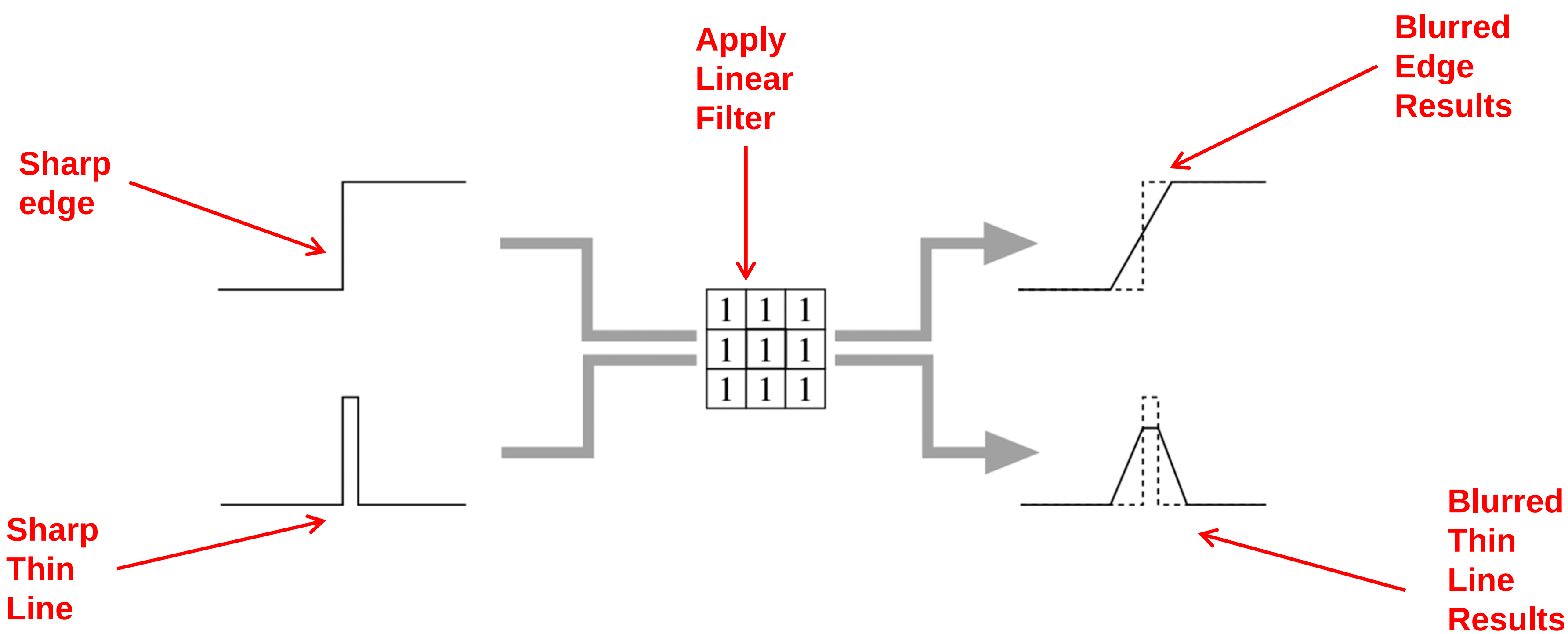


Figure 5.3: The twins image corrupted by periodic noise

Courtesy Allasdair McAndrews

Non-Linear Filters

- Linear filters blurs all image structures points, edges and lines, reduction of image quality (bad!)
- Linear filters thus not used a lot for removing noise



Using Linear Filter to Remove Noise?

- **Example:** Using linear filter to clean salt and pepper noise just causes smearing (not clean removal)
- Try non-linear filters?

Courtesy
Allasdair McAndrews

(a) 3×3 averaging

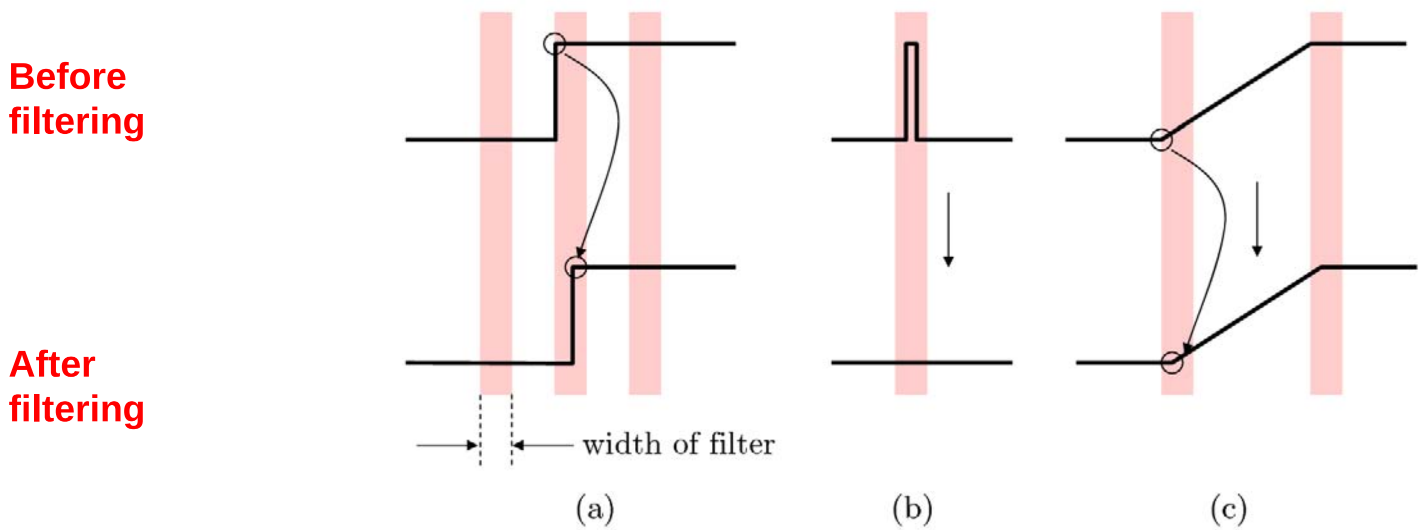
(b) 7×7 averaging

Non-Linear Filters

- Pixels in filter range combined by some non-linear function
- Simplest examples of nonlinear filters: Min and Max filters

$$I'(u,v) \leftarrow \min\{I(u+i, v+j) \mid (i,j) \in R\}$$

$$I'(u,v) \leftarrow \max\{I(u+i, v+j) \mid (i,j) \in R\}$$



Effect of Minimum filter

Step Edge (shifted to right)

Narrow Pulse (removed)

Linear Ramp (shifted to right)

Non-Linear Filters

(a) (b) (c)

Original Image with Salt-and-pepper noise

Minimum filter removes bright spots (maxima) and widens dark image structures

Maximum filter (opposite effect): Removes dark spots (minima) and widens bright image structures

Median Filter

- Much better at removing noise and keeping the structures

$$I'(u,v) \leftarrow \text{median}\,\{I(u+i, v+j) \mid (i,j) \in R\}$$

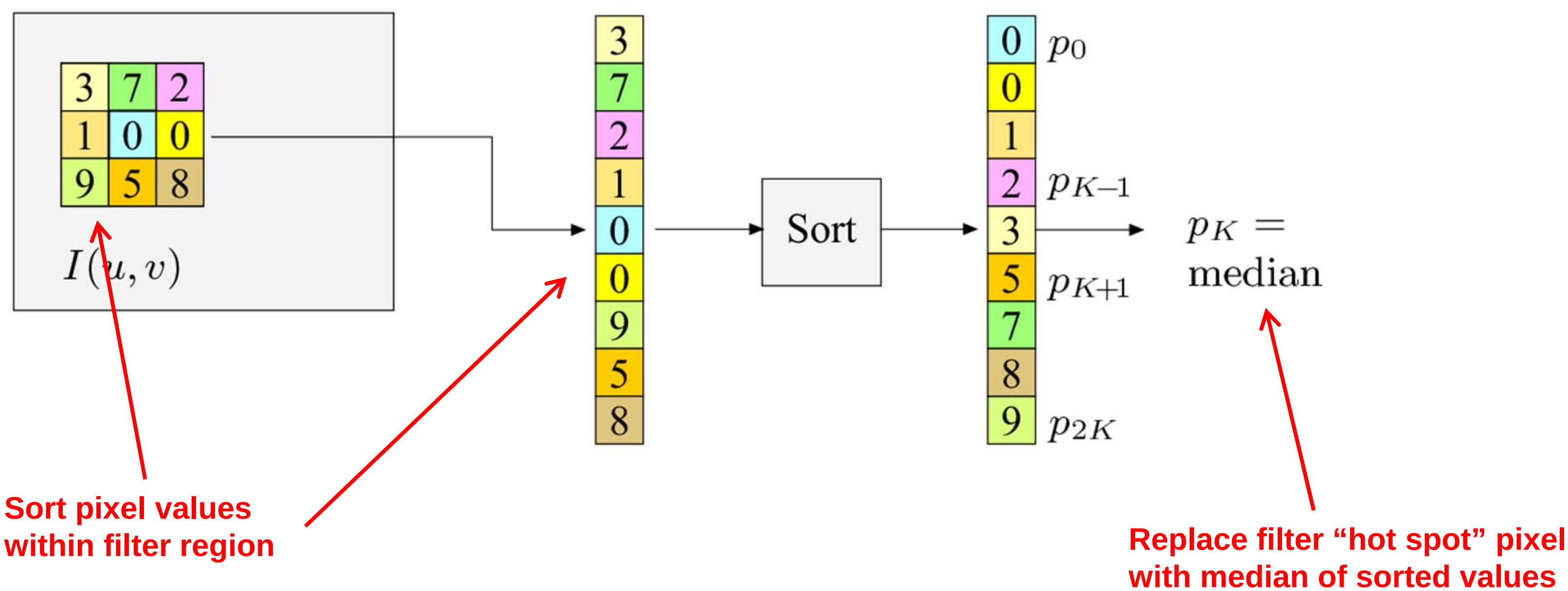
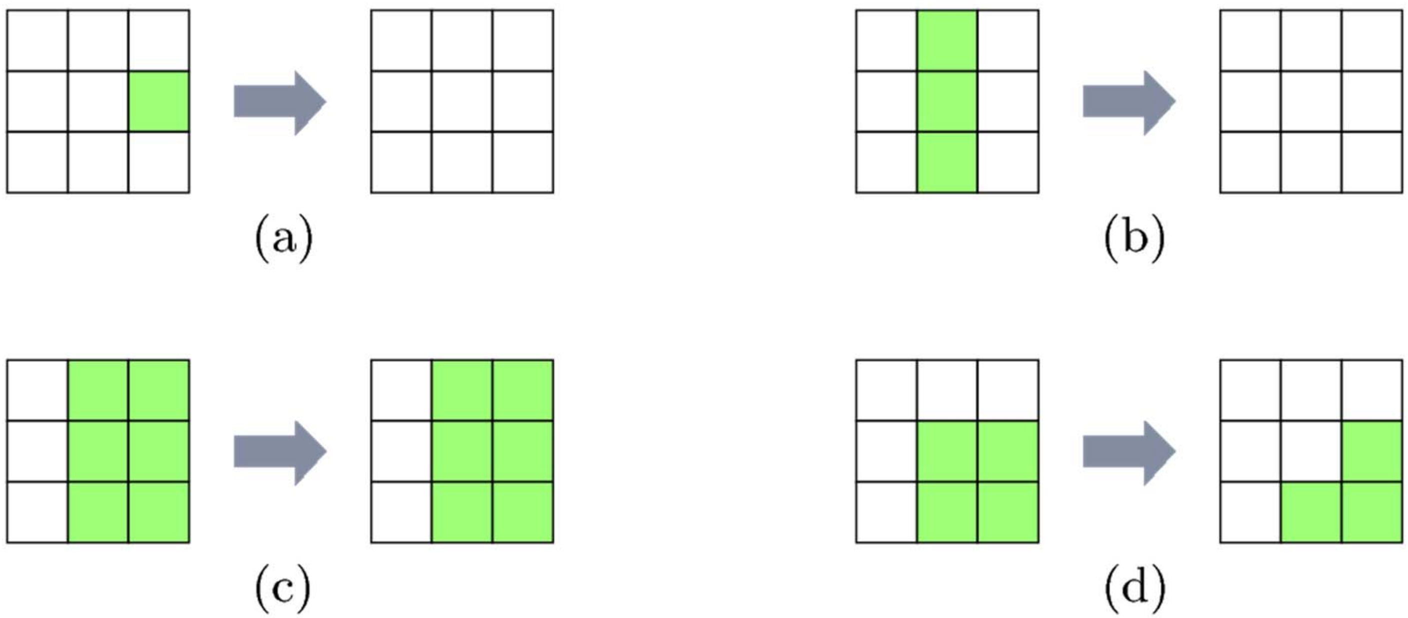


Illustration: Effects of Median Filter

Isolated pixels
are eliminated

Thin lines
are eliminated



A step edge is
unchanged

A corner is
rounded off

Effects of Median Filter

(a) Original Image with Salt-and-pepper noise

(b) Linear filter removes some of the noise, but not completely. Smears noise

(c) Median filter salt-and-pepper noise and keeps image structures largely intact. But also creates small spots of flat intensity, that affect sharpness

Median Filter ImageJ Plugin

```java
import ij.*;
import ij.plugin.filter.PlugInFilter;
import ij.process.*;
import java.util.Arrays;

public class Filter_Median3x3 implements PlugInFilter {
    final int K = 4; // filter size

    public void run(ImageProcessor orig) {
        int w = orig.getWidth();
        int h = orig.getHeight();
        ImageProcessor copy = orig.duplicate();

        // vector to hold pixels from 3×3 neighborhood
        int[] P = new int[2*K+1];

        for (int v = 1; v <= h-2; v++) {
            for (int u = 1; u <= w-2; u++) {
                // fill the pixel vector P for filter position u, v
                int k = 0;
                for (int j = -1; j <= 1; j++) {
                    for (int i = -1; i <= 1; i++) {
                        P[k] = copy.getPixel(u+i, v+j);
                        k++;
                    }
                }
                // sort pixel vector and take the center element
                Arrays.sort(P);
                orig.putPixel(u, v, P[K]);
            }
        }
    }

} // end of class Filter_Median3x3
```

Get Image width + height, and Make copy of image

Array to store pixels to be filtered. Good data structure in which to find median

Copy pixels within filter region into array

Sort pixels within filter using java utility Arrays.sort( )

Middle (k) element of sorted array assumed to be middle. Return as median

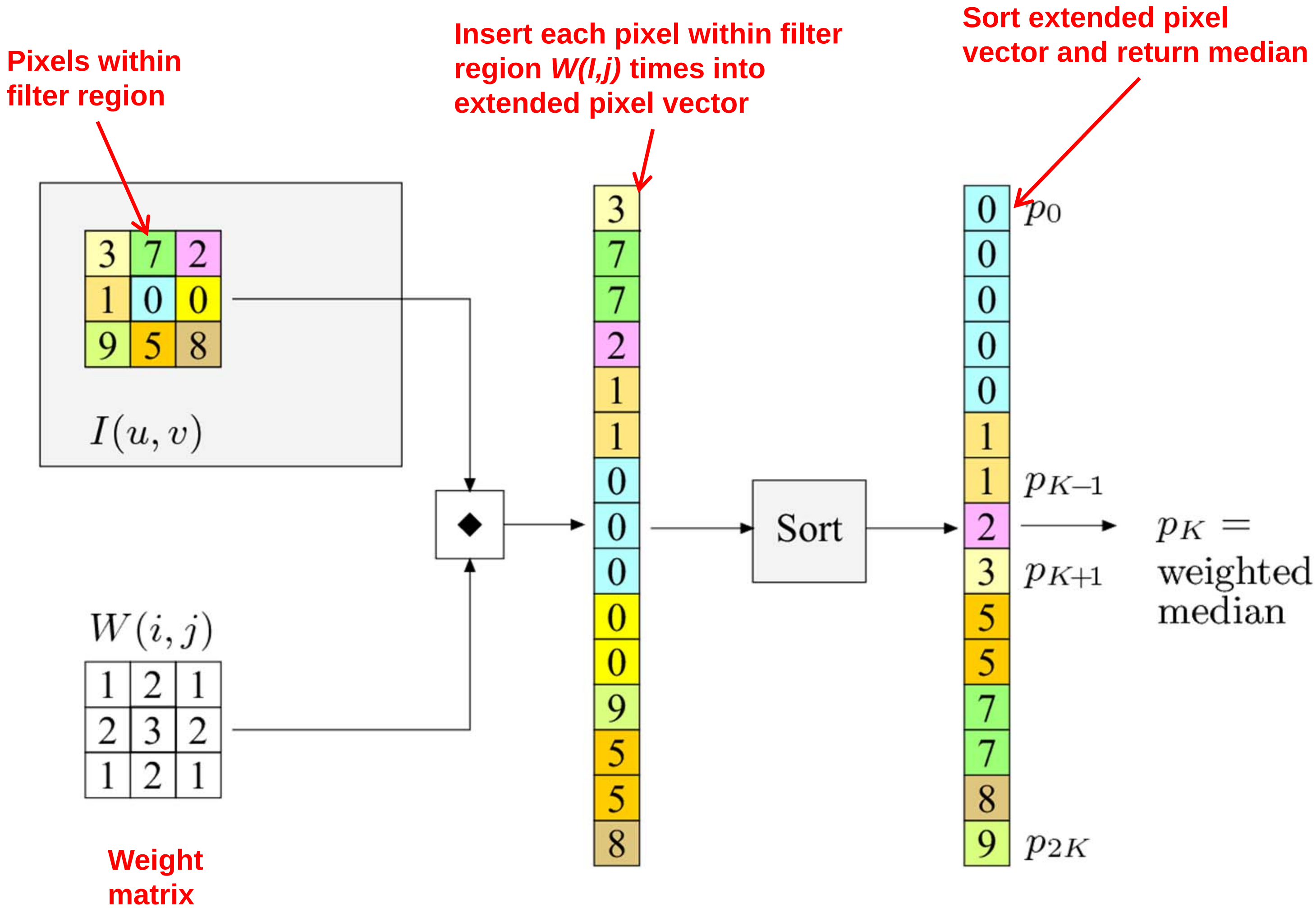
Weighted Median Filter

- Color assigned by median filter determined by colors of "the majority" of pixels within the filter region
- Considered robust since single high or low value cannot influence result (unlike linear average)
- Median filter assigns weights (number of "votes") to filter positions

$$W(i,j) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & \mathbf{3} & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- To compute result, each pixel value within filter region is inserted *W(i,j)* times to create **extended pixel vector**
- Extended pixel vector then sorted and median returned

Weighted Median Filter



Note: assigning weight to center pixel larger than sum of all other pixel weights inhibits any filter effect (center pixel always carries majority)!!

Weighted Median Filter

- More formally, **extended pixel vector** defined as

$$Q = (p_0, \ldots, p_{L-1}) \quad \text{of length} \quad L = \sum_{(i,j) \in R} W(i,j)$$

- For example, following weight matrix yields extended pixel vector of length 15 (sum of weights)

$$W(i,j) = \begin{bmatrix} 1 & 2 & 1 \\ 2 & \mathbf{3} & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- Weighting can be applied to non-rectangular filters
- Example: *cross-shaped* median filter may have weights

$$W^{+}(i,j) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \mathbf{1} & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

An Outlier Method of Filtering

- Algorithm by Pratt, Ref: Alasdair McAndrew, Page 116
- Median filter does sorting per pixel (computationally expensive)
- Alternate method for removing salt-and-pepper noise
  - Define noisy pixels as **outliers** (different from neighboring pixels by an amount > D)
- Algorithm:
  - Choose threshold value D
  - For given pixel, compare its value p to mean m of 8 neighboring pixels
  - If $|p - m| > D$, classifiy pixel as noise, otherwise not
  - If pixel is noise, replace its value with m; Otherwise leave its value unchanged
- Method not automatic. Generate multiple images with different values of D, choose the best looking one

Outlier Method Example

- Effects of choosing different values of *D*

(a) $D = 0.2$

D value too small: removes noise from dark regions

(b) $D = 0.4$

D value too large: removes noise from light regions

Courtesy Allasdair McAndrews

- *D* value of 0.3 performs best
- Overall outlier method not as good as median filter

Other Non-Linear Filters

- Any filter operation that is not linear (summation), is considered linear
- Min, max and median are simple examples
- More examples later:
  - Morphological filters (Chapter 10)
  - Corner detection filters (Chapter 8)
- Also, filtering shall be discussed in frequency domain

Extending Image Along Borders

Pad: **Set pixels outside border to a constant**

(a)

(b)

Extend: **pixels outside border take on value of closest border pixel**

Mirror: **pixels around image border**

(c)

(d)

Wrap: **repeat pixels periodically along coordinate axes**

Filter Operations in ImageJ

- Linear filters implemented by ImageJ plugin class **`ij.plugin.filter.Convolver`**
- Has several methods in addition to **`run( )`**

```
import ij.plugin.filter.Convolver;
...
public void run(ImageProcessor I) {
  float[] H = {              // filter array is one-dimensional!
      0.075f, 0.125f, 0.075f,
      0.125f, 0.200f, 0.125f,
      0.075f, 0.125f, 0.075f };
  Convolver cv = new Convolver();
  cv.setNormalize(false);  // do not use filter normalization
  cv.convolve(I, H, 3, 3); // apply the filter H to I
}
```

$$H(i,j) = \begin{bmatrix} 0.075 & 0.125 & 0.075 \\ 0.125 & \mathbf{0.2} & 0.125 \\ 0.075 & 0.125 & 0.075 \end{bmatrix}$$

Define filter matrix

Create new instance of Convolver class

Apply filter (Modifies Image I destructively)

Gaussian Filters

- **`ij.plugin.filter.GaussianBlur`** implements gaussian filter with radius (σ)
- Uses separable 1d gaussians

```
import ij.plugin.filter.GaussianBlur;
...
public void run(ImageProcessor ip) {
  GaussianBlur gb = new GaussianBlur();
  double radius = 2.5;
  gb.blur(ip, radius);
}
```

Create new instance of GaussianBlur class

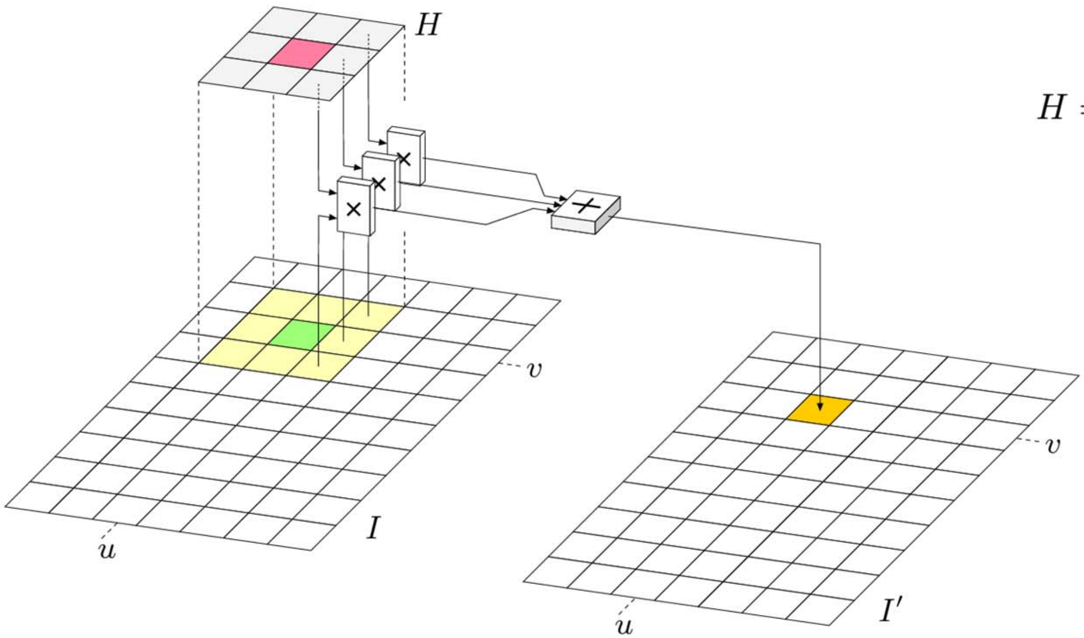
Blur image ip with gaussian filter of radius r

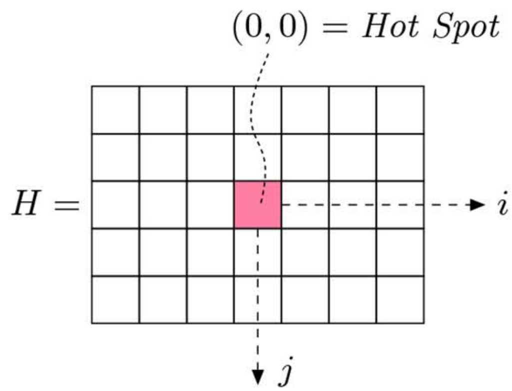
Non-Linear Filters

- A few non-linear filters (minimum, maximum and median filters implemented in **`ij.plugin.filter.RankFilters`**
- Filter region is approximately circular with variable radius
- Example usage:

```
import ij.plugin.filter.RankFilters;
...
public void run(ImageProcessor ip) {
  RankFilters rf = new RankFilters();
  double radius = 3.5;
  rf.rank(ip, radius, RankFilters.MIN);   // minimum filter
  rf.rank(ip, radius, RankFilters.MAX);   // maximum filter
  rf.rank(ip, radius, RankFilters.MEDIAN);   // median filter
}
```

Recall: Linear Filters: Convolution



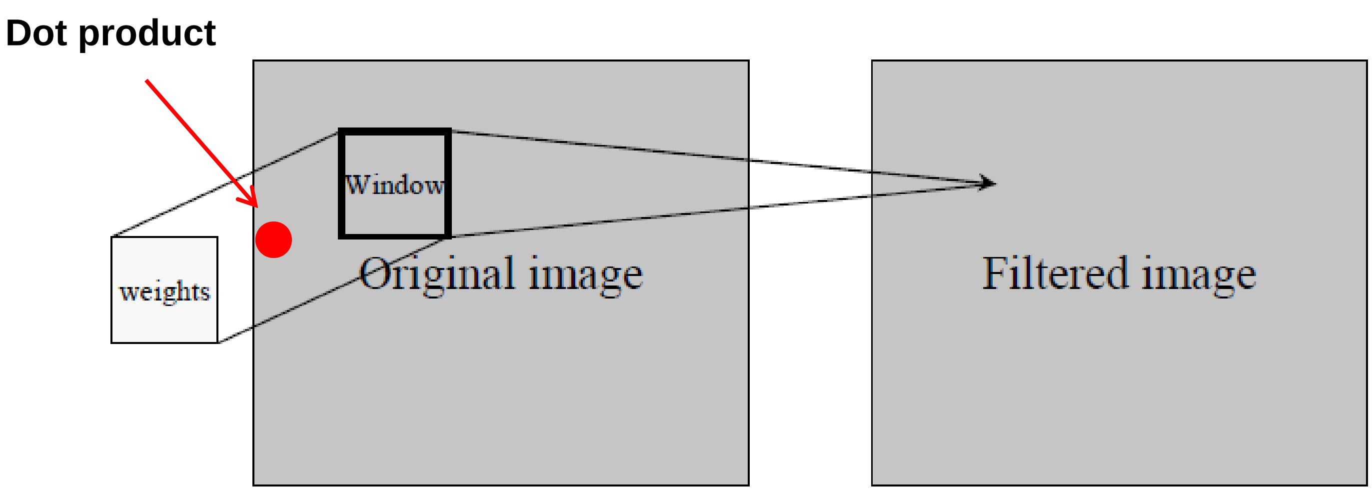


$$I'(u,v) \leftarrow \sum_{(i,j)\in R_H} I(u+i, v+j) \cdot H(i,j)$$

$$I'(u,v) \leftarrow \sum_{i=-1}^{i=1} \sum_{j=-1}^{j=1} I(u+i, v+j) \cdot H(i,j)$$

Convolution as a Dot Product

- Applying a filter at a given pixel is done by taking dot-product between the image and some vector
- Convolving an image with a filter equal to:
  - Filter ● each image window (moves through image)



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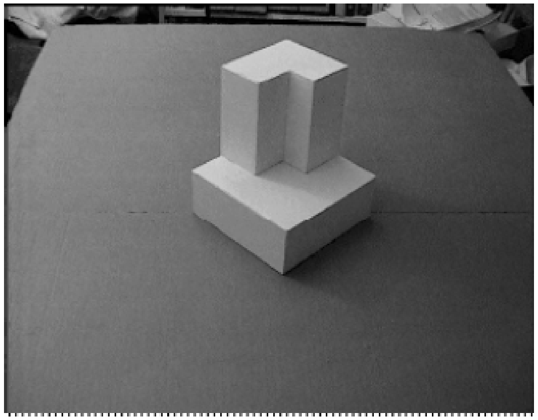
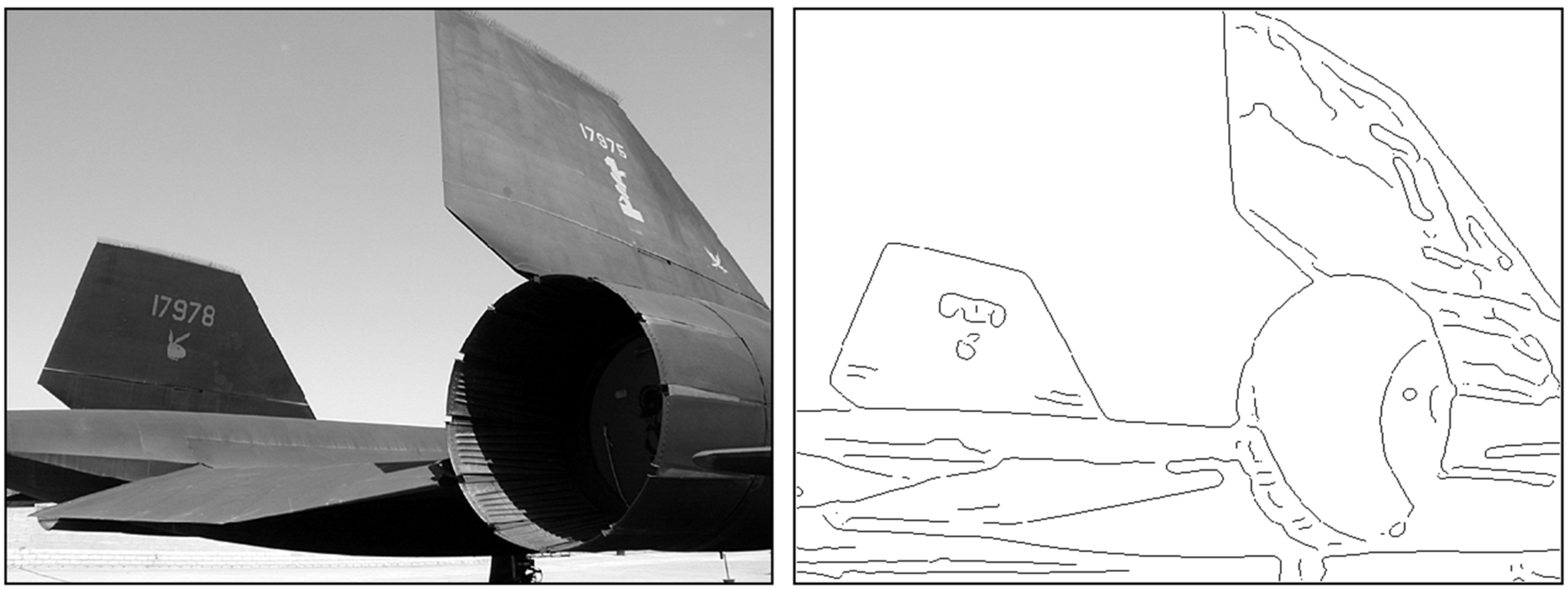
What is an Edge?

- Edge? sharp change in brightness (discontinuities)
- Where do edges occur?
  - **Actual edges:** Boundaries between objects
  - Sharp change in brightness can also occur within object
    - Reflectance changes
    - Change in surface orientation
    - Illumination changes. E.g. Cast shadow boundary



Edge Detection

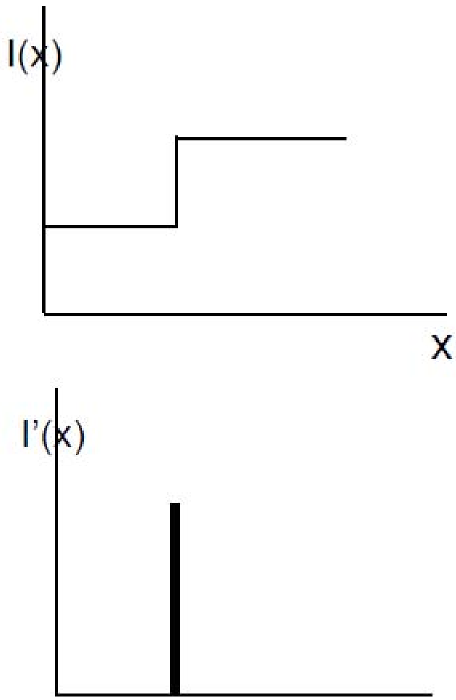
- Image processing task that finds edges and contours in images
- Edges so important that human vision can reconstruct edge lines

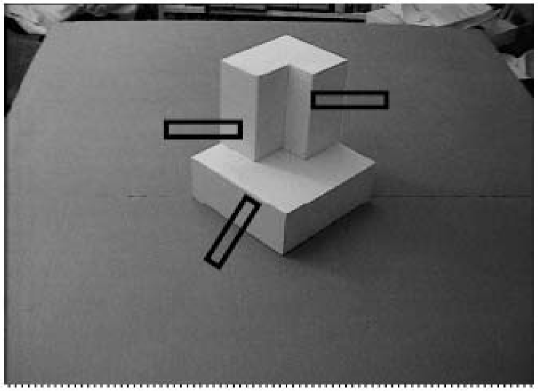


(a) (b)

Characteristics of an Edge

- Edge: A sharp change in brightness
- Ideal edge is a step function in some direction

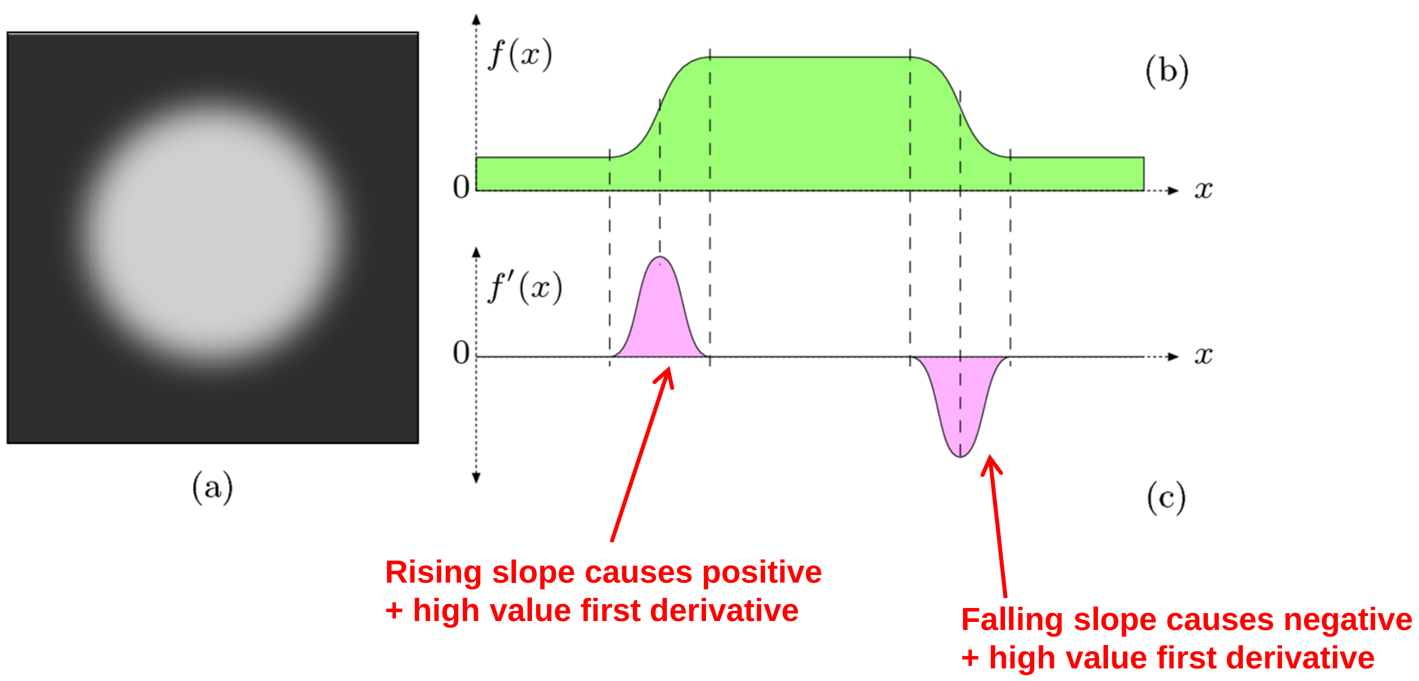




Characteristics of an Edge

- Real (non-ideal) edge is a slightly blurred step function
- Edges can be characterized by high value first derivative

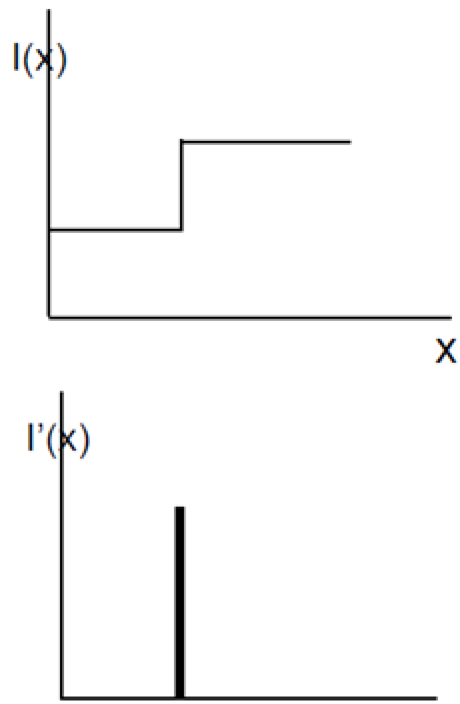
$$f'(x) = \frac{df}{dx}(x)$$

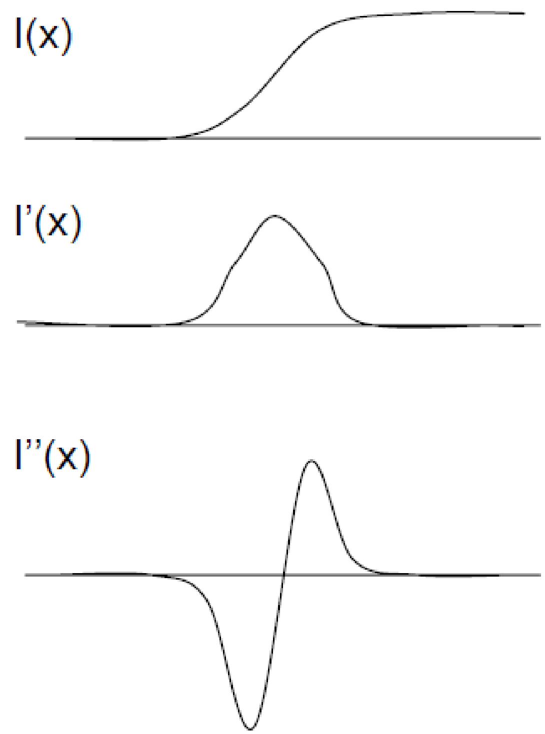


Characteristics of an Edge

- Ideal edge is a step function in certain direction.
- First derivative of I(x) has a **peak** at the edge
- Second derivative of I(x) has a **zero crossing** at edge

Ideal edge





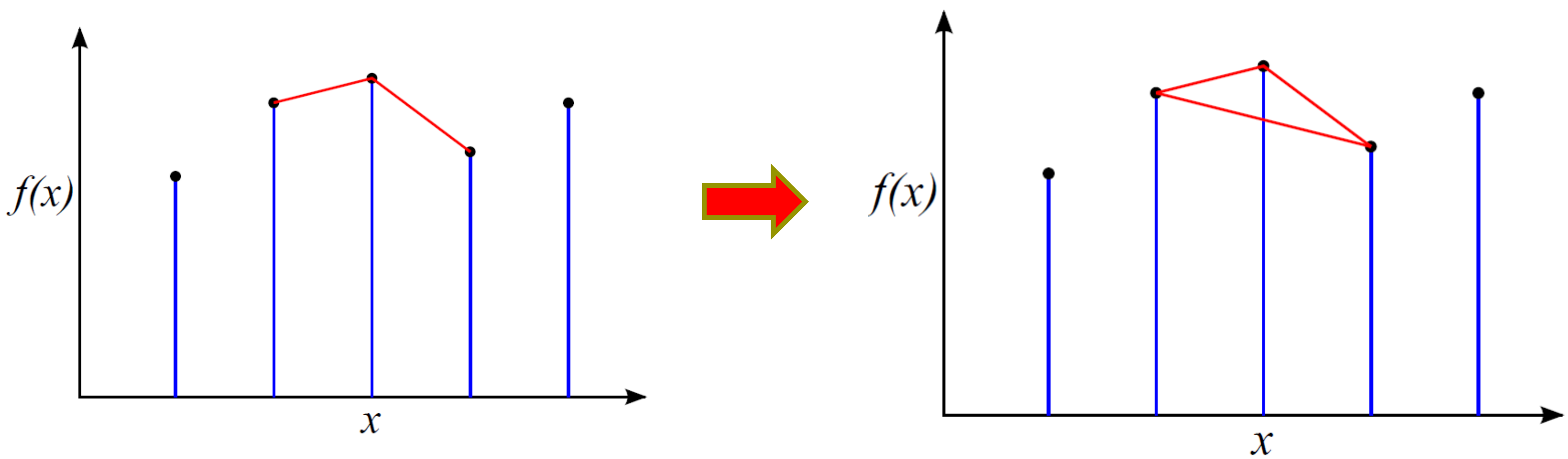
Real edge

First derivative shows peak

Second derivative shows zero crossing

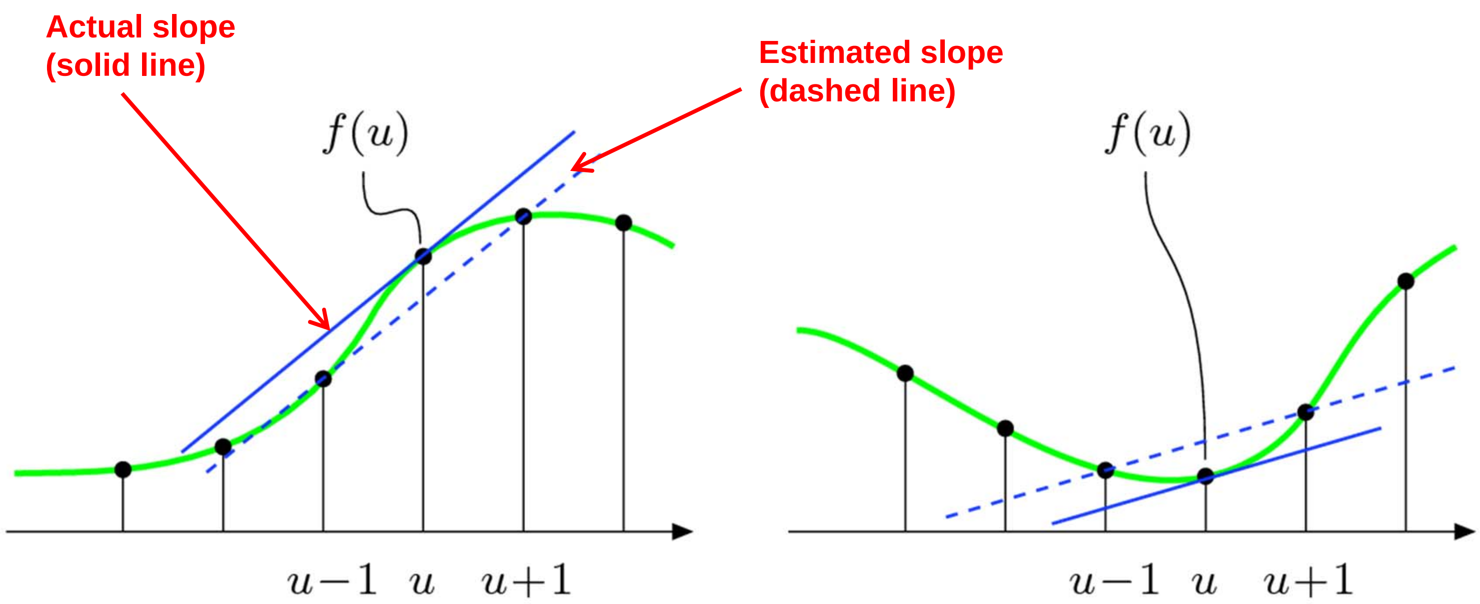
Slopes of Discrete Functions

- Left and right slope may not be same
- Solution? Take average of left and right slope



Computing Derivative of Discrete Function

$$\frac{df}{du}(u) \approx \frac{f(u+1) - f(u-1)}{2} = 0.5 \cdot \big(f(u+1) - f(u-1)\big)$$



Finite Differences

- Forward difference (right slope)

$$\Delta_+ f(x) = f(x+1) - f(x)$$

- Backward difference (left slope)

$$\Delta_- f(x) = f(x) - f(x-1)$$

- Central Difference (average slope)

$$\Delta f(x) = \frac{1}{2}\left(f(x+1) - f(x-1)\right)$$

Definition: Function Gradient

- Let $f(x,y)$ be a 2D function
- **Gradient:** Vector whose direction is in direction of maximum rate of change of f and whose magnitude is maximum rate of change of f
- Gradient is perpendicular to edge contour

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]^T$$

- magnitude = $[(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2]^{1/2}$
- direction = $\tan^{-1}(\frac{\partial f / \partial y}{\partial f / \partial x})$

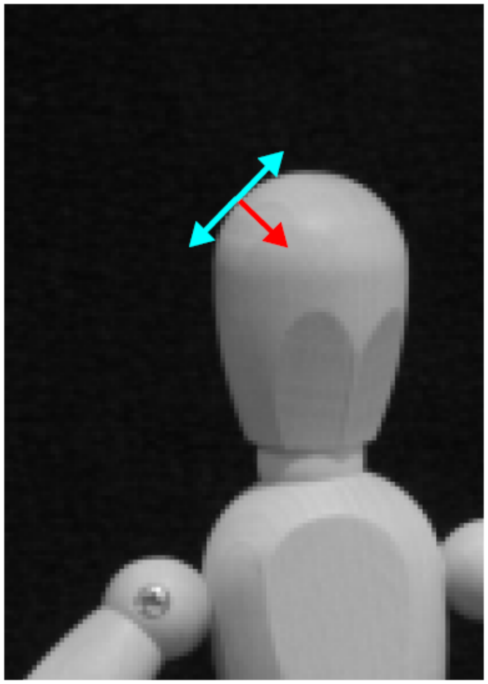


Image Gradient

- Image is 2D discrete function
- Image derivatives in horizontal and vertical directions

$$\frac{\partial I}{\partial u}(u,v) \quad \text{and} \quad \frac{\partial I}{\partial v}(u,v)$$

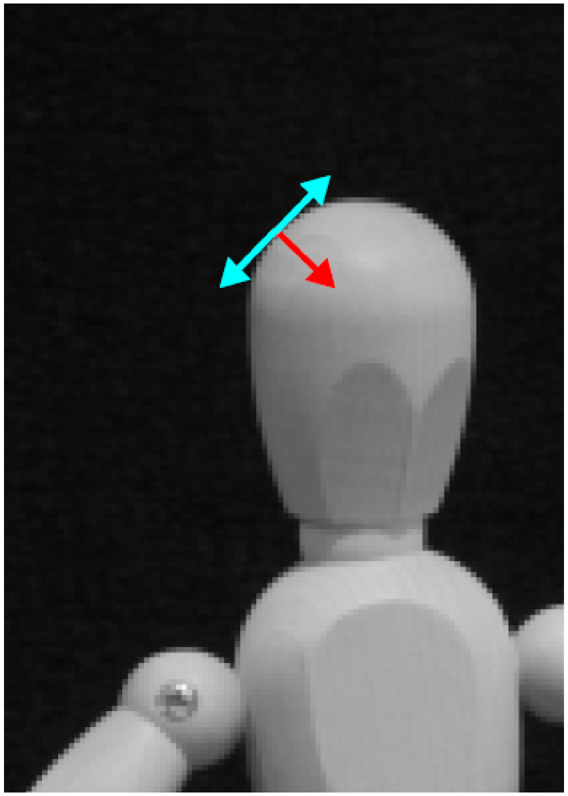
- Image gradient at location (u,v)

$$\nabla I(u,v) = \begin{bmatrix} \frac{\partial I}{\partial u}(u,v) \\ \frac{\partial I}{\partial v}(u,v) \end{bmatrix}$$

- Gradient magnitude

$$|\nabla I|(u,v) = \sqrt{\left(\frac{\partial I}{\partial u}(u,v)\right)^2 + \left(\frac{\partial I}{\partial v}(u,v)\right)^2}$$

- Magnitude is invariant under image rotation, used in edge detection



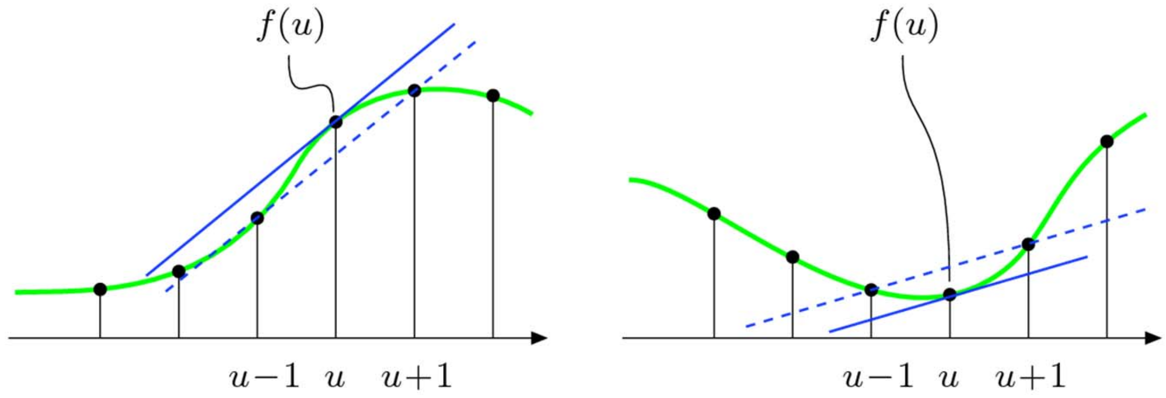
Derivative Filters

- Recall that we can compute derivative of discrete function as

$$\frac{df}{du}(u) \approx \frac{f(u+1) - f(u-1)}{2} = 0.5 \cdot \big(f(u+1) - f(u-1)\big)$$

- Can we make linear filter that computes central differences

$$H_x^D = \begin{bmatrix} -0.5 & \mathbf{0} & 0.5 \end{bmatrix} = 0.5 \cdot \begin{bmatrix} -1 & \mathbf{0} & 1 \end{bmatrix}$$



Finite Differences as Convolutions

- Forward difference

$$\Delta_+ f(x) = f(x+1) - f(x)$$

- Take a convolution kernel $H = [0 \quad -1 \quad 1]$

$$\Delta_+ f = f * H$$

Finite Differences as Convolutions

- Central difference

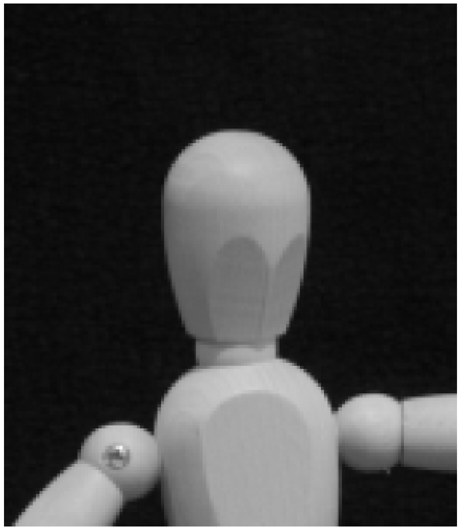
$$\Delta f(x) = \frac{1}{2}\left(f(x+1) - f(x-1)\right)$$

- Convolution kernel is: ***H* =** $\begin{bmatrix} -0.5 & \mathbf{0} & 0.5 \end{bmatrix}$

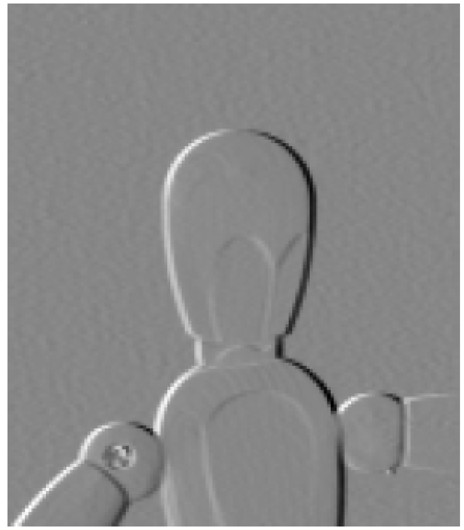
$$\Delta f(x) = f * H$$

- **Notice:** Derivative kernels sum to zero

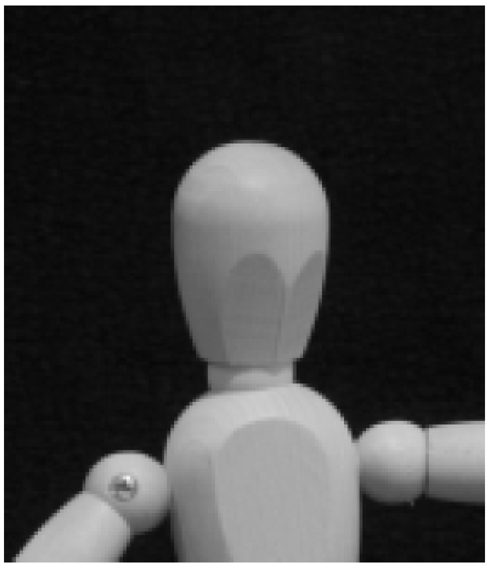
x-Derivative of Image using Central Difference



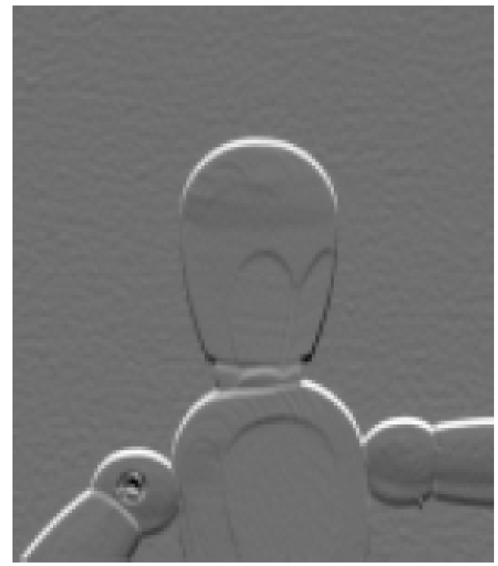
$$* \begin{bmatrix} -0.5 & 0 & 0.5 \end{bmatrix} =$$



y-Derivative of Image using Central Difference



$$* \begin{bmatrix} -0.5 \\ \mathbf{0} \\ 0.5 \end{bmatrix} =$$

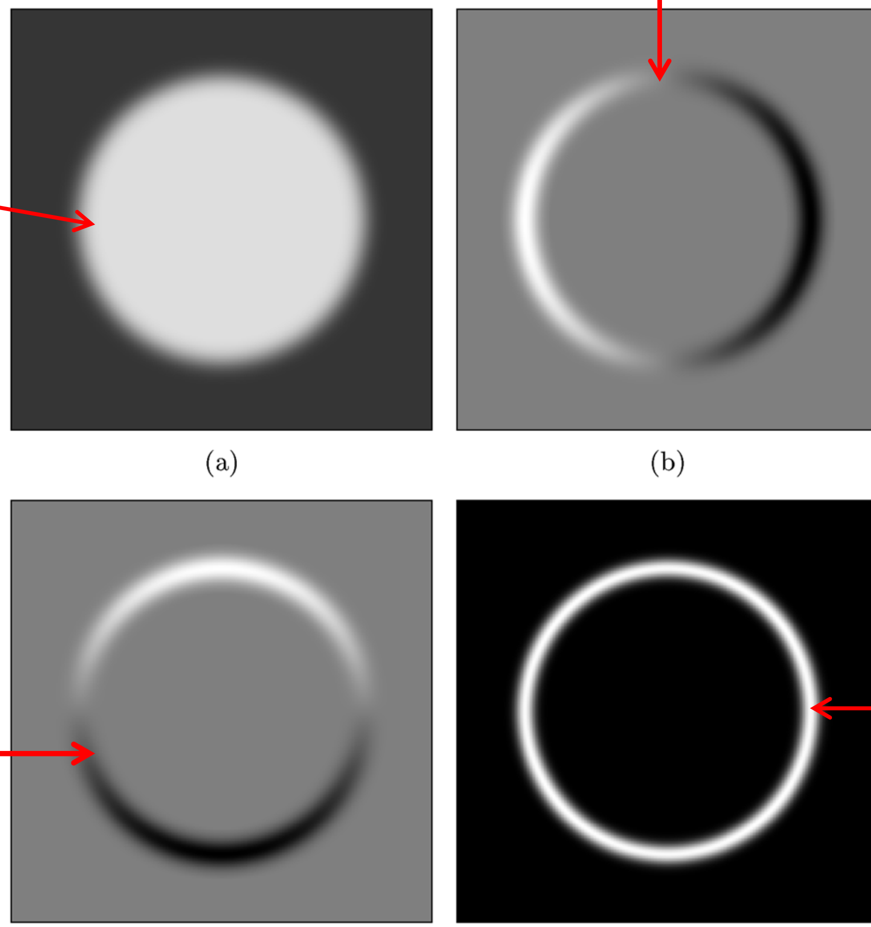


Derivative Filters

Gradient slope in horizontal direction

$$H_x^D = \begin{bmatrix} -0.5 & \mathbf{0} & 0.5 \end{bmatrix} = 0.5 \cdot \begin{bmatrix} -1 & \mathbf{0} & 1 \end{bmatrix}$$

A synthetic image



(a) (b) (c) (d)

$$H_y^D = \begin{bmatrix} -0.5 \\ \mathbf{0} \\ 0.5 \end{bmatrix} = 0.5 \cdot \begin{bmatrix} -1 \\ \mathbf{0} \\ 1 \end{bmatrix}$$

Gradient slope in vertical direction

Magnitude of gradient

Edge Operators

- Approximating local gradients in image is basis of many classical edge-detection operators
- Main differences?
  - Type of filter used to estimate gradient components
  - How gradient components are combined
- We are typically interested in
  - Local edge direction
  - Local edge magnitude

Partial Image Derivatives

- Partial derivatives of images replaced by finite differences

$$\Delta_x f = f(x,y) - f(x-1,y) \qquad \begin{bmatrix} -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Delta_y f = f(x,y) - f(x,y-1)$$

- Alternatives are:

$$\Delta_{2x} f = f(x+1,y) - f(x-1,y) \qquad \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\Delta_{2y} f = f(x,y+1) - f(x,y-1)$$

- Robert's gradient

$$\Delta_+ f = f(x+1,y+1) - f(x,y) \rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Delta_- f = f(x,y+1) - f(x+1,y) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Prewitt

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Sobel

Using Averaging with Derivatives

- Finite difference operator is sensitive to noise
- Derivates more robust if derivative computations are averaged in a neighborhood
- Prewitt operator: derivative in *x*, then average in *y*

$$H_x^P = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} * \begin{bmatrix}0.5 & 0 & -0.5\end{bmatrix} = \frac{1}{6}\begin{bmatrix}1 & 0 & -1\\1 & 0 & -1\\1 & 0 & -1\end{bmatrix}$$

Derivative in *x* direction

Average in *y* direction

Note: Filter kernel is flipped in convolution

- *y*-derivative kernel, H_y^P defined similarly

Sobel Operator

- Similar to Prewitt, but averaging kernel is higher in middle

$$H_x^S = \frac{1}{4}\begin{bmatrix}1\\2\\1\end{bmatrix} * [0.5 \quad 0 \quad -0.5] = \frac{1}{8}\begin{bmatrix}1 & 0 & -1\\2 & 0 & -2\\1 & 0 & -1\end{bmatrix}$$

$$H_y^S = \frac{1}{4}[1 \quad 2 \quad 1] * \begin{bmatrix}0.5\\0\\-0.5\end{bmatrix} = \frac{1}{8}\begin{bmatrix}1 & 2 & 1\\0 & 0 & 0\\-1 & -2 & -1\end{bmatrix}$$

Average in *x* direction

Derivative in *y* direction

Note: Filter kernel is flipped in convolution

Prewitt and Sobel Edge Operators

- Prewitt Operator

$$H_x^P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & \mathbf{0} & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_y^P = \begin{bmatrix} -1 & -1 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Written in separable form →

$$H_x^P = \begin{bmatrix} 1 \\ \mathbf{1} \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & \mathbf{0} & 1 \end{bmatrix} \quad \text{and} \quad H_y^P = \begin{bmatrix} 1 & \mathbf{1} & 1 \end{bmatrix} * \begin{bmatrix} -1 \\ \mathbf{0} \\ 1 \end{bmatrix}$$

- Sobel Operator

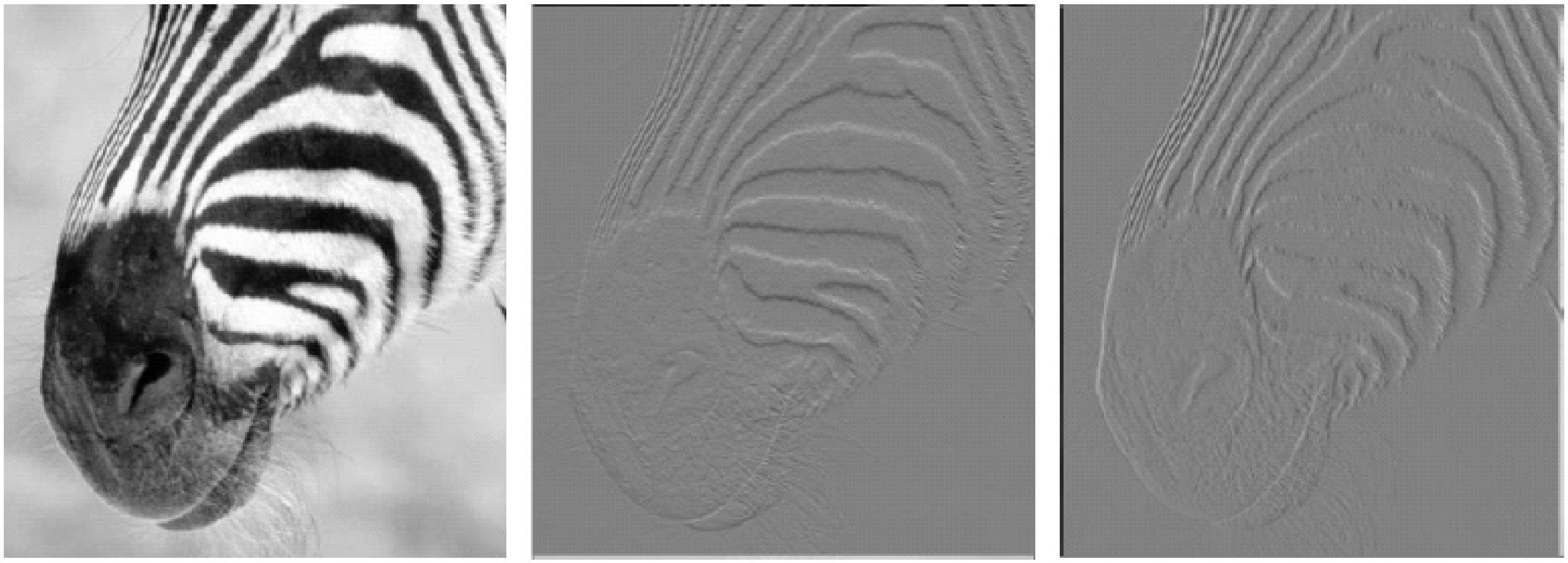
$$H_x^S = \begin{bmatrix} -1 & 0 & 1 \\ -2 & \mathbf{0} & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad H_y^S = \begin{bmatrix} -1 & -2 & -1 \\ 0 & \mathbf{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Improved Sobel Filter

- Original Sobel filter relatively inaccurate
- Improved versions proposed by Jahne

$$H_x^{S'} = \frac{1}{32}\begin{bmatrix} -3 & 0 & 3 \\ -10 & \mathbf{0} & 10 \\ -3 & 0 & 3 \end{bmatrix} \quad \text{and} \quad H_y^{S'} = \frac{1}{32}\begin{bmatrix} -3 & -10 & -3 \\ 0 & \mathbf{0} & 0 \\ 3 & 10 & 3 \end{bmatrix}$$

Prewitt and Sobel Edge Operators



Scaling Edge Components

- Estimates of local gradient components obtained from filter results by appropriate scaling

Scaling factor for Prewitt operator →

$$\nabla I(u,v) \approx \frac{1}{6} \cdot \begin{bmatrix} (I * H_x^P)(u,v) \\ (I * H_y^P)(u,v) \end{bmatrix}$$

Scaling factor for Sobel operator →

$$\nabla I(u,v) \approx \frac{1}{8} \cdot \begin{bmatrix} (I * H_x^S)(u,v) \\ (I * H_y^S)(u,v) \end{bmatrix}$$

Gradient-Based Edge Detection

- Compute image derivatives by convolution

$$D_x(u,v) = H_x * I \quad \text{and} \quad D_y(u,v) = H_y * I$$

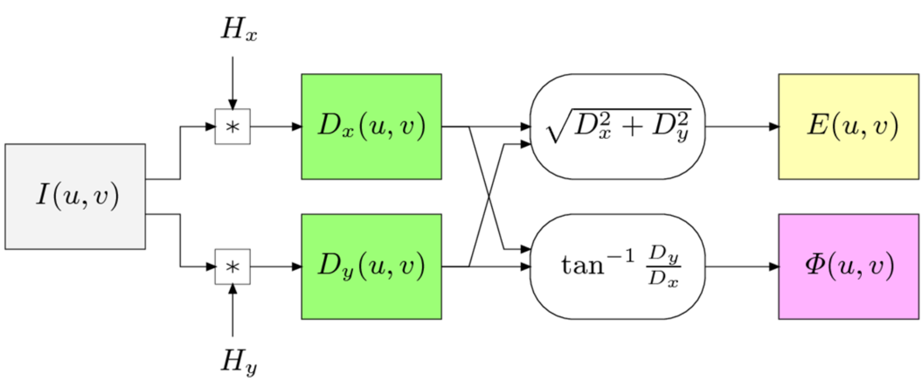
Scaled Filter results

- Compute edge gradient magnitude

$$E(u,v) = \sqrt{\left(D_x(u,v)\right)^2 + \left(D_y(u,v)\right)^2}$$

- Compute edge gradient direction

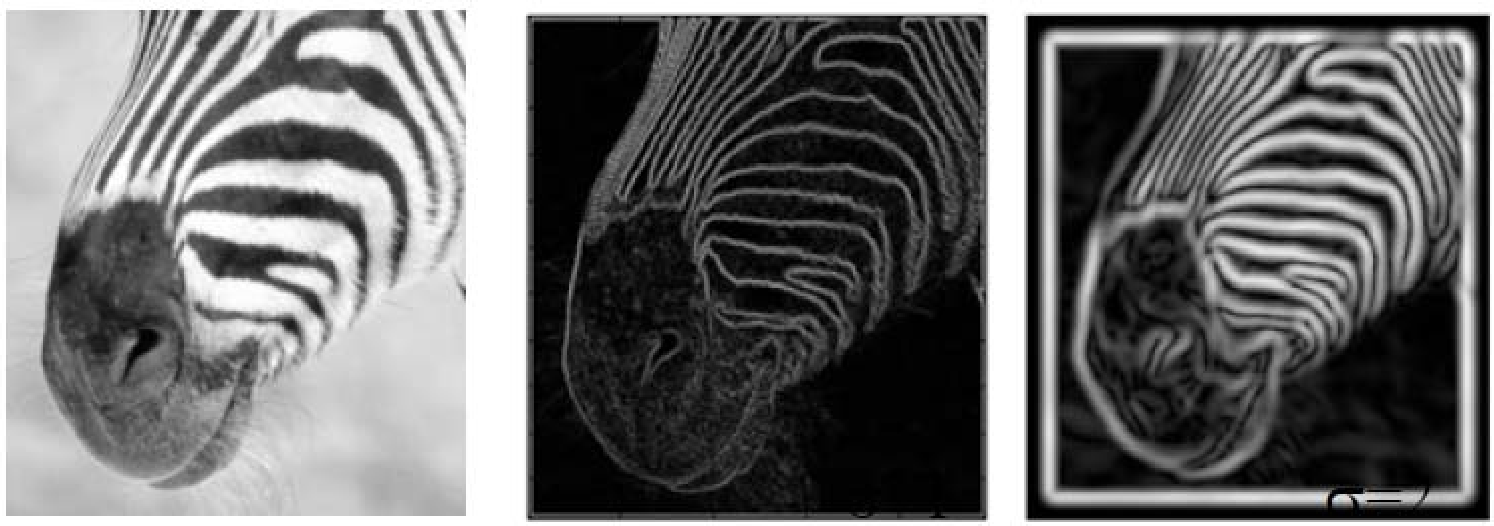
$$\Phi(u,v) = \tan^{-1}\left(\frac{D_y(u,v)}{D_x(u,v)}\right) = \mathrm{ArcTan}\left(D_x(u,v), D_y(u,v)\right)$$



Typical process of Gradient based edge detection

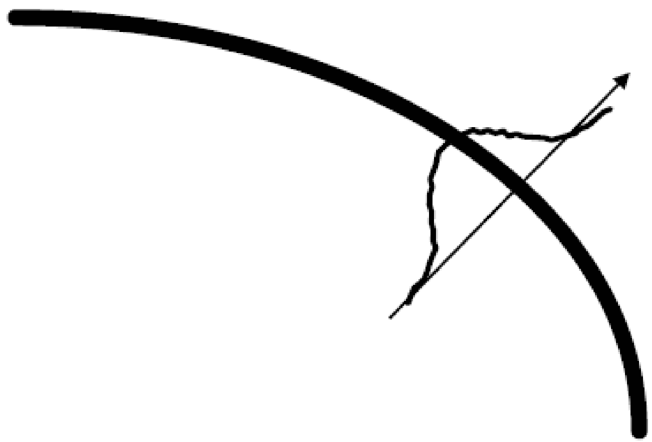
Gradient-Based Edge Detection

- After computing gradient magnitude and orientation then what?
- Mark points where gradient magnitude is large wrt neighbors



Non-Maxima Suppression

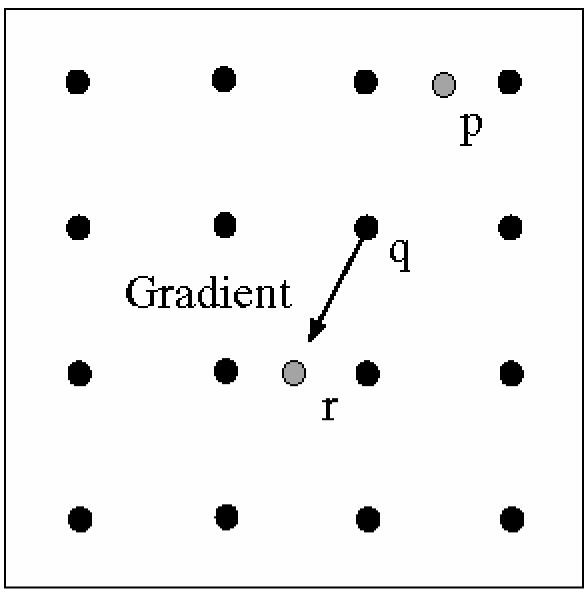
- Retain a point as an edge point if:
  - Its gradient magnitude is higher than a threshold
  - Its gradient magnitude is a local maxima in gradient direction

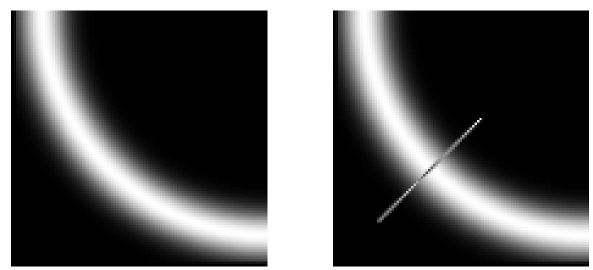


Simple thresholding will compute thick edges

Non-Maxima Suppression

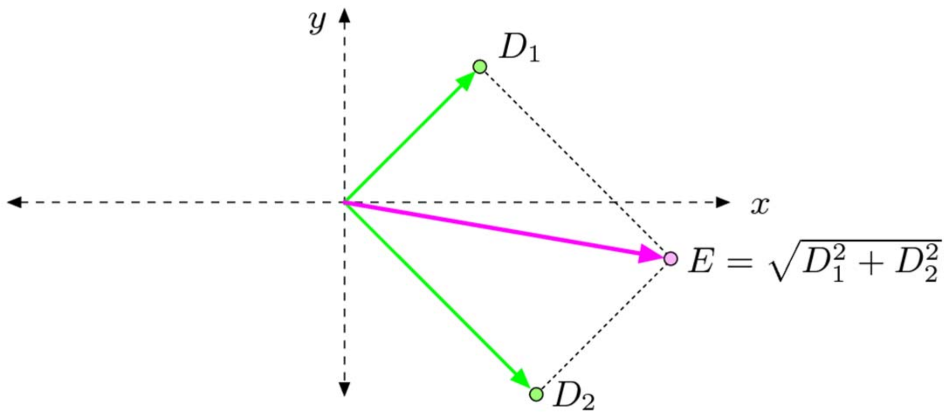
- A maxima occurs at *q*, if its magnitude is larger than those at *p* and *r*





Roberts Edge Operators

- Estimates directional gradient along 2 image diagonals
- Edge strength E(u,v): length of vector obtained by adding 2 orthogonal gradient components $D_1(u,v)$ and $D_2(u,v)$



- Filters for edge components

$$H_1^R = \begin{bmatrix} 0 & \mathbf{1} \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad H_2^R = \begin{bmatrix} -1 & 0 \\ 0 & \mathbf{1} \end{bmatrix}$$

Roberts Edge Operators

- Diagonal gradient components produced by 2 Robert filters

$D_1 = I * H_1^R$

$D_2 = I * H_2^R$

Compass Operators

- Linear edge filters involve trade-off

Sensitivity to Edge magnitude ↑ **=** ↓ **Sensitivity to orientation**

- Example: Prewitt and Sobel operators detect edge magnitudes but use only 2 directions (insensitive to orientation)
- Solution? Use many filters, each sensitive to narrow range of orientations **(compass operators)**

Compass Operators

- Edge operators proposed by Kirsh uses 8 filters with orientations spaced at 45 degrees

$$H_0^K = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad H_4^K = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Need only to compute 4 filters
Since H_4 = - H_0, etc

$$H_1^K = \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \qquad H_5^K = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$H_2^K = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad H_6^K = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$H_3^K = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \qquad H_7^K = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

Compass Operators

- Edge strength E^K at position(*u,v*) is max of the 8 filters

$$E^K(u,v) \triangleq \max\big(D_0(u,v), D_1(u,v), \ldots D_7(u,v)\big)$$
$$= \max\big(|D_0(u,v)|, |D_1(u,v)|, |D_2(u,v)|, |D_3(u,v)|\big)$$

- Strongest-responding filter also determines edge orientation at a position(*u,v*)

$$\Phi^K(u,v) \triangleq \frac{\pi}{4} \qquad \text{with} \;\; j = \underset{0 \le i \le 7}{\operatorname{argmax}} \; D_i(u,v)$$

Edge operators in ImageJ

- ImageJ implements Sobel operator
- Can be invoked via menu **Process -> Find Edges**
- Also available through method `void findEdges( )` for objects of type `ImageProcessor`

References

- Wilhelm Burger and Mark J. Burge, Digital Image Processing, Springer, 2008
- University of Utah, CS 4640: Image Processing Basics, Spring 2012
- Rutgers University, CS 334, Introduction to Imaging and Multimedia, Fall 2012