Sampling Theory
Image Reconstruction

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Fourier Analysis

- Fourier Transform defines a spatial function in the *frequency domain*
  - $F(?) = {}_{-8}?^{8} f(x)e^{-i2p?x}dx$
- Fourier Synthesis defines a frequency function in the *spatial domain*
  - $f(x) = {}_{-8}?^{8} F(?)e^{-i2p?x}d?$
- Fourier Analysis reveals error from sampling and reconstruction

Ideal Sampling

- Sampling requires equal spacing
  - Shah function is an infinite sum of equally spaced delta functions
    - $III_T(x) = T\ {}_{i=-8}?^{8} d(x - iT)$
    - T is the period or sampling rate
- Convolution is defined as $f(x)$ c $g(x) =$ -
  ${}_{8}?^{8} f(x')g(x - x')dx'$
- Reconstruction is done by $(III_T(x)\ f(x))$ c $r(x)$
  - $f'(x) = T_{i=-8}?^{8} f(iT)r(x - iT)$

Ideal Sampling

- Frequency space representation of *f*(x) provides better information
- Need to transform the Shah function to frequency domain
  - $III_{1/T}(?\ ) = 1/T \ _{i=-8}?^{\,8} d(?\ - i/T)$
- $F(?\ )$ c $III_{1/T}(?\ )$ gives representation of samples
- Reconstruction similar to spatial domain

Aliasing

- Undersampling causes overlapping of F(?)
- Reconstruction causes aliases
- *Nyquist frequency* tells us sampling frequency
- What about non-band limited signals?

Antialiasing

- Nonuniform Sampling
  - Reduces impact of aliasing by varying spacing of samples
  - Turns aliasing artifacts in noise
- Adaptive Sampling
  - More samples in higher frequencies
  - Sample adjacent values searching for significant changes
- Prefiltering
  - Blur out high frequencies

Aliasing in Rendering

- Geometry creates a step function
  - Sinc filter causes oscillations
- Small objects may flicker
- Textures and materials
  - Shading

References

- *Physically Based Rendering: From Theory to Implementation*; Pharr and Humphreys; 2004; pp 280-302