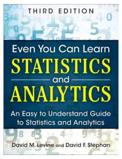
IMGD 2905

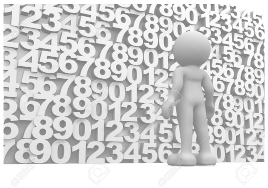
Descriptive Statistics

Chapter 3



Summarizing Data

- With lots of playtesting, there will be a lot of data
  - This is a good thing!
- But raw data is just a pile of numbers
  - Rarely of interest
  - Or even sensible
- Q: How to summarize all this information?



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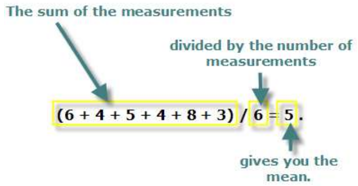
Measures of central tendency

Measure of Central Tendency: Mean



The sum of the measurements

divided by the number of measurements

(6 + 4 + 5 + 4 + 8 + 3) / 6 = 5.

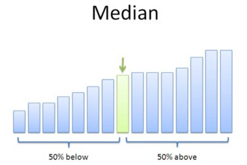
gives you the mean.

http://www.cdn.sciencebuddies.org/Files/463/9/MeanEquation.jpg

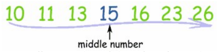
- Also called the "arithmetic mean" or "average"
- In Excel, =AVERAGE(range)
  - =AVERAGEIF() – averages if numbers meet certain condition

Measure of Central Tendency: Median

- Sort values low to high and take middle value



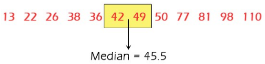
https://betterexplained.com/wp-content/uploads/average/median.png



10 11 13 15 16 23 26

middle number

https://www.mathsisfun.com/definitions/images/median.gif



13 22 26 38 36 42 49 50 77 81 98 110

Median = 45.5

http://www.nedarc.org/statisticalHelp/basicStatistics/measuresOfCenter/images/median.gif

- In Excel, =MEDIAN(range)

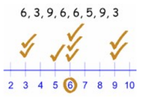
Measure of Central Tendency: Mode

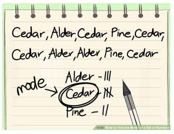
- Number which occurs most frequently
- Not so useful in many cases

→ Best use for categorical data

  - e.g., most played champion in League

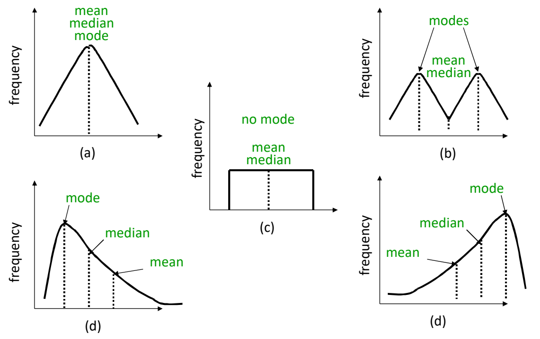
- In Excel, =MODE()




Cedar, Alder, Cedar, Pine, Cedar,
Cedar, Alder, Alder, Pine, Cedar
mode
Alder - III
Cedar - 𝍸
Pine - II

http://pad3.whstatic.com/images/thumb/c/cd/Find-the-Mode-of-a-Set-of-Numbers-Step-7.jpg/aid130521-v4-728px-Find-the-Mode-of-a-Set-of-Numbers-Step-7.jpg

Depiction: Mean, Median, Mode



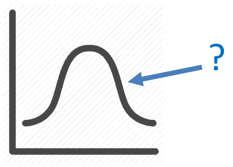
Which to Use, Mean, Median, Mode?

Which to Use, Mean, Median, Mode

- Mean many statistical tests with sample
  - Estimator of population mean
  - Uses all data
- Median is useful for skewed data
  - e.g., income data (US Census) or housing prices (Zillo)
  - e.g., Overwatch team (6 players): 5 people level 5, 1 person level 275
    - Mean is 50 - not so useful since no one at this level
    - Median is 5 - more representative
  - Does not use all data. “Resistant” to extremes (e.g., 275)
  - But what if were exam scores? Hard to “bring up” grade
- Mode is useful primarily for categorical data only
  - Most played League champion, most popular TagPro map, …

Other Measures of Position

- May not always want center
  - e.g., want to know best Champions
- What other positions may be desired?

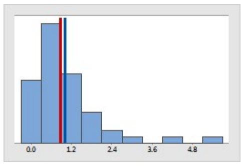


Other Measures of Position

- Maximum / Minimum
  - Not discussed more
- Trimmed Mean
- Quartiles
- Percentiles

Trimmed Mean

- Take “trimming” off top and bottom (typically 5% or 10%)
  - Reduces effects of extreme values, like median
- In Excel, =`TRIMMEAN(array,percent)`

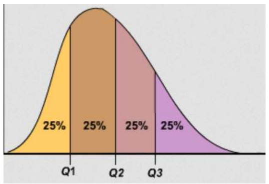


http://support.minitab.com/en-us/minitab/17/histogram_mean_vs_trimmed_mean.png

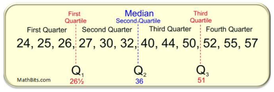
Blue – original mean
Red – trimmed mean

Quartiles

- Sort values
- First quartile (Q1) is 25% from bottom
- Third quartile (Q3) is 75% from bottom
- (What is second quartile?)
- In Excel, =`QUARTILE(array,n)`



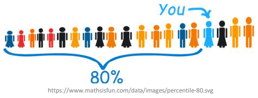
https://www.hackmath.net/images/quartiles.png



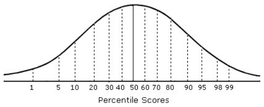
https://mathbitsnotebook.com/Algebra1/StatisticsData/quartileboxview2.png

Percentiles

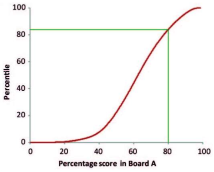
- Generalization of quartiles
- *N*th percentile is data point *n*% from bottom of data
- Interpolate as for first quartile
- In Excel, =`PERCENTILE(array,k)` (k: 0 to 1)



https://www.mathsisfun.com/data/images/percentile-80.svg



http://www.psychometric-success.com/images/AA1301.gif



http://www.isical.ac.in/~jeexiiscore_normal/PercentilesAdvantages.htm

Summarizing Data, Part 2

- Ok, pile of numbers can now be summarized as *one* number
  - Mean, median, mode
- But is that enough?
- Q: What other major aspect of numbers haven't we summarized?

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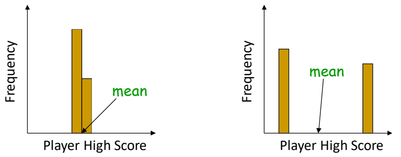
Measures of variation
(*aka* measures of *dispersion,* or
measures of *spread*)

Summarizing Data, Part 2

> *"Then there is the man who drowned crossing a stream with an average depth of six inches."* – W.I.E. Gates

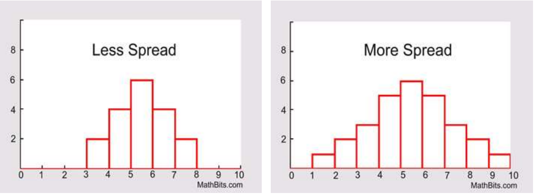
- Summarizing by single number rarely enough → need statement about variation



Above: does single number (mean) tell you enough about data?

Variation Overview (1 of 3)

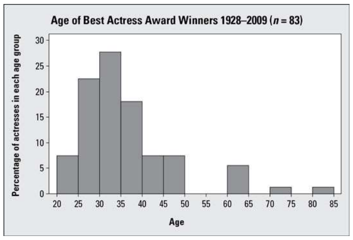
- Is data clumped or spread out?



https://mathbitsnotebook.com/Algebra1/StatisticsData/STSpread.html

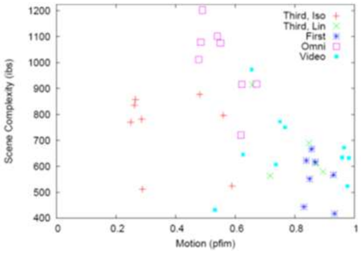
Variation Overview (2 of 3)

- Is data clumped or spread out?



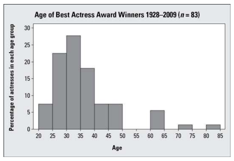
Variation Overview (3 of 3)

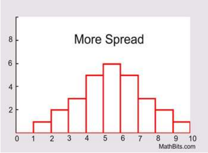
- Is data clumped or spread out?

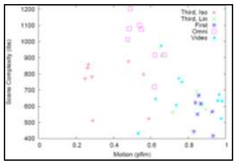


"Motion and Scene Complexity for Streaming Video Games"

What are Some Measures of Variation?



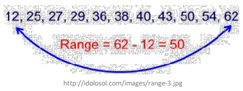




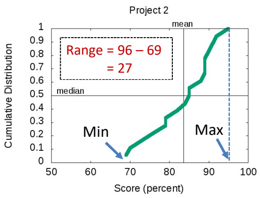
Range

- Difference between smallest and largest value
- Somewhat obvious, but doesn't tell you much about "clumping"
  - Minimum may be zero
  - Maximum can be from outlier
    - Event not related to phenomena studied
  - Maximum gets larger with # samples, so no "stable" point

In Excel, =MAX(array)-MIN(array)



http://idolosol.com/images/range-3.jpg



Variance

- Compute mean of sample
- Compute how far each value in sample is from mean
  - Some can be less than mean, some greater
  - → So square this difference
- Divide by number of sample values – 1
  - The "-1" corrects "bias" when trying to estimate population variance

"sum up all" "mean"

$$\text{Sample Variance} = s^2 = \frac{\Sigma(X - \bar{X})^2}{n - 1}$$

Variance Example

- Sample kills in League of Legends match
  - 12, 20, 16, 18, 19
  - What is sample variance?

- First, mean = 85 / 5 = 17

Kills	X – mean	(X – mean)^2
12	-5	25
20	3	9
16	-1	1
18	1	1
19	2	4

s^2 = (25 + 9 + 1 + 1 + 4) / (5 – 1) = 40 / 4 = 10 kills squared

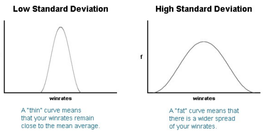
- In Excel, =VAR(array)

"Larger" means
"more spread"
… but units odd

Standard Deviation

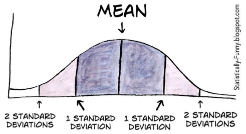
- Square-root of variance
- Usually, use standard deviation instead of variance
  - Why? → Same units as data (e.g., "kills" in previous example)
- Can compare standard deviation to mean (coefficient of variation, next)
- But first:
  - Mendenhall's Empirical Rule
  - Z-score

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$



A "thin" curve means that your winrates remain close to the mean average.

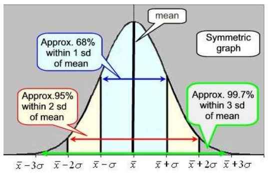
A "fat" curve means that there is a wider spread of your winrates.



Statistically-Funny.blogspot.com

Mendenhall's Empirical Rule

- About 68% data within one standard deviation of mean
  - interval between mean-s and mean+s contains about 68% of data
- About 95% within 2 standard deviations of mean
- Almost all data within 3 standard deviations of mean
- (Rules based on normal distribution)

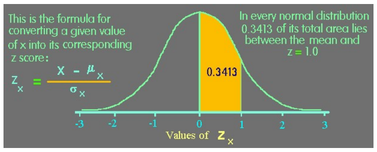


https://mathbitsnotebook.com/Algebra1/StatisticsData/normalgrapha.jpg

Z-Score

- Measure of how "far" from center (mean) single data point is
  - *Not* measure of dispersion for whole data set

$$z = \frac{X - \overline{X}}{s}$$


This is the formula for converting a given value of x into its corresponding z score:
$z_x = \frac{X - \mu_x}{\sigma_x}$
In every normal distribution 0.3413 of its total area lies between the mean and z = 1.0
0.3413
-3
-2
-1
0
1
2
3
Values of z_x

https://www.animatedsoftware.com/pics/stats/sgzscor2.gif

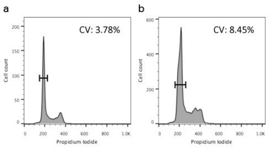
Example	
Mean	469
Std dev	119
X	650
Z-score for X?	
(650 – 469)/119	1.52

Coefficient of Variation (CV)

- Size of the standard deviation relative to the mean
  - e.g., large sd & large mean, not so spread
  - but large sd & small mean, more spread
- Standard deviation divided by mean
  - Can do this since same units!
- CV is "unit-less", so measure of spread independent of quantity
  - E.g. seconds, clicks, spaces

Shown as percent (multiply by 100)

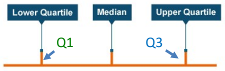
$$CV = \frac{s}{\bar{x}} \times 100$$



http://bitesizebio.s3.amazonaws.com/wp-content/uploads/2015/01/Spread1.jpg

Semi-Interquartile Range

- ½ distance between Q3 (75th percentile) and Q1 (24th percentile)



http://www.bbc.co.uk/staticarchive/9629000486ef4b1a40efa565c162cb779e0bd82c.png

$$\frac{Q3 - Q1}{2}$$

- Use semi-interquantile (SIQR) for index of dispersion whenever using median as index of central tendency

Index of Variation Example

(sorted) Lap Times
1.9
2.7
3.9
4.1
4.2
4.2
4.4
4.5
4.5
4.8
4.9
5.1
5.1
5.3
5.6
5.9

- First, sort
- Mean = 4.4
- Min = 1.9, Max = 5.9
- Median = [16 / 2] = 8th = 4.5
- Q1 = 16 / 4 = 8th = 4.1
- Q3 = 3 * 16 / 4 = 12th = 5.1

- *SIQR* = (Q3–Q1) / 2 = 0.5
- *Variance* = 0.96
- *Stddev* = 0.98
- *CV* = stddev/mean = 0.22
- *Range* = max – min = 4

Ranking of Affect by Outliers?

Measure of Variation
- Variance
- Range
- Standard Deviation
- Coefficient of Variation
- Semi-interquartile Range

Most to Least

Ranking of Affect by Outliers?

Measure of Variation
- Variance
- Range
- Standard Deviation
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- Semi-interquartile Range

Most to Least
- Range susceptible
- Variance
  - Standard Deviation
  - Coefficient of Variation
- SIQR resistant

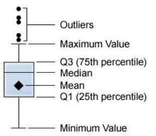
Index of Variation Summary

- Ranking of affect by outliers
  - Range susceptible
  - Variance
    - Standard deviation
    - Coefficient of variation
  - Semi-interquartile range resistant
- Note, all only applied to quantitative data!
  - For categorical data, can't quantify spread since no 'distance' between
  - Instead, give number of categories for given percentile of samples
    - e.g., "90% of samples are in 3 categories"

Depicting Variation in Charts

- Histogram (done)
- Cumulative distribution (done)
- Box-and-Whiskers (new)
- Error Bars (new)

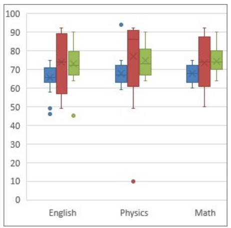
Box-and-Whiskers Chart



http://support.sas.com/documentation/cdl/en/vaug/65747/HTML/default/images/boxplot.png

- Way of showing variation
- Highlight middle 50% (interquartile range, IQR)
  - "Box"
- Lines go to smallest non-outlier
  - "Whiskers"
- Points indicate outliers
- Middle line shows median
- Sometimes with mean
- Outlier? → Data value "way out there", "far" from the rest
  - Formally, 1.5+ IQRs away from quartile
- Available in Excel 2016

Sometimes called "boxplot"



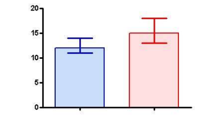
https://support.office.com/en-us/article/Create-a-box-and-whisker-chart-62f4219f-db4b-4754-aca8-4743f6190f0d

Error Bars

- Line through graph point parallel to axis with "caps"
- Denotes uncertainty (variation) in value
- Excel: click "+" → "Error Bars" → "type"

- Often:
  - 1 standard deviation
- Can be (discuss later):
  - 1 standard error
  - 1 confidence interval

State clearly!



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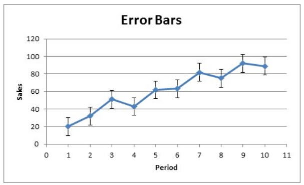

http://www.excel-easy.com/examples/images/error-bars/error-bars.png