CS 521, HW 4: Normal Derivations and the Subformula Property

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Non-derivability in Intuitionist Propositional Logic. Use the theorem that every normal derivation has the subformula property to show that the following judgments are not derivable in our system. It is Thm. 23 in the current version of the lecture notes.[1] Say which of these formulas are classically valid, i.e. true for every assignment of truth values to atomic formulas.

$$\vdash \; p \tag{1}$$

$$\vdash \; p \to \bot \tag{2}$$

$$\vdash \; p \vee (p \to \bot) \tag{3}$$

$$\vdash \; ((p \to \bot) \to \bot) \to p \tag{4}$$

Double Negation Elimination has the same strength as Excluded Middle. Formulas of the form given in Eqn. 3 are instances of a law called *the excluded middle*. The idea behind the name is that some middle position between p and $\neg p$ is impossible. Formulas of the form given in Eqn. 4 are instances of a law called *double negation elimination*. The instances of these two rules are equivalent in our logic. Prove:

$$p \vee (p \to \bot) \; \vdash \; ((p \to \bot) \to \bot) \to p \tag{5}$$

$$((p \to \bot) \to \bot) \to p \; \vdash \; p \vee (p \to \bot) \tag{6}$$

Proof of Lemma 22. Please choose two clauses of Lemma 22, about paths in normal derivation. For each, choose two relevant inference rules, and show that the assertion is correct for those inference rules.

[1] At URL `http://web.cs.wpi.edu/~guttman/cs521_website/16sep10_consequence.pdf`.