

**CS 3133 Foundations of Computer Science**  
**C term 2020**

**Solutions for Homework 5**

1. Exercise 17.b. on page 249.

**Solution:** (with the pumping lemma for context-free languages) Let us assume indirectly that the language  $L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$  is context-free. Then by the pumping lemma for context-free languages there exists a number  $k$ , such that every string  $z \in L$  of length  $k$  or more can be decomposed into substrings  $u, v, w, x$  and  $y$  such that  $length(vwx) \leq k$ ,  $length(v) + length(x) > 0$  and  $uv^i wx^i y \in L$  for all  $i \geq 0$ .

Consider the string  $z = a^k b^k c^k d^k$ . Clearly  $z \in L$  and  $length(z) \geq k$ . Using the pumping lemma we decompose  $z$  into substrings  $u, v, w, x$  and  $y$ , where  $0 < length(vwx) \leq k$ . But in this case  $uv^2 wx^2 y$  cannot be in  $L$ , a contradiction. Indeed, consider the possibilities for  $v$  and  $x$ . If either of these contains more than one type symbol, then  $uv^2 wx^2 y$  is not in  $L$ . So  $v$  and  $x$  must be substrings of one of  $a^k, b^k, c^k$ , or  $d^k$  and if they contain different symbols, then they have to be two “consecutive” symbols, so  $a$  and  $b$ , or  $b$  and  $c$ , or  $c$  and  $d$ . But then again  $uv^2 wx^2 y$  is not in  $L$ , since either the number of  $a$ ’s is different from the number of  $c$ ’s, or the number of  $b$ ’s is different from the number of  $d$ ’s. Thus  $L$  is not context-free. (20 points)

2. Let  $M$  be the Turing machine defined by

$\delta$	B	a	b	c
$q_0$	$(q_0, B, R)$	$(q_0, a, R)$	$(q_0, b, R)$	$(q_1, c, L)$
$q_1$	$(q_2, B, R)$	$(q_1, b, L)$	$(q_1, a, L)$	-
$q_2$	-	-	-	-

- (a) Trace the computation for the input string  $abcab$ .  
 (b) Trace the first six transitions of the computation for the input string  $abab$ .

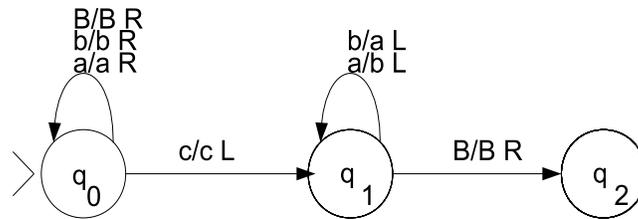
- (c) Give the state diagram of  $M$ .
- (d) Describe the result of a computation in  $M$ .

**Solution:**

a,b.)

$q_0Babcab$	$q_0Babab$
$\vdash q_0abcab$	$\vdash q_0abab$
$\vdash aq_0bcab$	$\vdash aq_0bab$
$\vdash abq_0cab$	$\vdash abq_0ab$
$\vdash aq_1bcab$	$\vdash abaq_0b$
$\vdash q_1aacab$	$\vdash ababq_0B$
$\vdash q_1Bbacab$	$\vdash ababBq_0B$
$\vdash q_2bacab$	

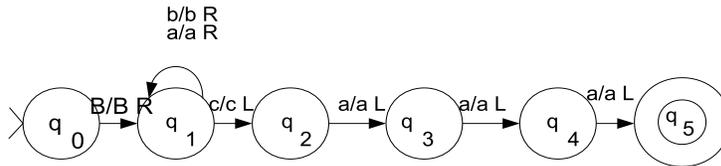
c.)



d.) If there is a  $c$ , then swap the  $a$ 's and  $b$ 's before the first  $c$ . Otherwise, if there is no  $c$ , then go to the right infinitely. (20 points)

3. Construct a Turing machine with input alphabet  $\{a, b, c\}$  that accepts strings in which the first  $c$  is immediately preceded by the substring  $aaa$ . A string must contain a  $c$  to be accepted by the machine.

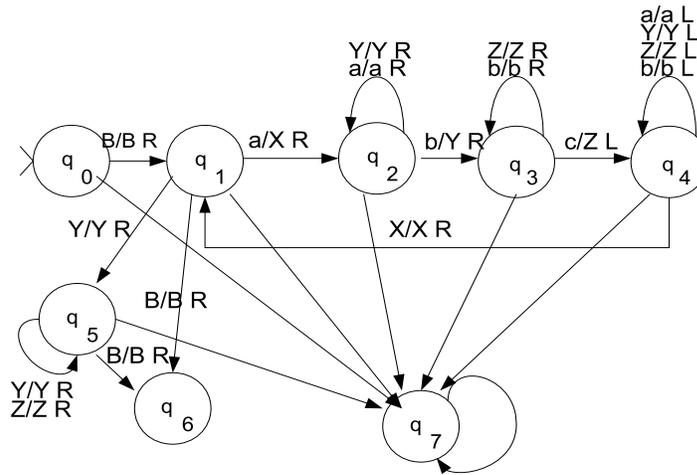
**Solution:**



(20 points)

4. Construct a Turing machine with input alphabet  $\{a, b, c\}$  that accepts the language  $L = \{a^i b^i c^i \mid i \geq 0\}$  by halting only.

**Solution:**

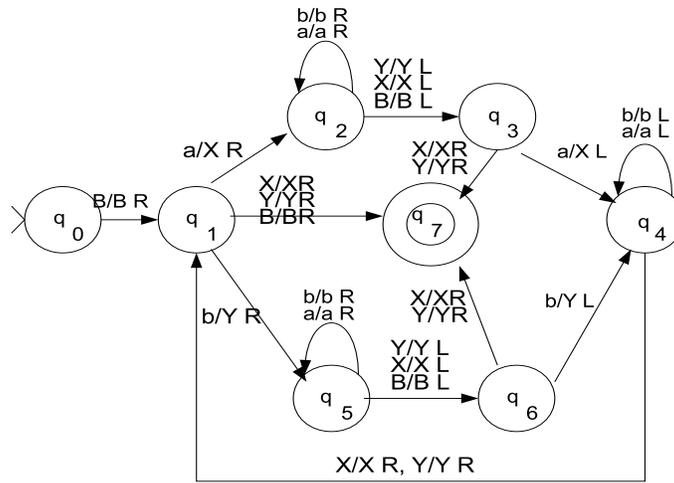


In this picture, if  $\delta(q_i, x)$  was not defined for  $q_i \notin F$  in the original machine, then we have  $\delta(q_i, x) = [q_7, x, R]$  for every  $1 \leq i \leq 5$  and  $\delta(q_7, x) = [q_7, x, R]$  for every  $x \in \Gamma$ . (20 points)

5. Construct a standard Turing machine that accepts the set of palin-

dromes over  $\{a, b\}$ .

**Solution:**



(20 points)