CS 563 Advanced Topics in Computer Graphics
Volume Scattering

by Paulo Gonçalves de Barros

Introduction

- Scenes in vacuum
- Real-life
  - Atmosphere
  - Smoke
  - Haze
  - Clouds

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- Scenes in vacuum
- Real-life
  - Atmosphere
  - Smoke
  - Haze
  - Clouds
- Volume scattering
  - Participating media
  - Its effect on light rays passing through it

Summary

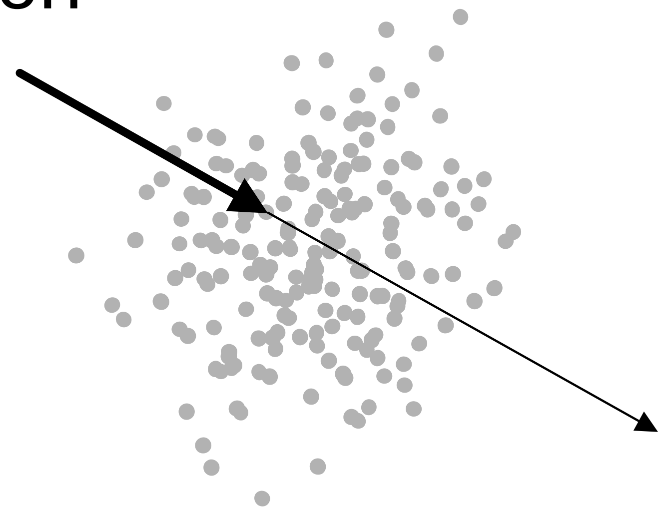
- Volume Scattering processes
- Phase Functions
- Volume Interface
  - Homogeneous Media
  - Varying Density Volumes
  - Volume Aggregates

Volume Scattering Processes

- Pariticipating media properties
  - Absorption
  - Emission
  - Scattering
    - Out-scattering
    - In-scattering
  - Homogeneous or Inhomogeneous

Volume Scattering Processes

- Absorption



Volume Scattering Processes

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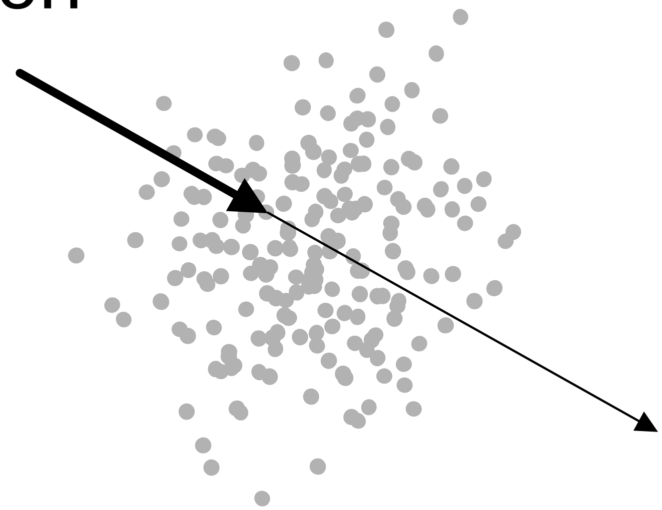
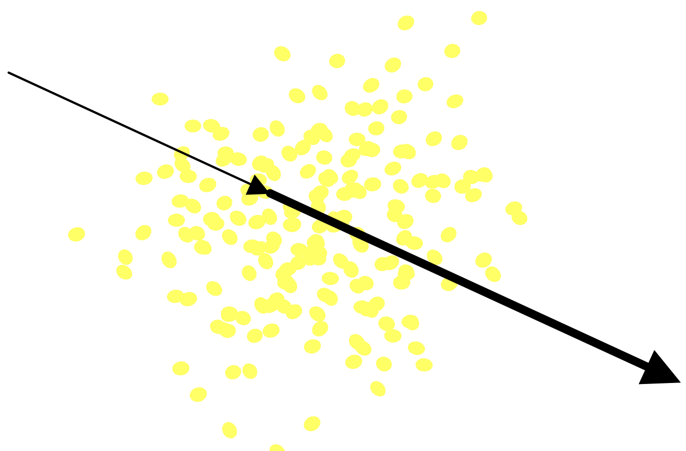
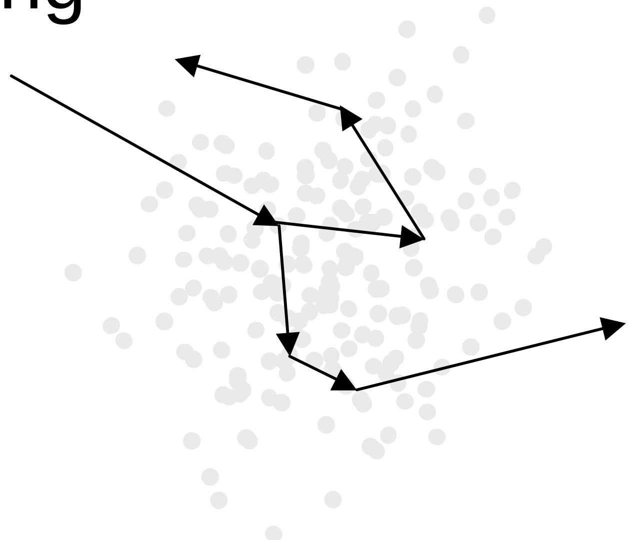


- Emission



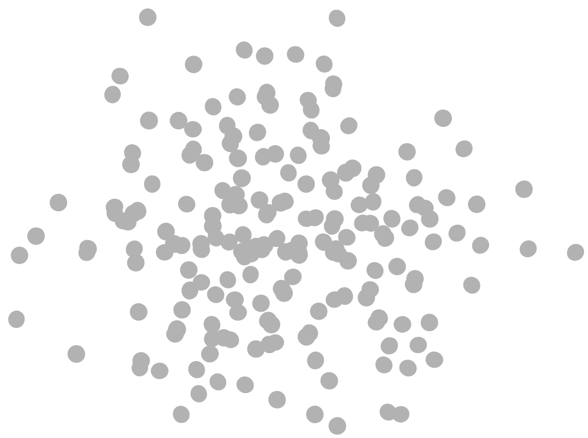
Volume Scattering Processes

- Scattering



Volume Scattering Processes

- Homogeneous
  - Constant particle density
  - Uniform particle types distribution



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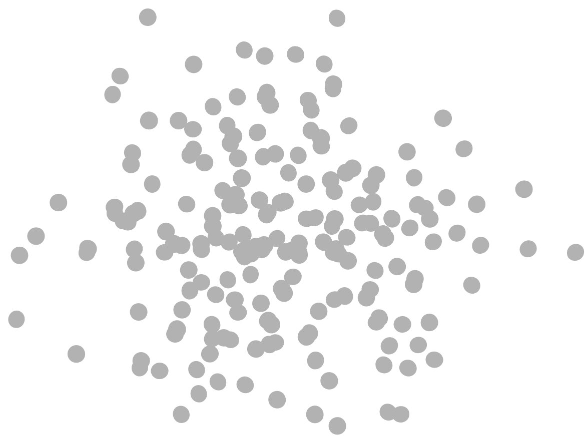
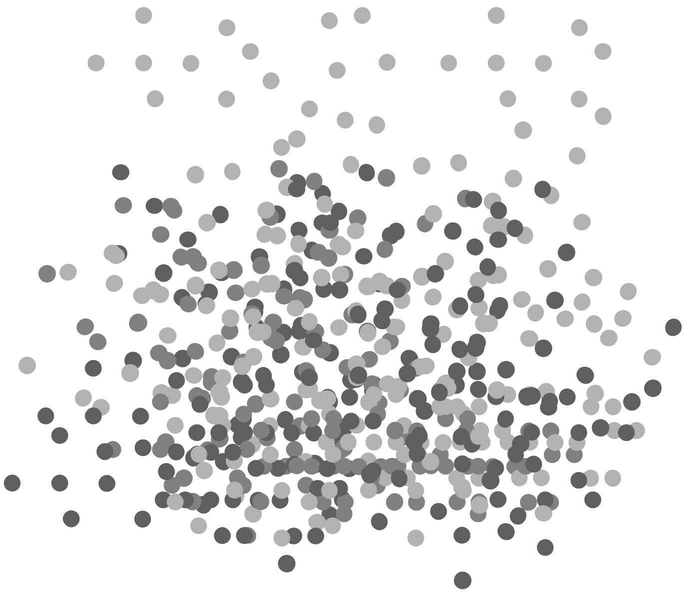


- Inhomogeneous
  - Varying particle density
  - Varying particle distribution



Volume Scattering Processes

- Absorption
  - Light is absorbed by medium
  - Ray radiance decreases through the medium

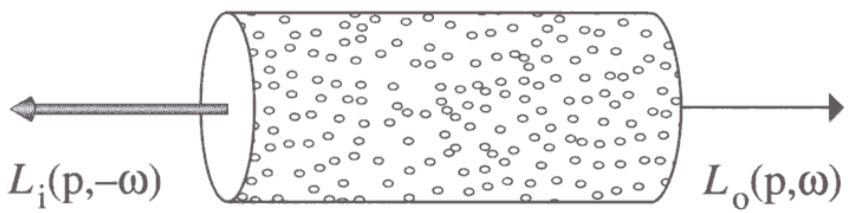
Volume Scattering Processes

- Absorption crossed section s $_a$
  - Light absorption probability density per unit distance traveled in medium
  - Units ? m^{-1}
    - dt ? through-medium-travel unit
    - Values may be larger than 1
  - Influence factors
    - Position (p)
    - Direction (?)
    - Spectrum

Volume Scattering Processes

- Change in radiance per unit
  - Difference between incoming and outgoing radiance

$$dL_o(p,w) = L_o(p,w) - L_i(p,w)$$



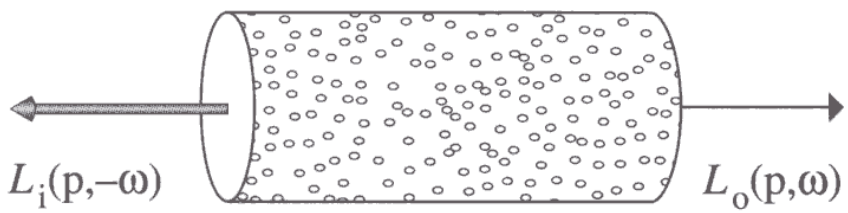
Volume Scattering Processes

- Change in radiance per unit
  - Difference between incoming and outgoing radiance

$$dL_o(p,w) = L_o(p,w) - L_i(p,w)$$

  - Negative fraction of L_i

$$dL_o(p,w) = -s_a(p,w)L_i(p,-w)dt$$



Volume Scattering Processes

- Absorbed radiance
  - Traveled a distance *d* through medium

$$\underbrace{L(\mathbf{p}+wd,w)}_{L_o}=\underbrace{L(\mathbf{p},w)}_{L_i}\underbrace{e^{-\overbrace{\int_0^d s_a(\mathbf{p}+wt,w)dt}^{s_a \text{ integrated along } d}}}_{\text{Probability density function}}$$

Volume Scattering Processes

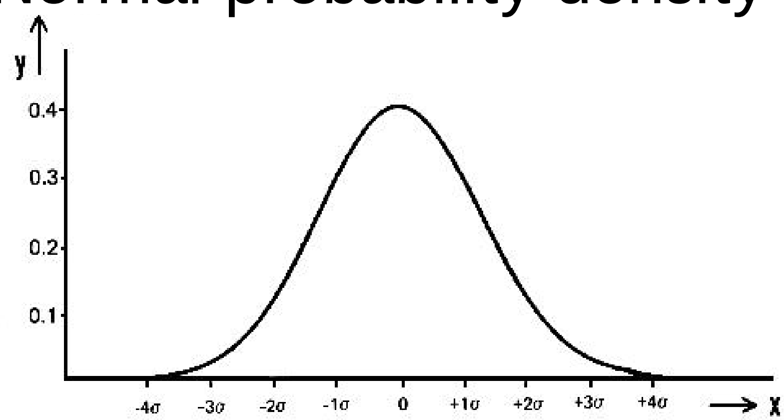
- Absorbed radiance
  - Traveled a distance *d* through medium

s_a integrated in *d*

$$\underbrace{L(\mathbf{p}+wd,w)}_{L_o} = \underbrace{L(\mathbf{p},w)}_{L_i}\underbrace{e^{-\overbrace{\int_0^d s_a(\mathbf{p}+wt,w)dt}}}_{\text{Probability density function}}$$

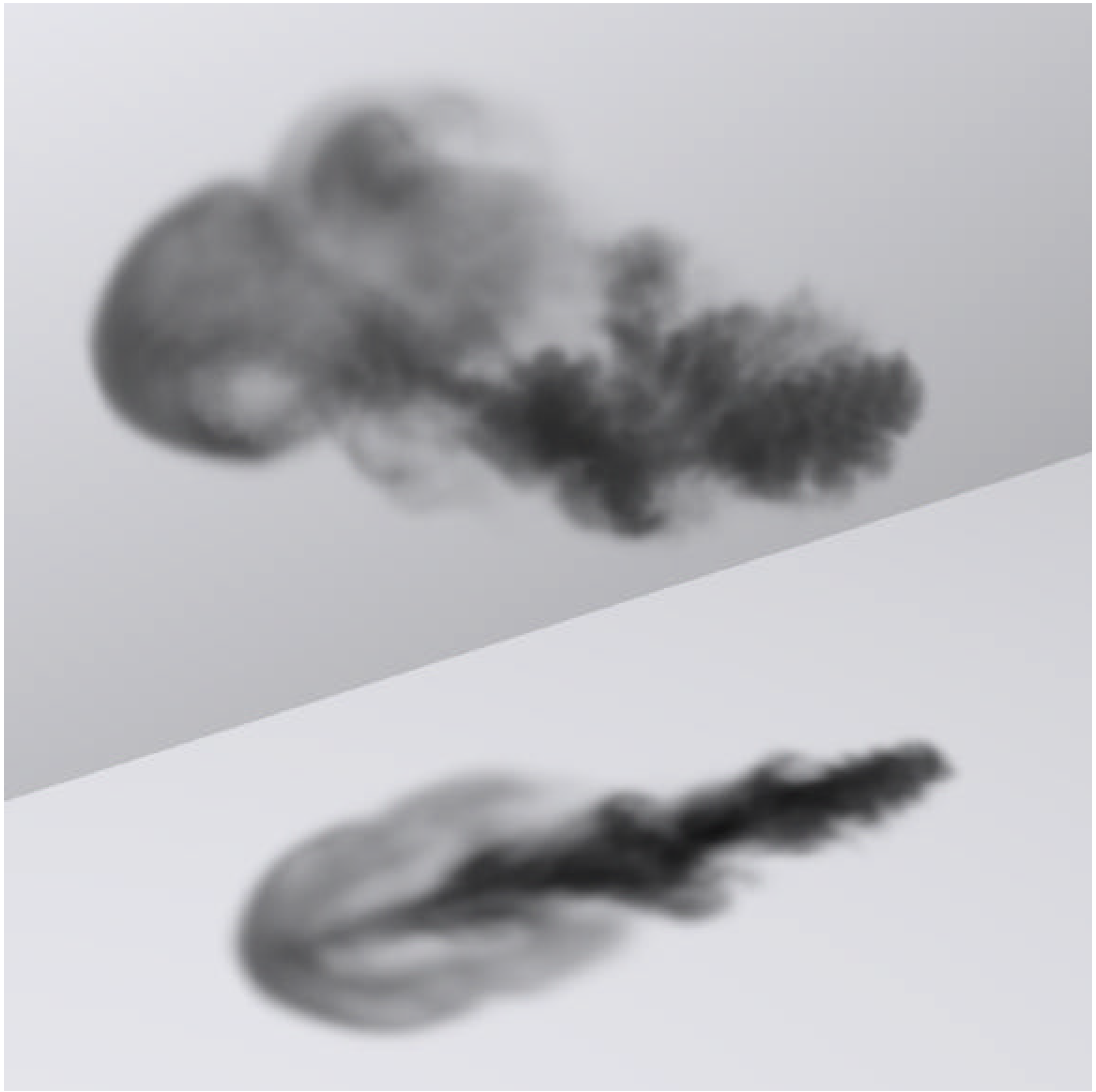
L_o L_i Probability density function

- Normal probability density function (Gaussian)



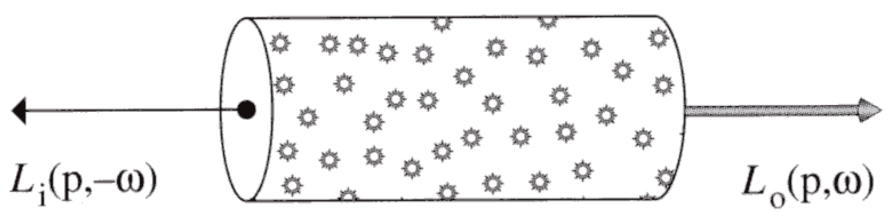
$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Volume Scattering Processes



Volume Scattering Processes

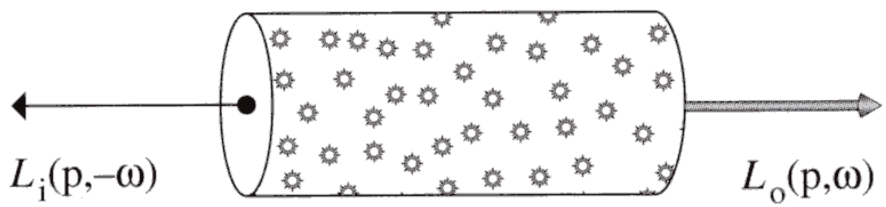
- Emission
  - Light is emitted by the medium
- Emitted radiance: $L_{ve}(p,w)$
  - Independent of incoming light



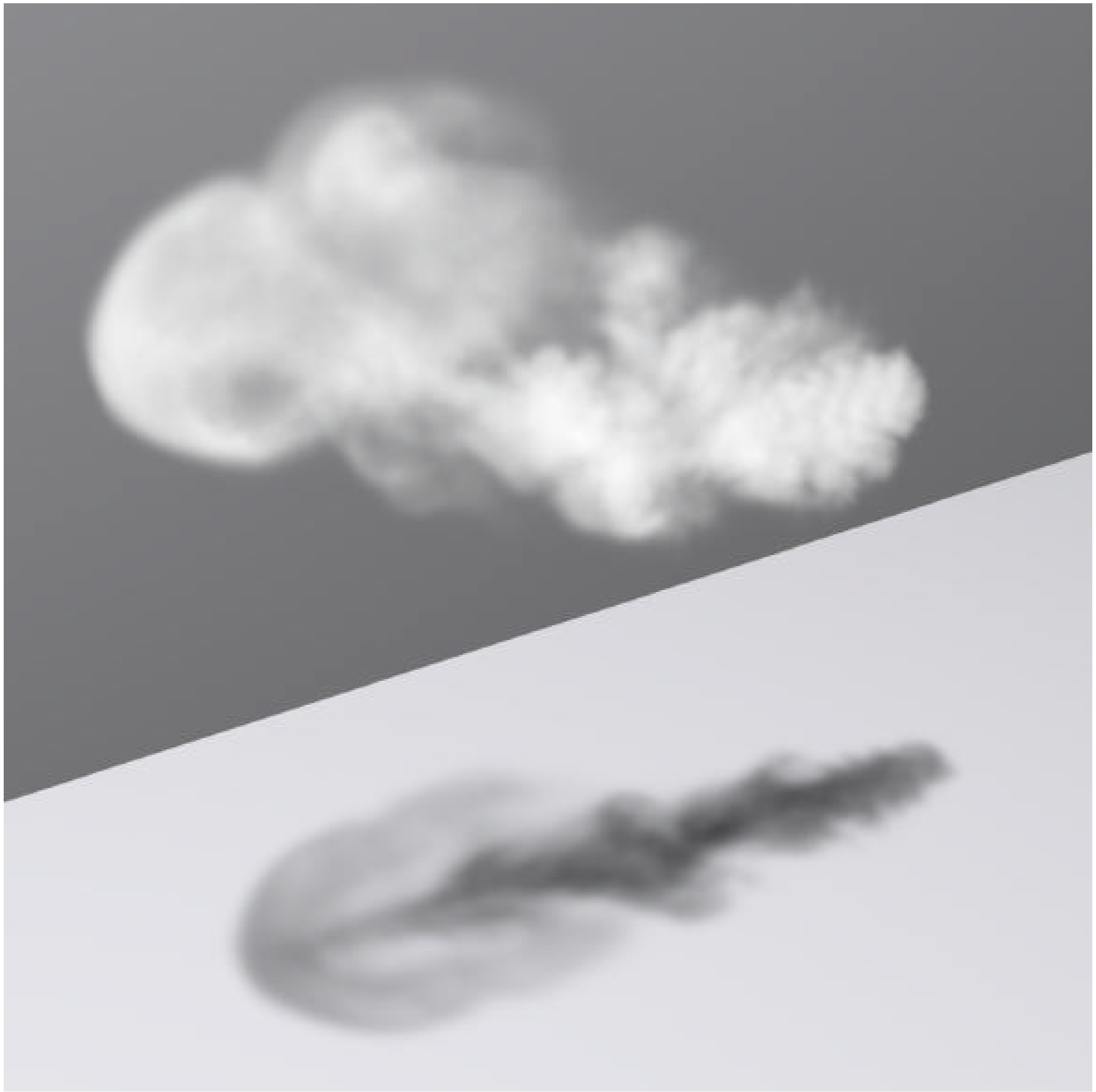
Volume Scattering Processes

- Emission
  - Light is emitted by the medium
- Emitted radiance: $L_{ve}(p,w)$
  - Independent of incoming light
- Change in radiance per unit

$$dL_o(p,w) = L_{ve}(p,w)dt$$



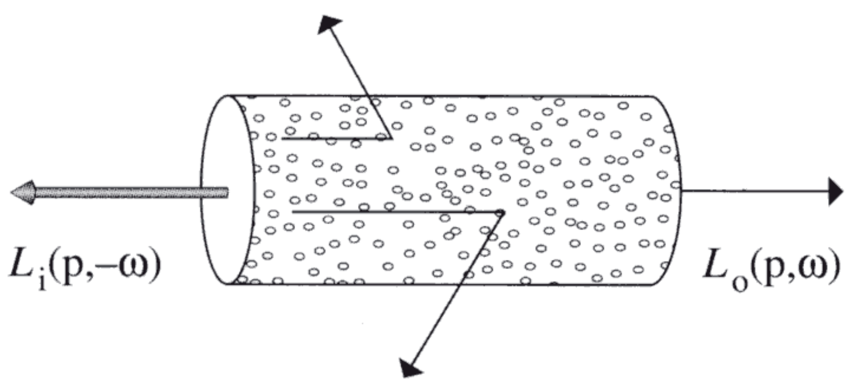
Volume Scattering Processes



Volume Scattering Processes

- Out-scattering
  - Light is scattered out of the path of the ray
  - Probability density for scattering: s_s
  - Reduction in radiance is given by

$$dL_o(p,w) = -s_s(p,w)L_i(p,-w)dt$$



Volume Scattering Processes

- Total radiance reduction
  - Absorption
  - Scattering
- Attenuation or extinction
  - Coefficient: s_t

$$s_t(p,w) = s_a(p,w) + s_s(p,w)$$

- Change in radiance per unit

$$dL_o(p,w) = -s_t(p,w)L_i(p,-w)dt$$

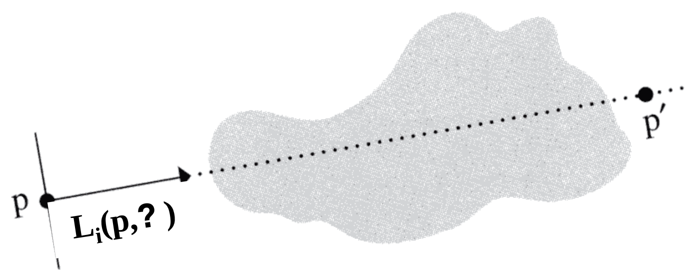
Volume Scattering Processes

- Beam transmittance T_r

s_t integrated along *d (p to p')*

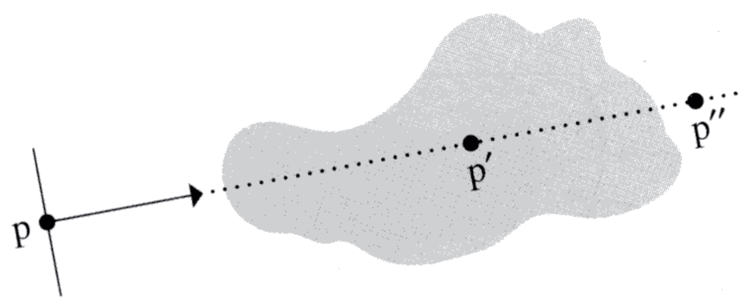
$$T_r(\mathbf{p} \rightarrow \mathbf{p}') = e^{\overbrace{-\int_0^d s_t(\mathbf{p}+wt,w)dt}}$$

$$\underbrace{L(\mathbf{p}')}_{L_o} = \underbrace{T_r(\mathbf{p} \rightarrow \mathbf{p}')}_{\text{Probability density function}} \underbrace{L(\mathbf{p},w)}_{L_i}$$



Volume Scattering Processes

- Transmittance
  - Fraction of light that is transmitted between two points
  - Values between 0 and 1
  - Properties
    - *Tr(p? p) = 1*
    - In vacuum: *Tr(p? p') = 1*, for all *p'*
    - *Tr(p? p'') = Tr(p? p') Tr(p'? p'')*



Volume Scattering Processes

$$T_r(\mathbf{p} \rightarrow \mathbf{p}') = e^{-\int_0^d s_t(\mathbf{p}+wt,w)dt}$$

- Optical thickness

$$t(\mathbf{p} \rightarrow \mathbf{p}') = \int_0^d s_t(\mathbf{p}+wt,w)dt$$

- Homogeneous medium
  - s_t is position independent
  - Transmittance reduced to Beer's Law

$$T_r(\mathbf{p} \rightarrow \mathbf{p}') = e^{-s_t d}$$

Volume Scattering Processes

- Beer's Law

$$A = alc$$

In PBRT:

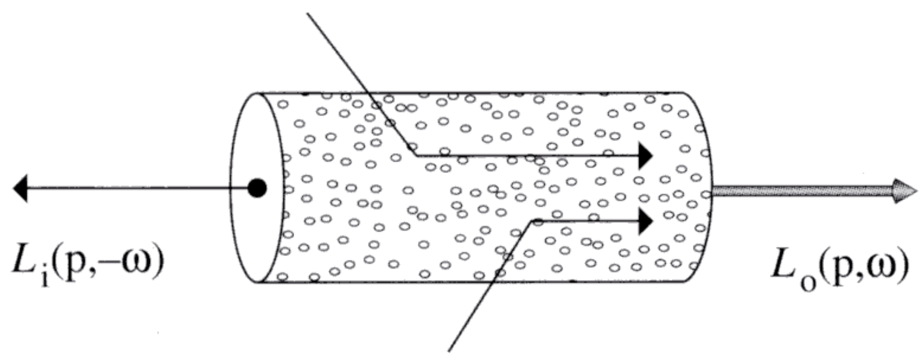
$$s_t = a(c)$$

$$d = l$$

- A = amount of light absorbed
- a = Absorption coefficient or molar absorptivity of medium
- *l* = distance light travels through medium
- c = Concentration or particle density

Volume Scattering Processes

- In-scattering
  - Outside light scatters converging to ray path
  - Phase functions to represent scattered radiation in a point



Volume Scattering Processes

- Phase function (PF)
  - Volumetric analog of BSDF
  - Normalization constraints
    - PF defines a direction's scattering probability distribution

$$\int_{S^2} p(w \to w')dw' = 1$$

- Change in radiance per unit

$$dL_o(p,w) = S(p,w)dt$$

Volume Scattering Processes

- S(p,w) includes volume emission

$$S(\mathbf{p},w)=\underbrace{L_{ve}(\mathbf{p},w)}_{\text{Emission}}+\underbrace{s_{s}(\mathbf{p},w)\int_{S^2}p(\mathbf{p},-w'\rightarrow w)L_i(\mathbf{p},w')dw'}_{\text{In-scattering}}$$

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In-scattering Probability

Amount of added radiance

Emission

In-scattering

Volume Scattering Processes

- S(p,w) includes volume emission

In-scattering Probability

Amount of added radiance

Phase function (range: 0 ? 1)

Incident radiance

$$S(\mathbf{p},w) = \underbrace{L_{ve}(\mathbf{p},w)}_{\text{Emission}} + \underbrace{s_s(\mathbf{p},w)\int_{S^2} p(\mathbf{p},-w' \rightarrow w) L_i(\mathbf{p},w')\,dw'}_{\text{In-scattering}}$$

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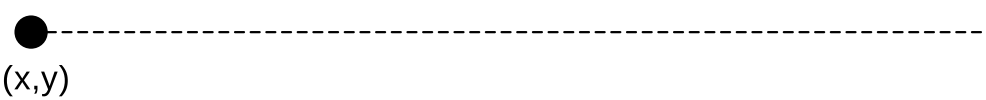

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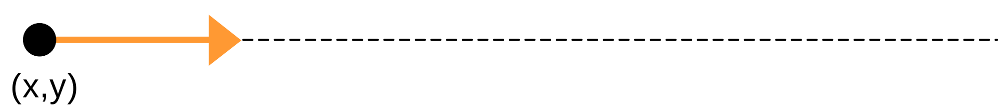

Volume Scattering Processes

- S(p,w) includes volume emission

In-scattering Probability

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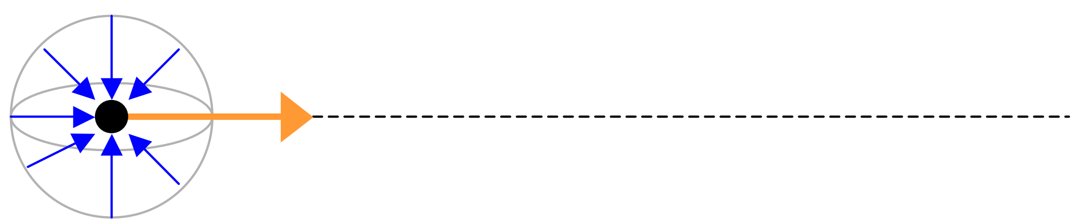
Emission

In-scattering



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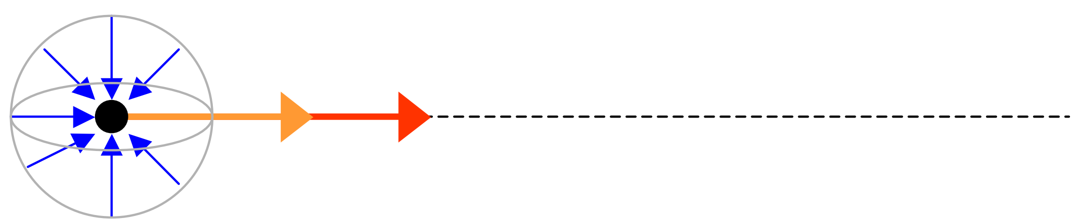
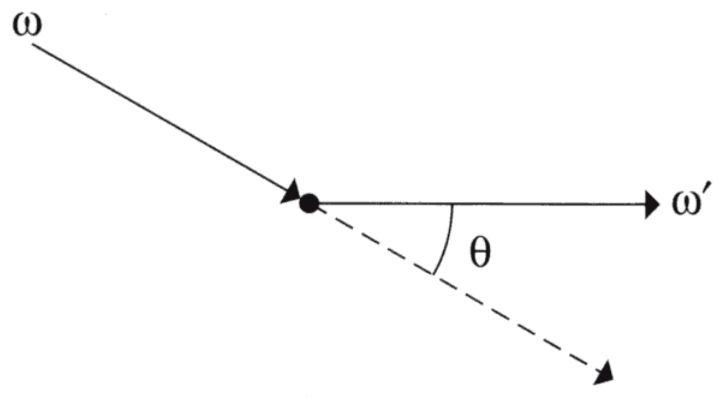
Emission

In-scattering



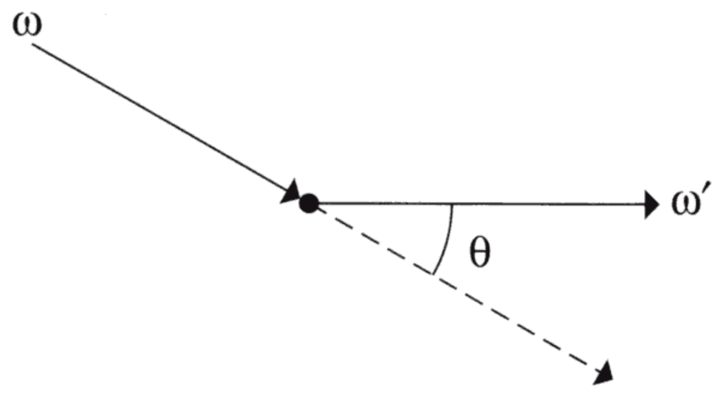
Phase Functions

- BSDFs for volume scattering
- Vary complexity according to medium
  - Isotropic
  - Anisotropic



Phase Functions

- BSDFs for volume scattering
- Vary complexity according to medium
  - Isotropic
  - Anisotropic
- Properties
  - Direction reciprocity
  - May also be classified as
    - Isotropic – uniform scattering
    - Anisotropic – variable scattering



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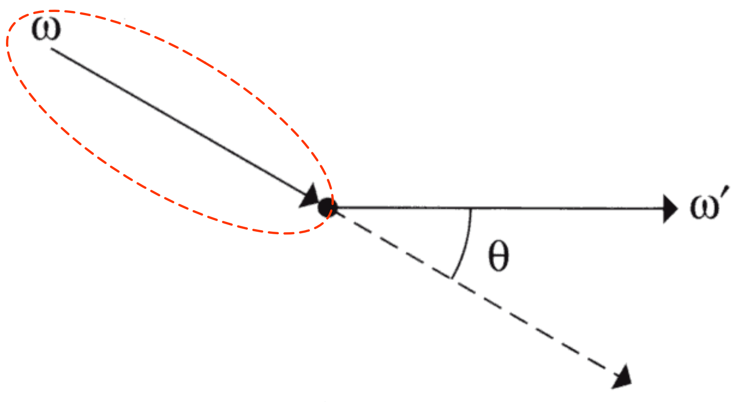
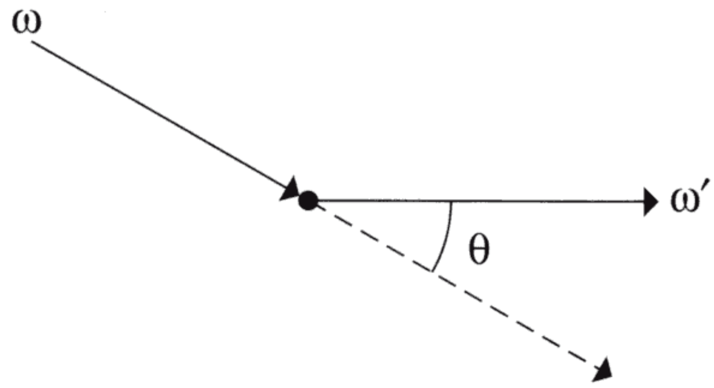


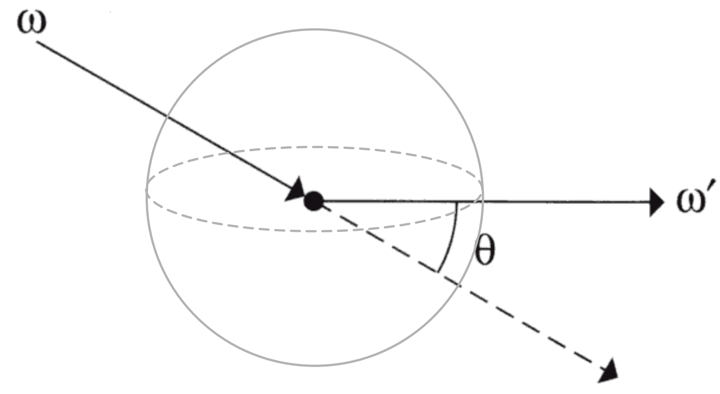
Phase Functions

- Isotropic
  - Basic PFs
  - PFs is constant



Phase Functions

- Isotropic
  - Basic PFs
  - PFs is constant
  - Since
    - Area of sphere = 4?*r2
    - pfS are normalized (r =1)



$$p_{isotropic}(w \rightarrow w') = \frac{1}{4p}$$

Phase Functions

- Rayleigh
  - Very small particles
  - Acurately describes light scattering when
    - Particle radii $<$ light wavelength
  - Good for atmospheric simulation

Phase Functions

- Mie
  - Based on Maxwell's equations
  - Broader range of particle sizes
  - Good for fog and water droplets simulation

Phase Functions

- Henyey and Greenstein
  - Easy to fit
  - Single control parameter
    - Controls relative proportion of forward backward scattering
    - $g \in (-1, 1)$
    - $g < 0$: back scattering

$$p_{HG}(\cos q : g) = \frac{1}{4p} \frac{1 - g^2}{\left(1 + g^2 - 2g(\cos q)\right)^{3/2}}$$

Phase Functions

- Increase complexity by combination

$$p(\cos\theta) = \sum_{i=1}^{n} w_i \, p_{HG}(\cos\theta : g_i)$$

Phase Functions

- Increase complexity by combination

$$p(\cos\theta) = \sum_{i=1}^{n} w_i \, p_{HG}(\cos\theta : g_i)$$

- More efficient version
  - Avoids 3/2 power computation
  - $k \sim 1.55g - 0.55g^3$

$$p_{Schlick}(\cos\theta) = \frac{1}{4\pi} \frac{1 - k^2}{(1 - k\cos\theta)^2}$$

Volume Interface and Homogeneous Media

- VolumeRegion class
  - Volume Scattering Interface in PBRT
  - General methods
    - BBox
    - IntersectP
  - Scattering information methods
    - Sigma_a (absorption)
    - Sigma_s (scattering)
    - Lve (emission)
    - P (phase value)
    - Sigma_t (attenuation)
    - Tau (optical thickness)

Volume Interface and Homogeneous Media

- Homogeneous volumes
  - Uniform particle density
  - One type of particle
- HomogeneousVolume : VolumeRegion
  - s_s And s_a are constant

Varying-Density Volumes

Varying-Density Volumes

- Inhomogeneous volumes
  - Still one type of particle
  - Spatially variable density
  - Scales scattering properties according to density, except for Tau
- DensityRegion : VoumeRegion
  - Method for obtaining density added
  - Tau not implemented
    - Dependent on shape and density distribution function

Varying-Density Volumes

- 3D Grids
  - VolumeGrid : DensityRegion
  - Density values defined in a 3D matrix
  - Intermediary values are interpolated
    - Manhatan distance
      - p1 (x1, y1) and p2 (x2, y2)
        Manhatan distance is |x1 - x2| + |y1 - y2|.
    - Trilinear interpolation
  - Used in smoke pictures

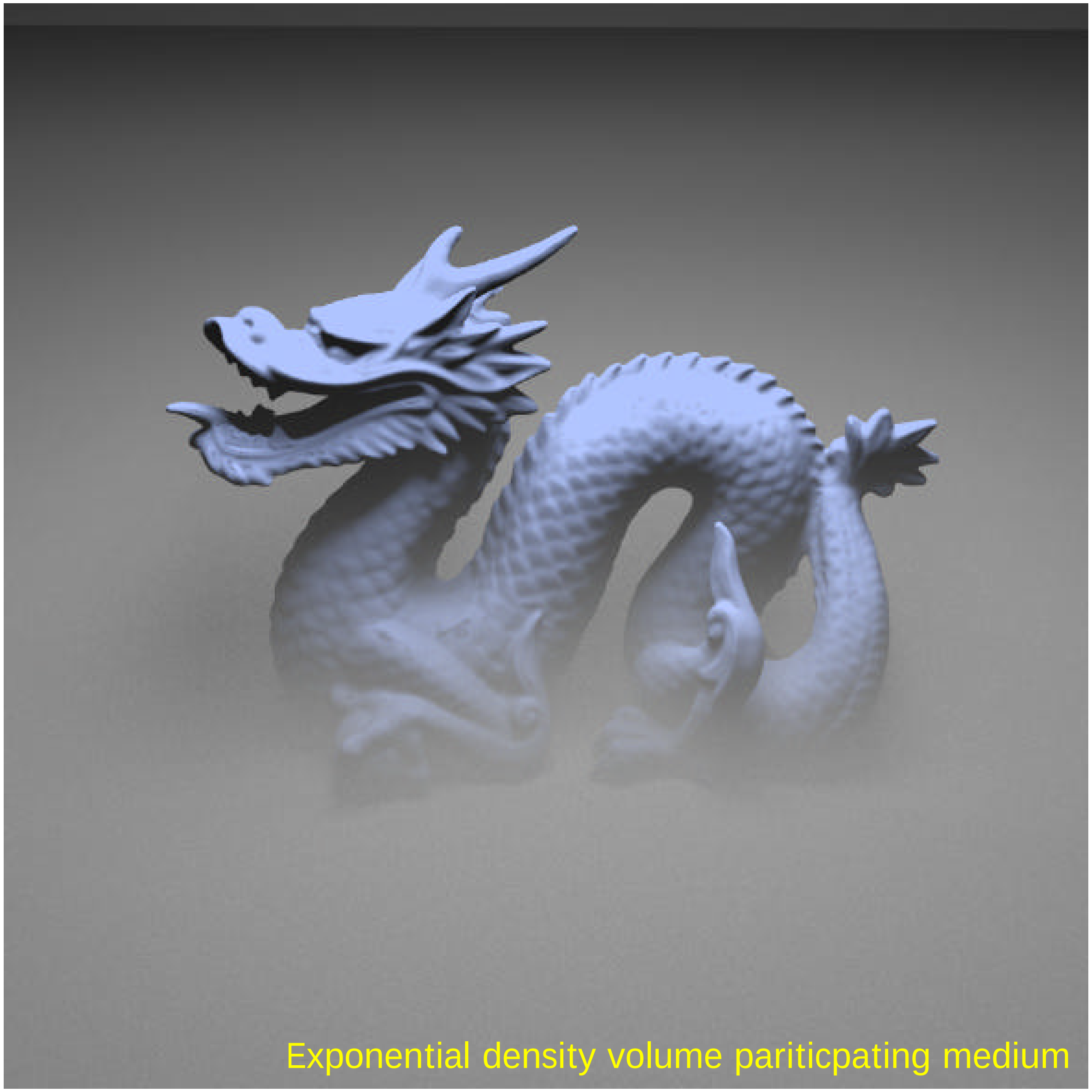
Varying-Density Volumes

- Exponential Density Volume
  - ExponentialDensity : Density Region
  - Density varies with height

$$d(h) = ae^{-bh}$$

  - Good for modelling Earth's atmosphere

Varying-Density Volumes



Volume Aggregates

- AggregateVolume
  - Many Volume Regions
- Advantages
  - Simplifies scene
  - Easy integration with 3D structures
- Parameters creation
  - Sum of individual volumes` parameters

References

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