CS 563 Advanced Topics in Computer Graphics
Light Transport: Volume Rendering

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Introduction

- The Light Transport Equation (LTE) – equation that describes distribution of radiance in the scene
- Integrators – objects (algorithms) that are responsible for finding numerical solution to the LTE
- Two basic classes of Integrators:
  - SurfaceIntegrator
  - VolumeIntegrator

The Equation of Transfer

- The equation of transfer – equation that governs behavior of light in a medium that absorbs, emits and scatters radiation

- Integro-differential form – describes how the radiance along a beam changes at a point in space

- Pure integral form – describes the effect of participating media from infinite number of points along a line

The Equation of Transfer

- Can be derived by subtracting the effects of processes that reduce energy along the beam from those processes that increase energy along it

- The source term:

$$S(p,w) = L_{ve}(p,w) + s_s(p,w) \int_{S^2} p(p,-w' \rightarrow w) L_i(p,w') dw'$$

$L_{ve}(p,w)$ - emitted radiance

$s_s(p,w)$ - scattering probability

$p(p,-w' \rightarrow w)$ - phase function

$L_i(p,w')$ - incident radiance

The Equation of Transfer

- The attenuation coefficient:

$$\sigma_t(p,\omega)$$

$$dL_o(p,\omega) = -\sigma_t(p,\omega)L_i(p,-\omega)dt$$

- The overall change in radiance at a point p' along a ray:

$$\frac{\partial}{\partial t}L_o(p,\omega) = -\sigma_t(p,\omega)L_i(p,-\omega) + S(p,\omega)$$

- To get pure integral form of the above equation assume that the rays have infinite length:

$$L_i(p,\omega) = \int_0^\infty T_r(p' \to p)\cdot S(p',-\omega)dt$$

The Equation of Transfer

$$L_i(p,w) = \int_0^{\infty} T_r(p' \to p) \cdot S(p', -w)\,dt$$

Where $p' = p + tw$

$T_r(p' \to p)$ - beam transmittance from p' to the ray's origin

$$T_r(p' \to p) = e^{-s_t d}$$

The Equation of Transfer

Basic terms of the equation of transfer

The Equation of Transfer

- More generally if a ray (p,w) intersects a surface at p_0 some point the integral equation of transfer is:

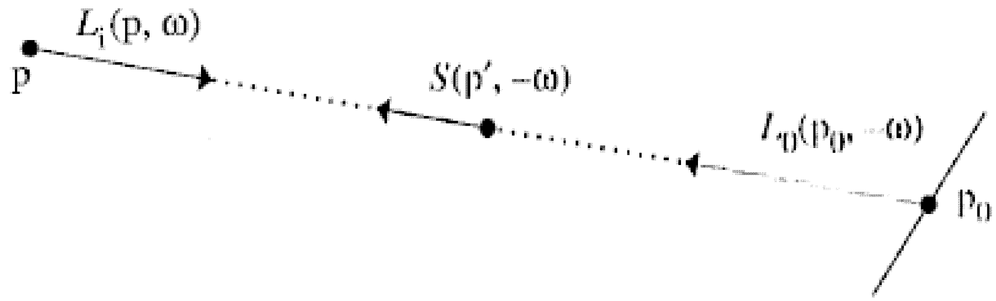
$$L_i(p,w) = T_r(p_0 \to p)L_0(p_0,-w) + \int_0^t T_r(p' \to p) \cdot S(p',-w)dt'$$

$p_0 = p + tw$ - the point on the surface

$p' = p + t'w$ - points along the ray

$L_0(p_0,-w)$ - radiance outgoing from the surface

The Equation of Transfer



For equation of transfer for a finite ray

Volume Integrator Interface

- Integrator → VolumeIntegrator
  - Preprocess()
  - RequestSamples()
  - Li()
  - Transmittance()

- To compute the total radiance arriving at the ray origin:
  - The surface integrator computes outgoing radiance L_0 at the ray's intersection point
  - The volume integrator's Transmittance() computes the beam transmission T_r
  - The volume integrator's Li() gives the radiance along the ray due to participating media
  - The sum of $L_0 T_r$ and the additional radiance from participating media gives the total radiance arriving at the ray origin

Emission-only Integrator

- Uses simplified equation of transfer
  - Ignoring in-scattering term

$$L_i(p,w) = T_r(p_0 \to p) L_0(p_0,-w) + \int_0^t T_r(p' \to p) \cdot S(p',-w)\,dt'$$

$$S(p,w) = L_{ve}(p,w) + s_s(p,w) \int_{S^2} p(p,-w' \to w) L_i(p,w')\,dw'$$

$$L_i(p,w) = T_r(p_0 \to p) L_0(p_0,-w) + \int_0^t T_r(p' \to p) \cdot L_{ve}(p',-w)\,dt'$$

Emission-only Integrator

- Implemented with EmissionIntegrator interface
- Monte-Carlo integration is used by Transmittance() and Li() methods
- Number of samples taken to evaluate estimates of integrals depends on the distance the ray travels in the volume
- The ray is divided into segments of the given length and a single sample is taken in each of the segments

Emission-only Integrator

- Transmittance() implementation
  - VolumeRegion's Tau() method computes optical thickness
  - Feed volumeRegion->Tau() with step size and sample value
  - Return Exp(-tau)

- Li() implementation
  - If the ray enters the volume at $t = t_0$ Li() can consider integral

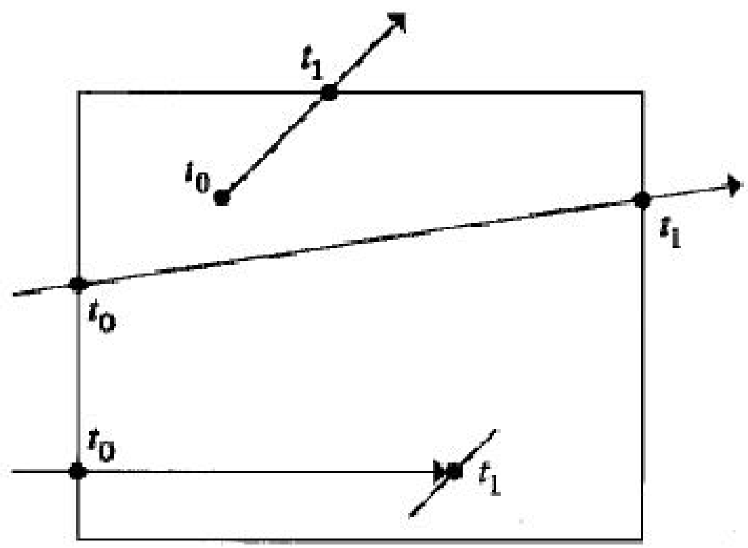
$$\int_{t_0}^{t_1} T_r(p' \to p) \cdot L_{ve}(p', -w)\,dt'$$

where t_1 is the minimum of the parametric offset where the ray exits the volume and the offset where it intersects a surface

Emission-only Integrator

- Li() implementation
  - The integral can be found by uniformly selecting random points along the ray between t_0 and t_1 and evaluating the estimator:

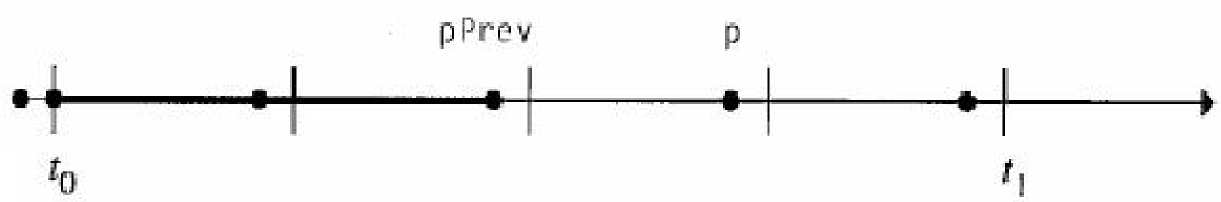
$$\frac{1}{N}\sum_i \frac{T_r(p_i \to p)L_{ve}(p_i,-w)}{p(p_i)} = \frac{t_1 - t_0}{N}\sum_i T_r(p_i \to p)L_{ve}(p_i,-w)$$



Emission-only Integrator

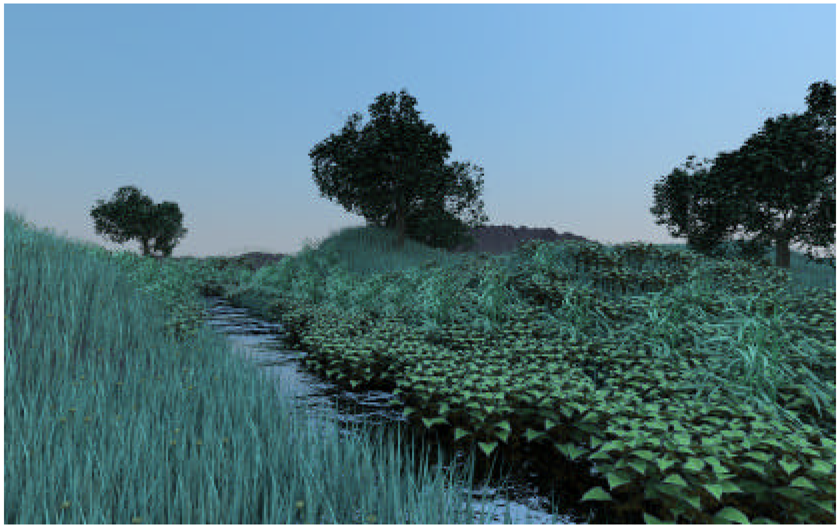
- Additional implementation details:
  - For efficient evaluation of beam transmittance T_r values the points p_i are sorted and multiplicative property of T_r is used to incrementally compute T_r from its value for the previous point:

$$T_r(p_i \to p) = T_r(p_{i-1} \to p) T_r(p_i \to p_{i-1})$$

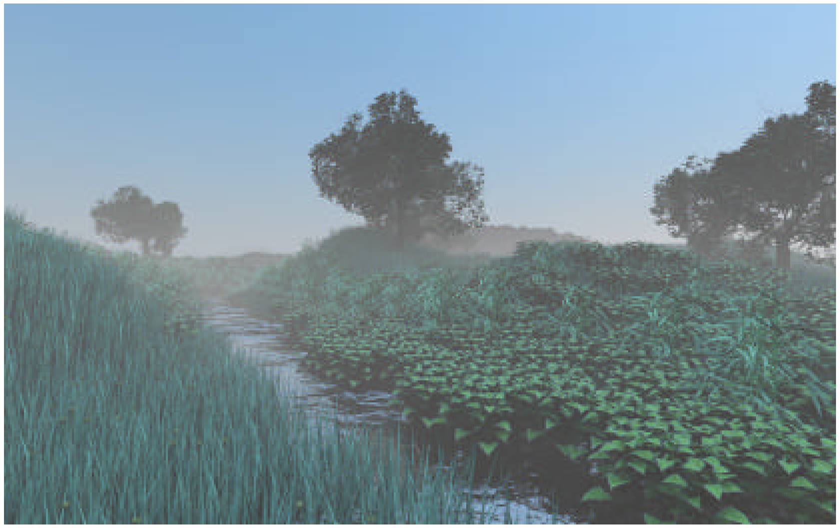


  - Ray stepping is randomly terminated with Russian roulette when transmittance is sufficiently small

Emission-only Integrator



a)



b)

The scene rendered (a) without any participating media and (b) with fog and EmissionIntegrator

Single Scattering Integrator

- SingleScattering integrator considers the incident radiance due to direct illumination ignoring one due to multiple scattering

- Li() method evaluates integral:

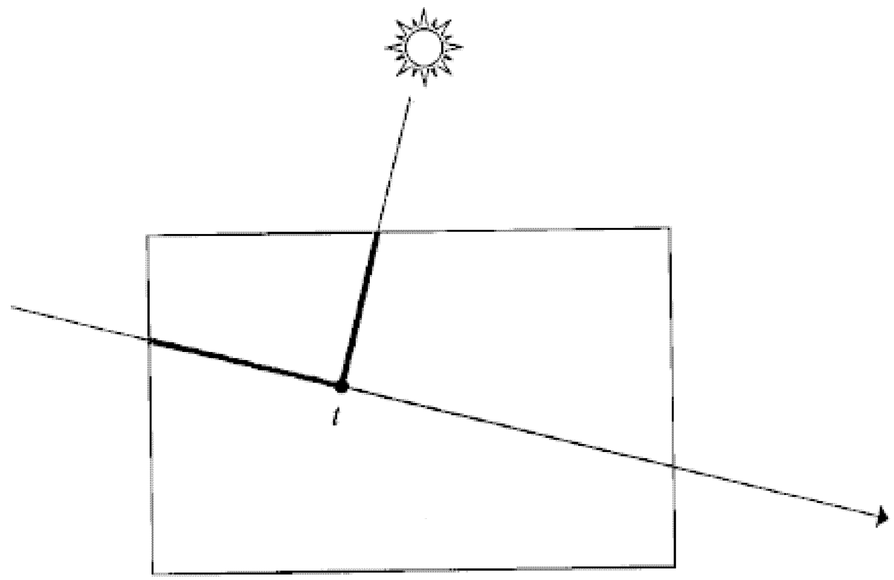
$$\int_0^t T_r(p' \to p) \cdot \left(L_{ve}(p', -w) + s_s(p', w) \int_{S^2} p(p', -w' \to -w) L_d(p', w') dw'\right) dt'$$

- More computationally expensive

- Allows “beams of light” effects

Single Scattering Integrator

$$\sigma_s(p,w)\int_{S^2} p(p,-w' \to -w)L_d(p,w')dw'$$



Evaluation of direct lighting contribution

Single Scattering Integrator

The scene rendered with Single scattering volume integrator

References

- Matt Pharr, Greg Humphreys

  "Physically Based Rendering: From Theory to Implementation"

- Images were taken from the companion CD or scanned from the book