CS 521, HW 3: Derivability and Reduction

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Can you derive this? Some of the following are derivable using our rules. Others are not. For each derivable judgment, give a derivation.

If a judgment is not derivable, say so. Also, try to find a propositional logic model in which it is false. If this is also impossible, say so.

Think of $p \to \bot$ as the negation of p, that is, $\neg p$.

$$\vdash\ (p \wedge (p \to \bot)) \to \bot \tag{1}$$

$$p \vee q\ \vdash\ p \wedge q \tag{2}$$

$$\vdash\ p \vee (p \to \bot) \tag{3}$$

$$\vdash\ ((p \to \bot) \to \bot) \to p \tag{4}$$

$$\vdash\ (((p \to \bot) \to \bot) \to \bot) \to (p \to \bot) \tag{5}$$

Reduce! Reduce each of the following explicit proof objects as much as possible. Use the rules in Fig. 12, p. 7 of the lecture notes[1].

Be careful about avoiding capture of free variables. Use the method from Section 4.1 of the lecture notes to avoid capture of free variables. Mark each step in which you've done so with the letter α.

$$(\lambda y\ .\ (\lambda f\ .\ \lambda y\ .\ f(y^2 + y))\ (\lambda z\ .\ z * y)) \tag{6}$$

$$(\lambda x\ .\ \lambda y\ .\ \mathsf{fst}(x, \lambda(z\ .\ \langle \mathsf{lft}, \lambda w\ .\ \langle x, y\rangle\rangle))))(\lambda z\ .\ \langle y, z\rangle)(\lambda w\ .\ z) \tag{7}$$

$$(\lambda x\ .\ (\lambda y\ .\ \mathsf{fst}(x, \lambda(z\ .\ \langle \mathsf{lft}, \lambda w\ .\ \langle x, y\rangle\rangle))))(\lambda z\ .\ \langle y, z\rangle))(\lambda w\ .\ z) \tag{8}$$

Reduce forever? Can you find a λ-expression which can be reduced, after which the result can be reduced, and so on forever? [**Hint:** a successful term will not be typable, so think about ways to prevent typability.]

[1] At URL `http://web.cs.wpi.edu/~guttman/cs521_website/9sep10_consequence.pdf`.