CS 563 Advanced Topics in Computer Graphics

Microfacet BRDFs and Scattering

by Emmanuel Agu

Recall

- BRDFs have evolved historically
- 1970's: Empirical models
  - Phong's illumination model
- 1980s:
  - Physically based models
  - Microfacet models (e.g. Cook Torrance model)
- 1990's
  - Physically-based appearance models of specific effects (materials, weathering, dust, etc)
- Early 2000's
  - Measurement & acquisition of static materials/lights (wood, translucence, etc)
- Late 2000's
  - Last week: Measurement & acquisition of time-varying BRDFs (ripening, etc)

Physically-Based Shading Models

- Phong model produces pretty pictures
- **Cons:** empirical fudge ($cos^n \phi$), plastic look
- First physically-based models were based on microfacet Theory
- Classic: Cook-Torrance shading model (TOGS 1982)

Cook-Torrance Shading Model

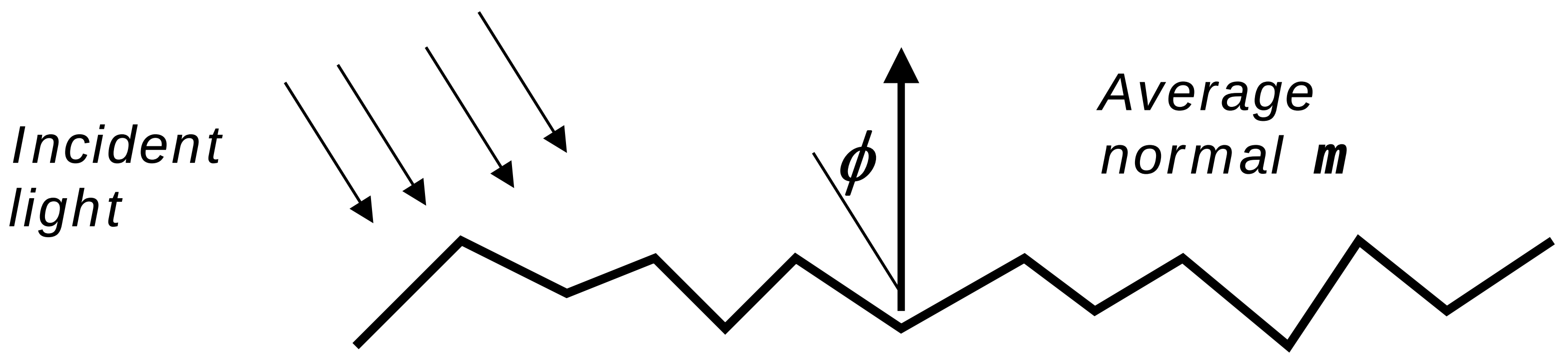
- Similar ambient and diffuse terms to Phong
- More complex specular component than ($cos^n \phi$)
- Define new specular term

$$\cos^n \phi \rightarrow \frac{F(\phi, \eta) DG}{(\mathbf{m} \cdot \mathbf{v})}$$

- where
  - D - Distribution term
  - G – Geometric term
  - F – Fresnel term
- Let’s explain each term

Distribution Term, D

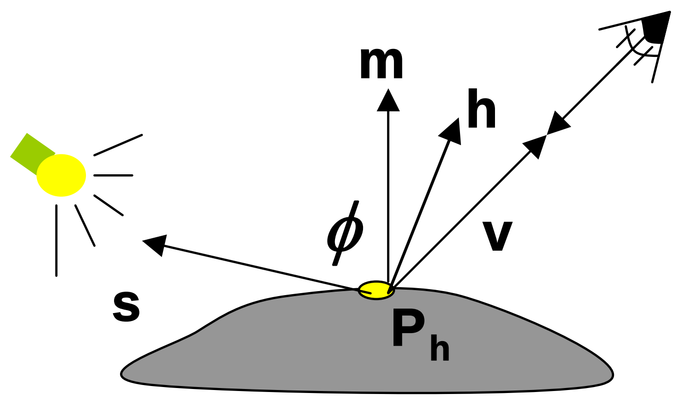
- Surface composed of **microfacets:** small V-shaped grooves



- Only grooves facing mirror directions reflect light
- D term expresses groove directions
- Note: several grooves occur at each hit point
- Technically, D expresses direction of aggregates (distribution)
- E.g. half of grooves at hit point face 30 degrees, etc

Cook-Torrance Shading Model

- Can define mirror direction using Blinn's halfway vector, **h = s + v**
- Only microfacets with V normal pointing in **h** direction contributes



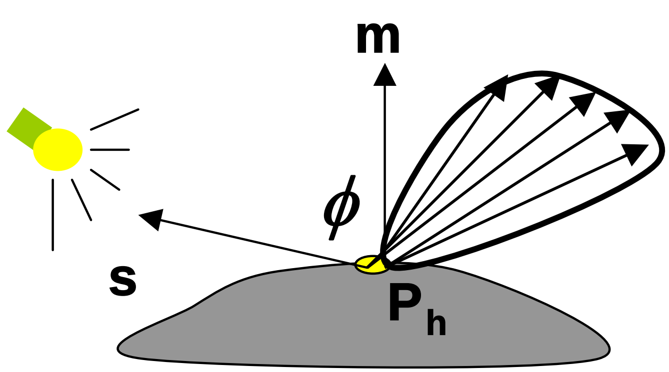
- Define angle δ as deviation of **h** from surface normal
- D(δ) is fraction of microfacets facing angle δ
- Can actually plug old Phong cosine ($cos^n\phi$), in as D
- More widely used is Beckmann distribution

$$D(\delta) = \frac{1}{4\mathbf{m}^2 \cos^4(\delta)} e^{-\left(\frac{\tan(\delta)}{\mathbf{m}}\right)^2}$$

- Where **m** expresses roughness of surface

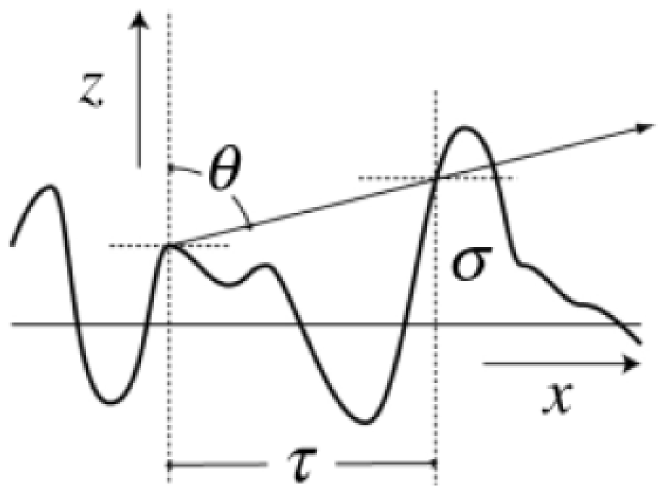
Cook-Torrance Shading Model

- **m** is Root-mean-square (RMS) value of slope of V-groove
- Basically, m exresses slope of V-groove
- m = 0.2 for nearly smooth
- m = 0.6 for very rough



Microfacet Slope

- Slope



- Beckmann Distribution of Microfacet Slope

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Beckmann

Other Microfacet Distributions

- Some popular distributions

- **Blinn** $D_1(\alpha) = \cos^{c_1} \alpha \qquad c_1 = \frac{\ln 2}{\ln \cos \beta}$
- **Torrance-Sparrow** $D_2(\alpha) = e^{-(c_2 \alpha)^2} \qquad c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $D_3(\alpha) = \frac{c_3^2}{(1 - c_3^2)\cos^2 \alpha - 1}$

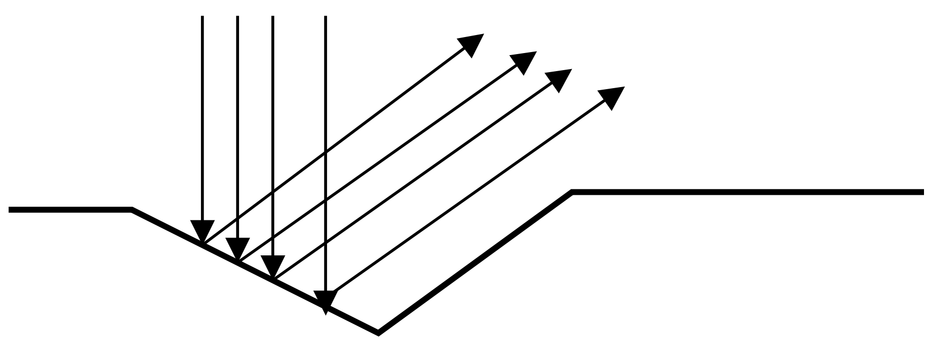
$$c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$$

Self-Shadowing

- Geometric Term, G

Geometric Term, G

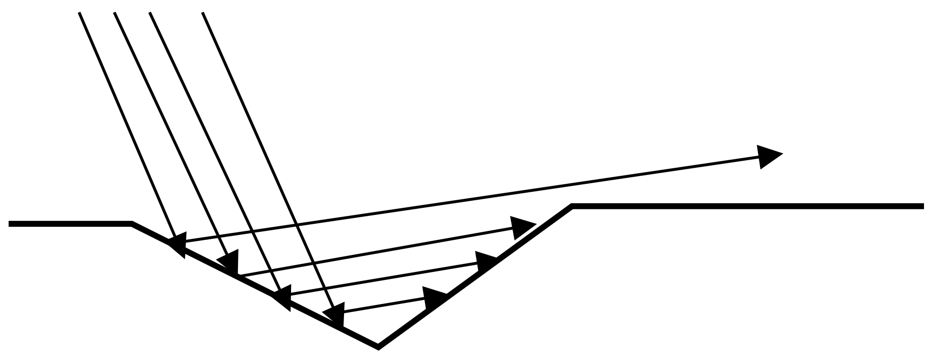
- Surface may be so rough that edges block groove interior from light
- This is known as **shadowing** or **masking**
- Geometric term G accounts for this
- Break G into 3 cases:
- **G, case a:** No self-shadowing



- Mathematically, G = 1

Geometric Term, G

- **G, case b:** No blocking of incident light, partial blocking of exitting light **(masking)**



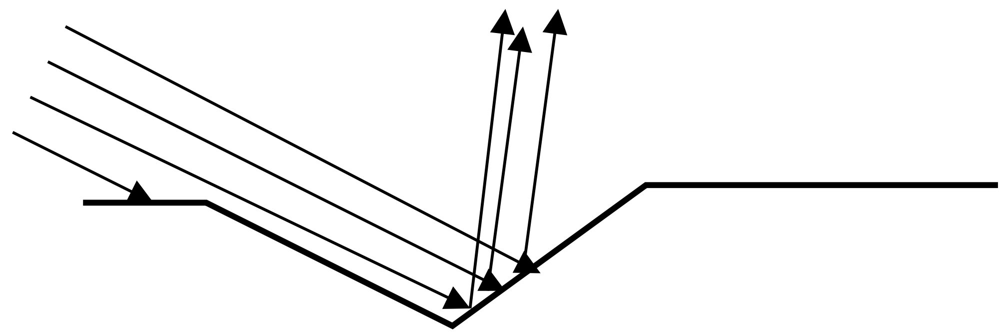
- Mathematically,

$$G_m = \frac{2(\mathbf{m} \cdot \mathbf{h})(\mathbf{m} \cdot \mathbf{h})}{\mathbf{h} \cdot \mathbf{s}}$$

Geometric Term, G

- G, case c: Partial blocking of incident light, no blocking of exitting light (**shadowing)**
- Mathematically,

$$G_m = \frac{2(\mathbf{m}\cdot\mathbf{h})(\mathbf{m}\cdot\mathbf{h})}{\mathbf{h}\cdot\mathbf{s}}$$



- G term is minimum of 3 cases, hence

$$G = (1, G_m, G_s)$$

Fresnel Term, F

- So, again recall that specular term

$$spec = \frac{F(\phi,\eta)DG}{(\mathbf{m}\cdot\mathbf{v})}$$

- Microfacets are not perfect mirrors
- $F(\phi,\eta)$ term gives fraction of incident light reflected (angle-dependent)
- ϕ is incident angle, η is refractive index of material

$$F = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left\{1+\left(\frac{c(g+c)-1}{c(g-c)-1}\right)^2\right\}$$

- where c = cos(ϕ) = **m.s** and $g^2 = \eta^{2\,+}\,c^2 + 1$

Fresnel Term, F

- Combining expressions

$$spec = \frac{F(\phi, \eta) DG}{(\mathbf{m} \cdot \mathbf{v})}$$

- In above expression for F, could simply use *FDG*
- Why divide by **m.v**?
- Accounts for why when eye is close to surface, more microfacets are seen per solid angle than when eye is close to normal

Fresnel Term, F

- Required that $k_d + k_s = 1$
- For spec, we need $F(\phi,\eta)$
- Usually, $F(0,\eta)$ is available from tables (Terloukian)
- Inserting $\phi = 0,\ c = 1$ in expression for F

$$F = \frac{(\eta - 1)^2}{(\eta + 1)^2}$$

- And

$$\eta = \frac{1 + \sqrt{F_0}}{1 - \sqrt{F_0}}$$

- So, use tabulated $F(0,\eta)$ values to calculate η
- Then use calculated η in original equation for F

Some Fresnel Values, F(0)

- At incident angle 0

Material	Fresnel Value (R,G,B)
Water	0.02, 0.02, 0.02
Plastic	0.05, 0.05, 0.05
Glass	0.08, 0.08, 0.08
Diamond	0.17, 0.17, 0.17
Copper	0.95, 0.64, 0.54
Aluminum	0.91, 0.92, 0.92

- Schlick approximation to get arbitrary F

$$F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$$

Final Words

- Oren-Nayar – Lambertian not specular
- Aishikhminn-Shirley – Grooves not v-shaped. Other Shapes
- BRDF viewer
- Microfacet generator

Subsurface Scattering in liquids and solids

- Examples

Jade

Marble

Skin

- Next week: scattering in atmosphere, clouds, gases, etc

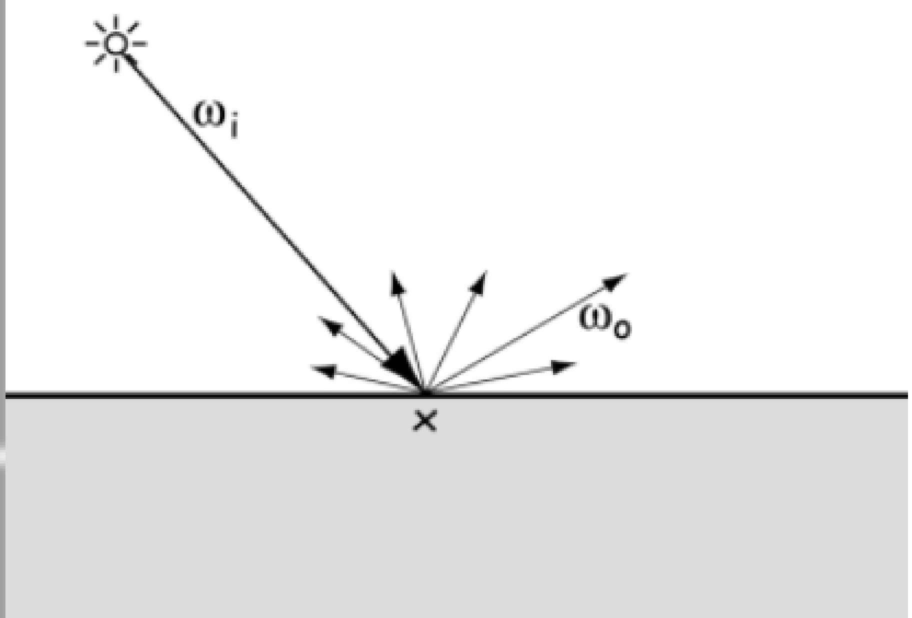
More Examples…

Leaves

Hair

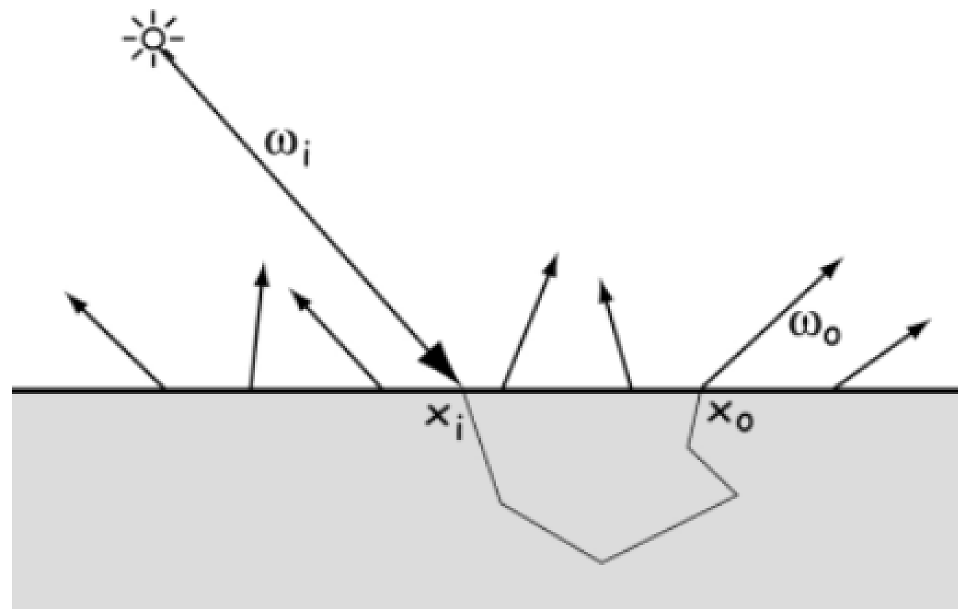
Milk

Subsurface Scattering



$$f_r(x,\omega_i,\omega_o) \equiv \frac{dL_r(x,\omega_o)}{dE_i(x,\omega_i)}$$

Reflection

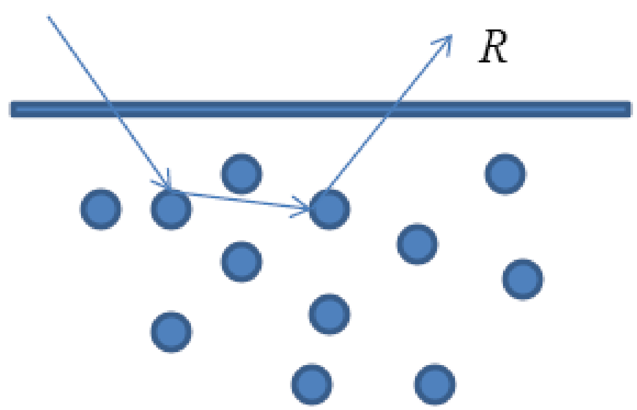


$$S(x_i,\omega_i;x_o,\omega_o) \equiv \frac{dL_r(x_o,\omega_o)}{d\Phi_i(x_i,\omega_i)}$$

Subsurface Scattering

Modeling Subsurface Scattering

- Multiple layers that reflect or transmit light
- Origin: Kubelka-Munk theory for modeling scattering of paint pigments
- K, fraction of light absorbed
- S, fraction of light scattered



$$R = 1 + \frac{K}{S} - \sqrt{\frac{K}{S} + 2\frac{K}{S}}$$

Dorsey and Hanrahan

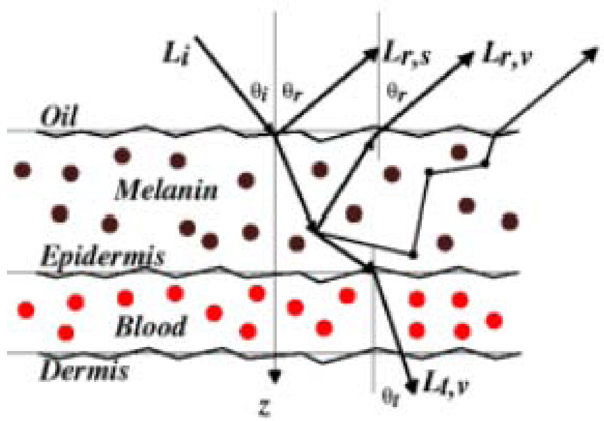
- Kubelka Munk extended by Dorsey, Hanrahan
- Multiple layers of scattering
- Considering material of thickness *d*

$$R = \frac{\sinh(bSd)}{a\sinh(bSd) + b\cosh(bSd)}$$

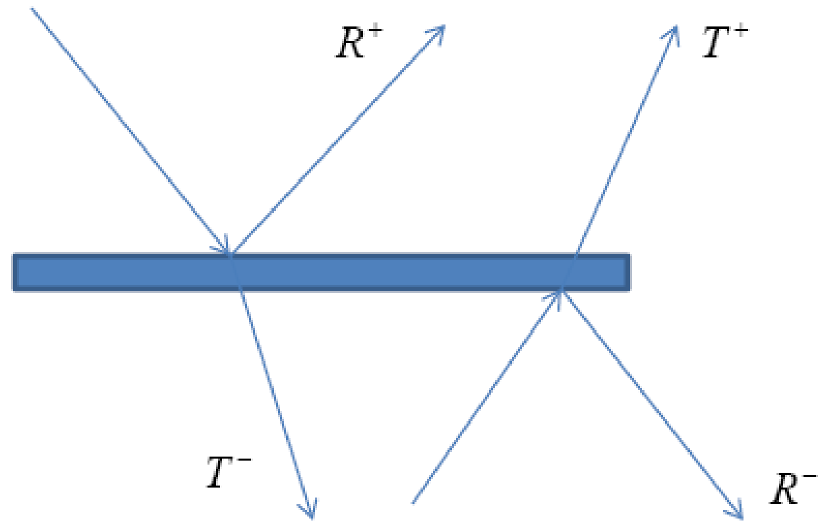
$$T = \frac{b}{a\sinh(bSd) + b\cosh(bSd)}$$

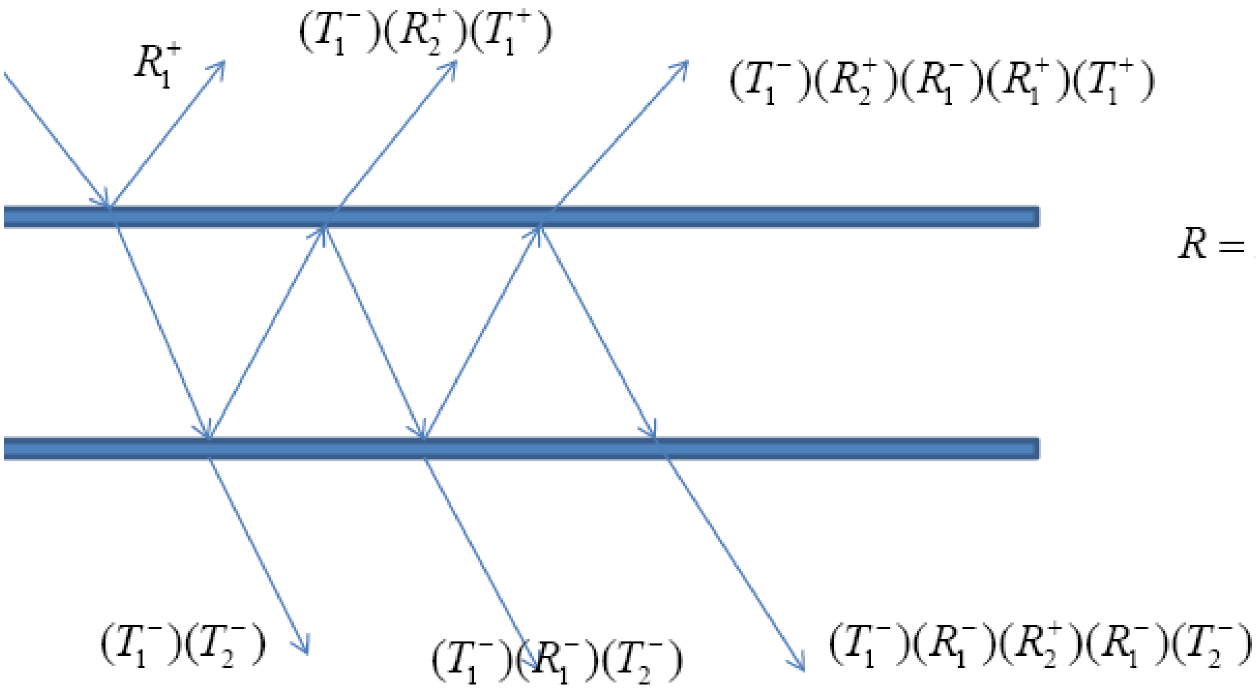
$$a = \frac{S+K}{S}$$

$$b = \sqrt{a^2 - 1}$$



Dorsey and Hanrahan Model





$$R = R_1^{+} + T_1^{-} R_2^{+} T_1^{+} + T_1^{-} R_2^{+} R_1^{-} R_2^{+} T_1^{+} + \ldots$$

$$R = R_1^{+} + \frac{T_1^{-} T_1^{+} R_2^{+}}{1 - R_2^{+} R_1^{-}}$$

References

- Pat Hanrahan, CS 348B slides, 2009
- Hill and Kelley, Computer Graphics using OpenGL (3rd edition)
- Matt Pharr, Greg Humphreys "Physically Based Rendering", Chapter 13
- Dorsey and Rushmeier, Modeling of Digital Materials
- Akenine-Moller, Haines and Hoffman, Real Time Rendering, 3rd edition