CS 2022/ MA 2201 Discrete Mathematics
A term 2014

Solutions for Homework 4

1. Exercise 18 on page 330.

   Solution:

   a.) Plugging in $n = 2$, we see that $P(2)$ is the statement $2! < 2^2$.

   b.) Since $2! = 2$, this is the true statement $2 < 4$.

   c.) The inductive hypothesis is the statement $n! < n^n$.

   d.) For the inductive step, we have to show for each $n \geq 2$ that $P(n)$ implies $P(n + 1)$. In other words, we have to show that assuming the inductive hypothesis we can prove that $(n + 1)! < (n + 1)^{n+1}$.

   e.)

   $$(n + 1)! = (n + 1)n! < (n + 1)n^n < (n + 1)(n + 1)^n = (n + 1)^{n+1}.$$

   f.) We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer greater than 1. (15 points)

2. Exercise 24 a-d.) on page 396.

   Solution: It is useful to note first that there are exactly 9000 numbers in this range.

   a.) Every ninth number is divisible by 9, so the answer is one ninth of 9000 or 1000.

   b.) Every other number is even, so the answer is one half of 9000 or 4500.

   c.) We can reason from left to right. There are 9 choices for the first (left-most) digit (since it cannot be 0), then 9 choices for the second digit (since it cannot equal the first digit but it can be 0), then, in a similar way, 8 choices for the third digit, and 7 choices for the right-most digit. Therefore there are $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ such numbers.

d.) Every third number is divisible by 3, so one third of 9000 or 3000 numbers in this range are divisible by 3. The remaining 6000 are not. (15 points)

3. Exercise 46 on page 397.

   Solution: (a) We first place the bride in any of the 6 positions. Then, from left to right in the remaining positions, we choose the other 5 people to be in the picture; this can be done in $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120$ ways. Therefore, the answer is $6 \cdot 15,120 = 90,720$.

   (b) We first place the bride in any of the 6 positions, and then place the groom in any of the remaining 5 positions. Then, from left to right in the remaining positions, we choose the other 4 people to be in the picture; this can be done in $8 \cdot 7 \cdot 6 \cdot 5 = 1,680$ ways. Therefore, the answer is $6 \cdot 5 \cdot 1,680 = 50,400$.

   (c) From part (a) there are $90,720$ ways for the bride to be in the picture. There are (from part (b)) $50,400$ ways for both the bride and groom to be in the picture. Therefore there are $90,720 - 50,400 = 40,320$ ways for just the bride to be in the picture. Symmetrically, there are $40,320$ ways for just the groom to be in the picture. Therefore, the answer is $40,320 + 40,320 = 80,640$.

   (15 points)

4. Exercise 36 on page 406.

   Solution: Let $C(x)$ be the number of other computers that computer x is connected to. The possible values for $C(x)$ are 1,2,3,4 and 5. Since there are 6 computers, the pigeonhole principle guarantees that at least two of the values $C(x)$ are the same, which is what we wanted to prove. (20 points)

5. Exercise 26 on page 414.

   Solution:

   (a) $C(13, 10) = \binom{13}{10} = 286$.

   (b) $P(13, 10) = 13 \cdot 12 \cdot \ldots 4$.

(c)

- First solution: There is only one way to choose the 10 players without choosing a woman, since there are exactly 10 men. Therefore using part a.) there are 286-1=285 ways to choose the players if at least one of them must be a woman.
- Second Solution: $C(3,1) \cdot C(10,9) + C(3,2) \cdot C(10,8) + C(3,3) \cdot C(10,7) = \binom{3}{1} \cdot \binom{10}{9} + \binom{3}{2} \cdot \binom{10}{8} + \binom{3}{3} \cdot \binom{10}{7} = 285.$

(20 points)

6. Exercise 8 on page 421.

   Solution: By the binomial theorem the coefficient is

$$\binom{17}{9} 3^8 2^9.$$

(15 points)