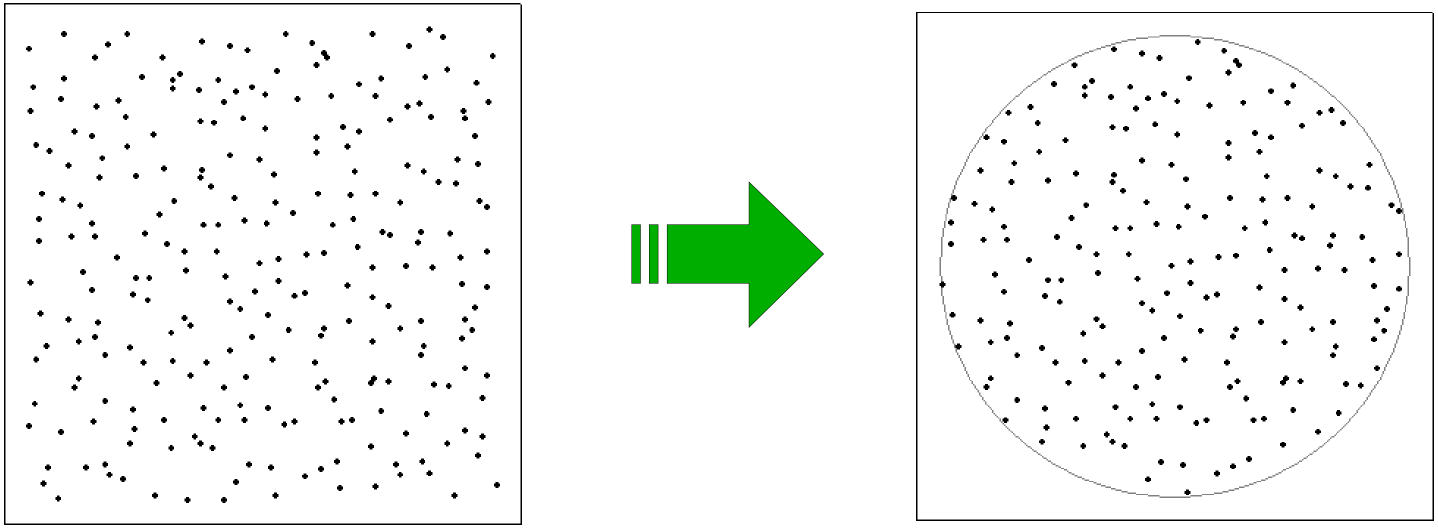
CS 563 Advanced Topics in Computer Graphics

Disc and Hemisphere Sampling

by Joe Miller

Mapping Samples to a Disc

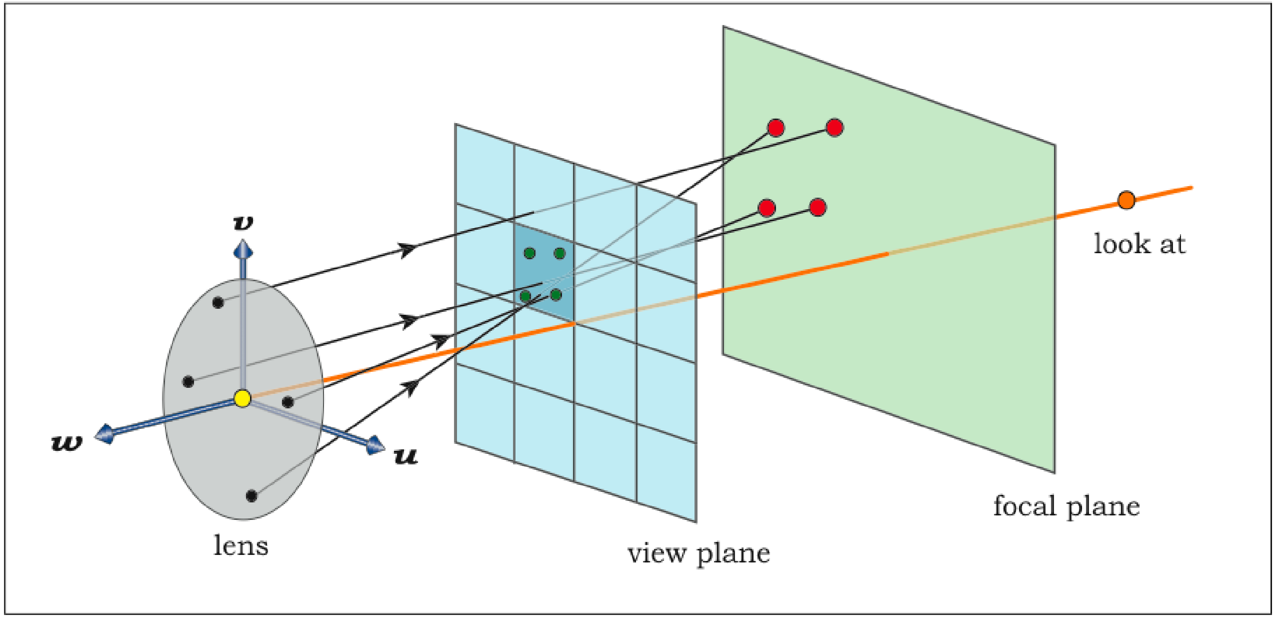
- Until now sampling has been done on the unit square



- Now we'd like to find samples in a circular area

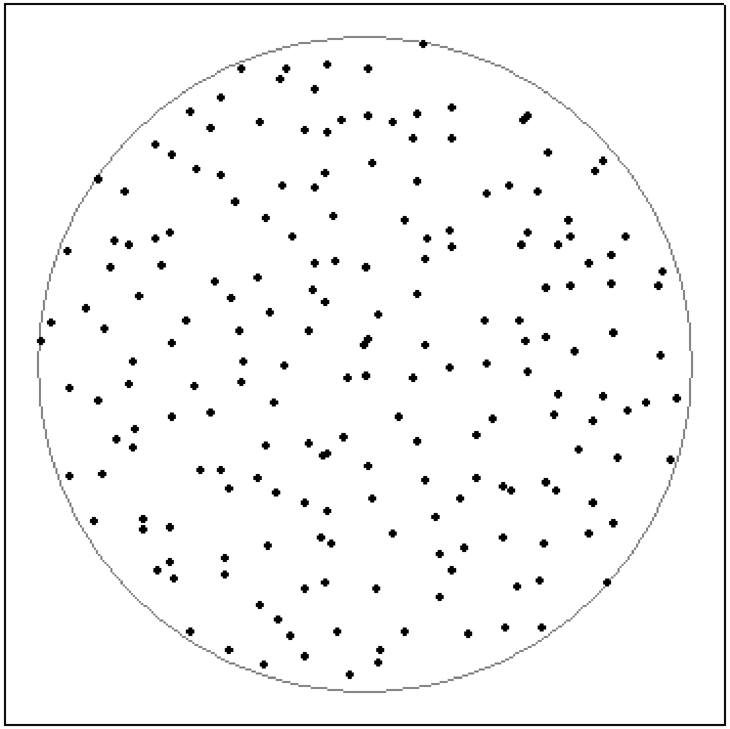
Application of Disc Sampling

- When is disc sampling used?
  - Sampling a circular lens
  - Shading with disc lights



Rejection Sampling

- Simplest approach is to use the same techniques for unit square and **reject** the samples that are not in the circle



Rejection Sampling

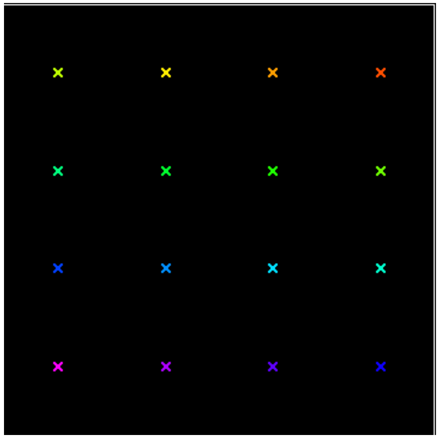
- Advantages
  - Simple, simple, simple
- Disadvantages
  - Breaks uniformity of some sampling techniques
    - N-rook, multi-jittered or Hammersley
  - Wastes time looking at many samples that are simply dropped
    - 2D uniform sampling loses ~21% of the samples
- Alternative is to map sample locations from the unit square to a unit circle

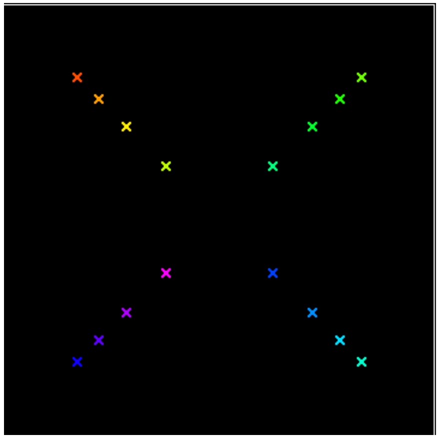
Polar Mapping

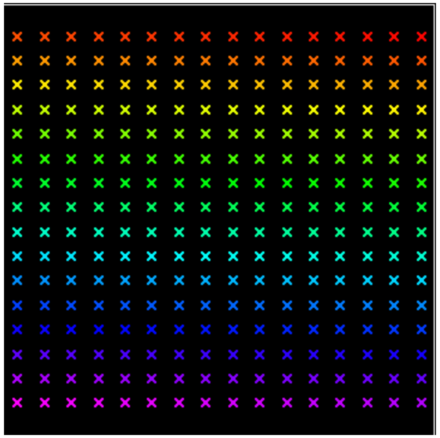
- One option is to use a polar mapping technique
- To do this convert unit square coordinates to polar coordinates: (x_s, y_s) => (r,□)
  - $r = □x_s$
  - □= 2 y_s
- Convert polar coordinates to sample coordinates
  - x = r * cos (□)
  - y = r * sin (□)

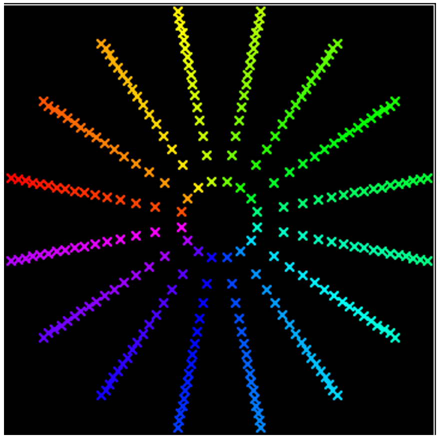
Polar Mapping

- Example using 64 points and 256 points







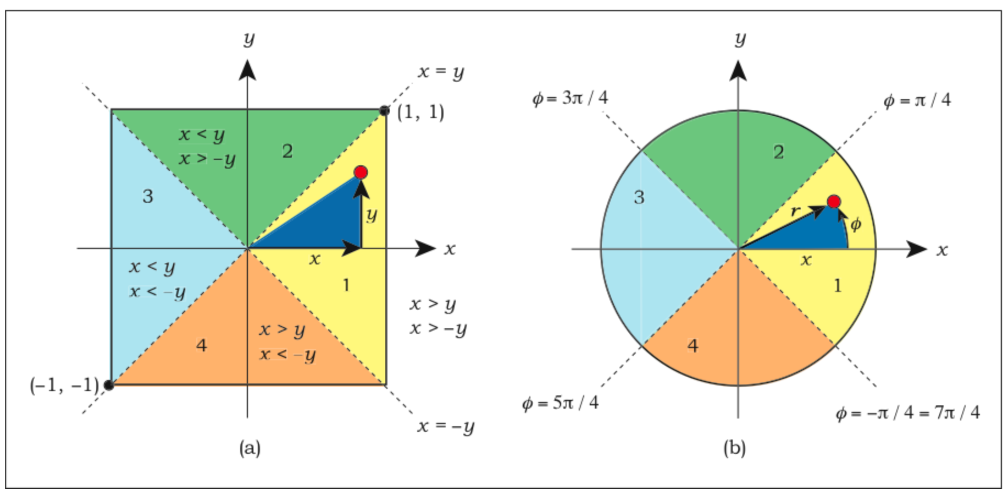


Polar Mapping

- One problem with polar mapping is that the many parts of the disc are under-sampled
  - Doesn't maintain the minimum distance between points
- Another problem is the mapping grossly distorts the original point
- So....

Concentric Mapping

- Another approach is to use a concentric mapping of the unit square to a unit circle
- Divide the unit square and circle into 4 quadrants along the 45° lines
- Set r to x or y and □to a ratio of x and y based on the quadrant



Concentric Mapping

- Here's a table of the radius and angle for each of the quadrants

Quadrant	Angular Range	(x,y) Range	Map Equations
1	315° < □<= 45°	x > -y x > y	r = x □= □/4 * y/x
2	45° < □<= 135°	x > -y x < y	r = y □= □/4 * (2 - x/y)
3	135° < □<= 225°	x < y x < -y	r = -x □= □/4 * (4+y/x)
4	225° < □<= 315°	x > y x < -y	r = -y □= □/4 * (6 - x/y)

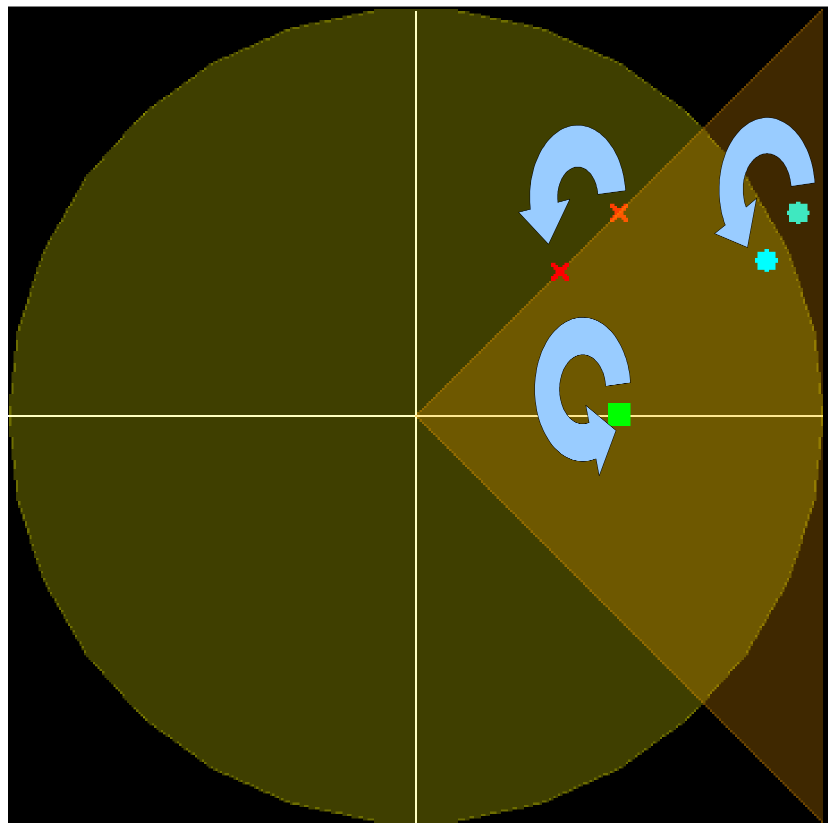
- Calculate x,y same as before
  - x = r * cos (□)
  - y = r * sin (□)

Concentric Mapping

- Here's an example of how the unit square maps back to the unit circle

- Unit Circle

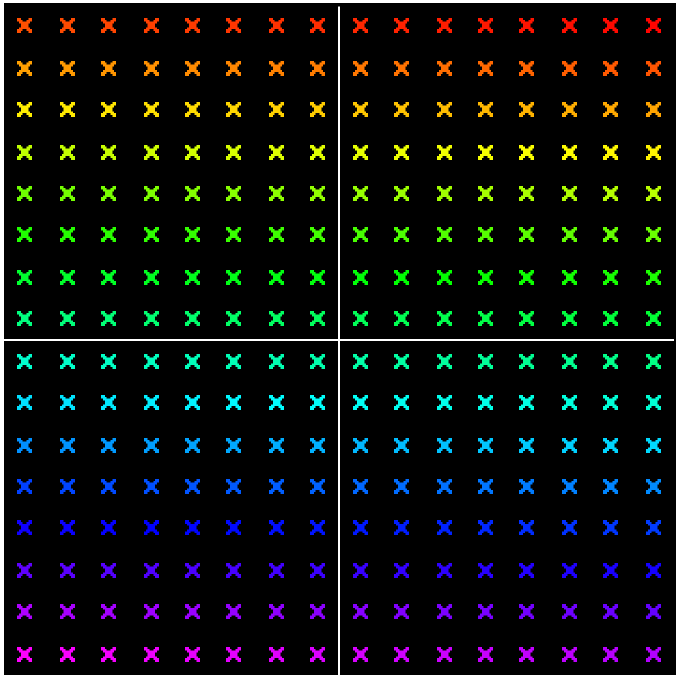
- Quadrant 1

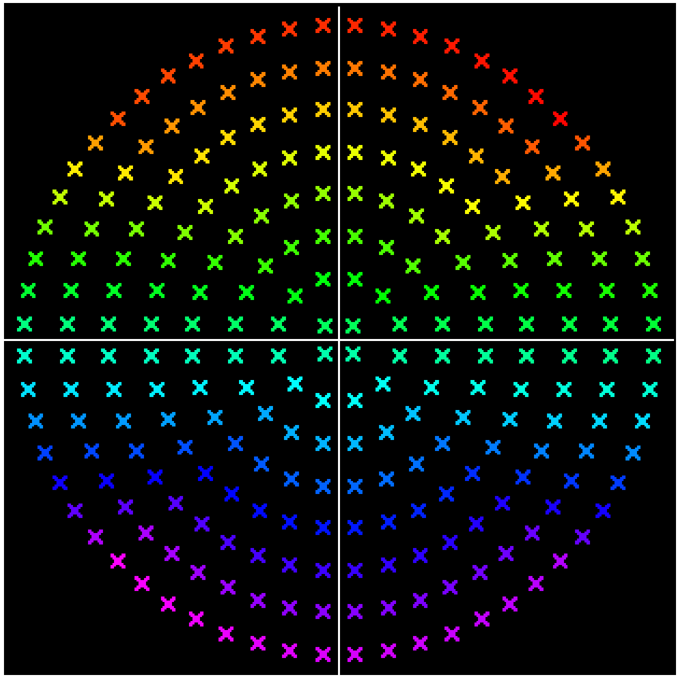


- r = x
- □= □/4*y/x
- x = r*cos(□)
- y = r*sin (□)

Concentric Mapping

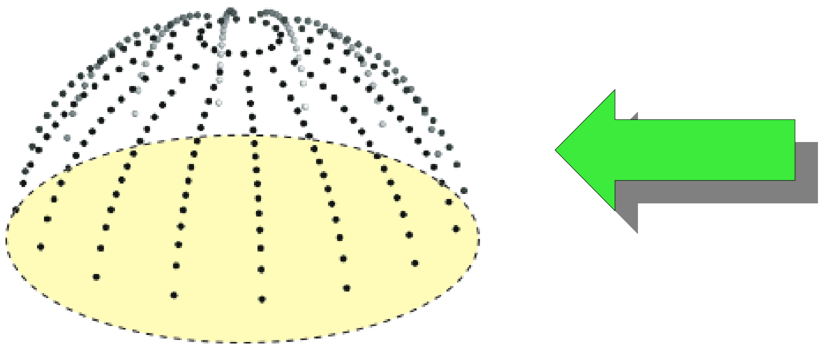
- Here's the projection of 256 points





Mapping Samples to a Hemisphere

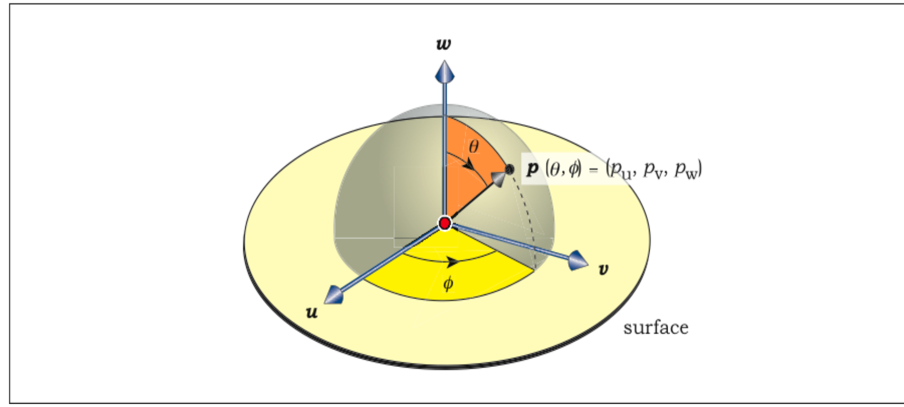
- The next challenge is to get samples on a unit hemisphere
- Use the same concept as disc mapping of using the algorithms derived for a unit square and remapping them to the unit hemisphere



256 regular samples mapped to a hemisphere

Hemisphere Mapping Algorithm

- In this case we'll define a hemisphere in spherical coordinates (□,□) and then revert that back to a cartesian 3D point



- The equation for (□,□) where (x_s, y_s) is a position in the unit square with range [0,1]
  - □= 2ξ□ σ
  - $= \cos^{-1}[(1-y_s)^{1/(e+1)}]$
- And then as a point in 3D space
  - **p** = sin □cos □**u** + sin □sin □**v** + cos □**w**

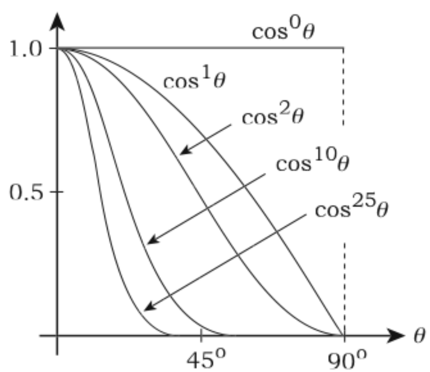
Hemisphere Mapping Algorithm

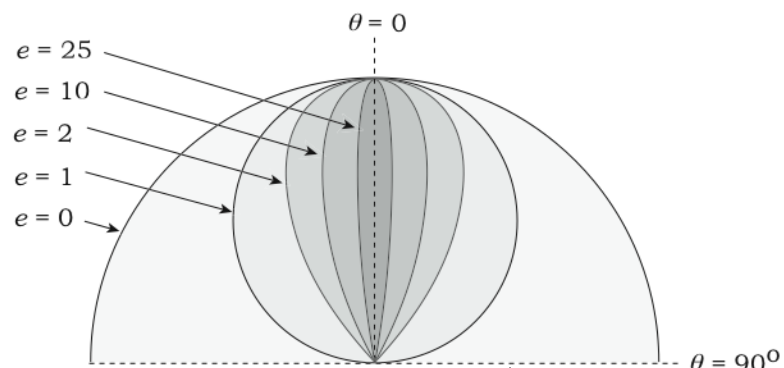
- □= 2ξ□ σ
  - As x_s increases from 0 → 1 then □goes from 0° → 360°
- = $\cos^{-1}[(1-y_s)^{1/(e+1)}]$
  - Ignore the e for right now
  - As y_s increases from 0 → 1 then □goes from 0° → 90°
- **p** = sin □cos □**u** + sin □sin □**v** + cos □**w**
  - General equation for spherical coordinates

Cosine Distribution

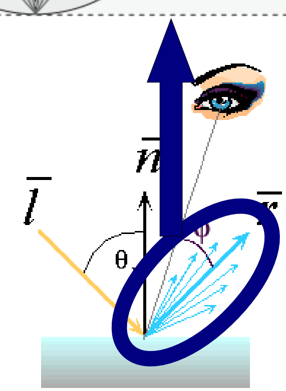
- What about that e in □= $\cos^{-1}[(1-y_s)^{1/(e+1)}]$

- The book describes this as the cosine distribution function
  - As general function $d = \cos^e$ (□)



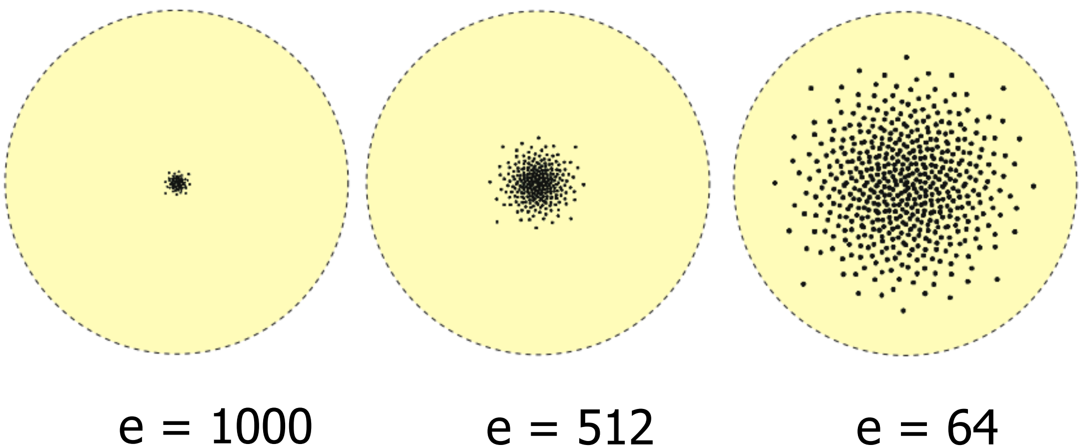


Phong → $I_s = I_L \cos^{n_{shiny}}$ □



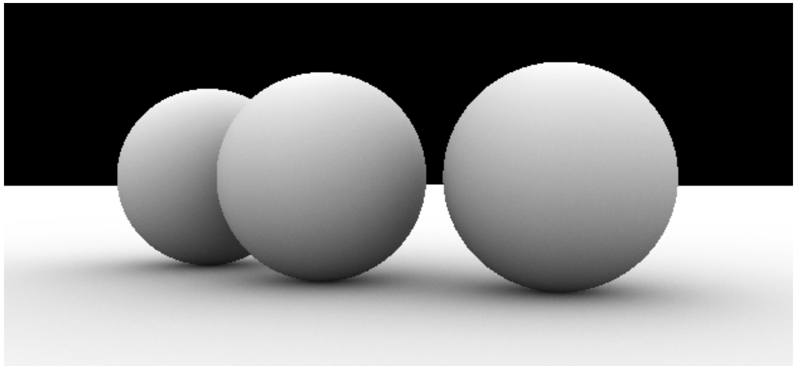
Cosine Distribution

- So what does it do for us?
- The distribution allows the system to tighten the sample distribution
  - As e gets larger the sample distribution gets tighter
    - e = 0 is a the unit hemisphere – radius of 1
    - e = 1 is a hemisphere is a radius of .5 (shifted)



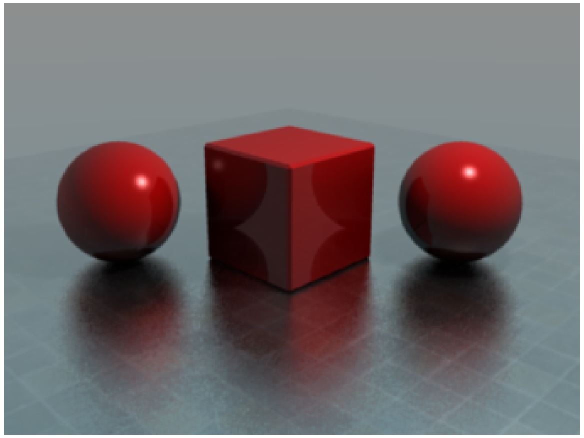
Applications of Hemisphere Mapping

- Here are a few places where hemisphere mapping is used in ray tracing



Ambient occlusion (chapter 17)

Glossy Reflection (chapter 25)



And many more ...

References

- Suffern, Kevin (2007). *Ray Tracing from the Ground Up.* pp. 119-131 Wellesley, MA: A K Peters, Ltd.
- Shirley, P. and K. Chiu (1997). A low distortion map between disk and square. *journal of graphics tools*, 2(3), 45-52