Homework #1 Solutions
Due Thursday, January 27

True or False:

a) Given a language (set of strings) L, the question: "Is string w ε L" is a decision problem: T F
b) Φ = {ε} T F
c) For sets A and C. ~ (A ∪ C) = ~ A ∪ ~ C   T F
d) There is only 1 dfa that accepts a*   T F
e) Given an alphabet Σ and a regular language L ⊆ Σ*, the strings in L' = Σ* - L form a regular language   T F

Proofs:

#2. Given that an integer *n* is even if there is an integer *i* such that *n = 2 * i* and an integer *n* is odd if there is an integer *i* such that *n = 2 * i + 1*, prove that for every integer *n*≥0, *n* is either even or odd, but not both.

<u>Solution</u>

There are actually 2 things to prove: 1) an integer must be one of {even,odd} and 2) a number cannot be both even and odd.

1) All numbers *n* can be written as *n* = 2q + r for 0 ≤ r ≤ 2
   So *r* must be 0 or 1.
   If *r* is 0 then *n = 2q* (i.e., *n* is even).
   If *r* is 1, then *n = 2 q + 1* (i.e., *n* is odd)

2) If *n* is both even and odd, then
   n = 2i
   and *n = 2j + 1*

   Then we have *2i = 2 j + 1*
   Case 1) *i =j : then 0 = 1 (impossiblej*
   Case 2) i ≠ j: then (dividing by 2) i = j + ½ (impossible)

Therefore, an integer *n* must be even or odd, but not both

#3. Given an alphabet Σ, and a string *x* in Σ *, define the reversal of *x*, denoted x^R as:

a) If length(*x*) = 0, then *x* = ε and $\varepsilon^R = \varepsilon$
b) If length(*x*) = n>0, then *x* = *wa* for some string *w* with length n – 1 and some *a* in Σ, and $x^R = aw^R$.

Using this definition, the definition of concatenation and associativity, prove by induction that: $(xy)^R = y^R x^R$.

Proof by induction on |y|

Basis:

Left:

if $|y| = 0$, then $(xy)^R = (x\varepsilon)^R = x^R$

Right:

if $|y| = 0$, then $y^R x^R = \varepsilon^R x^R = \varepsilon x^R = \mathbf{x}^R$

Induction Hypothesis: $(xy)^R =$, when $0 \leq |y| \leq n$, $n \geq 0$

Induction Step:

If $|y| = n + 1$, then $y = wa$, where $a \varepsilon \Sigma$ and $|w| = n$

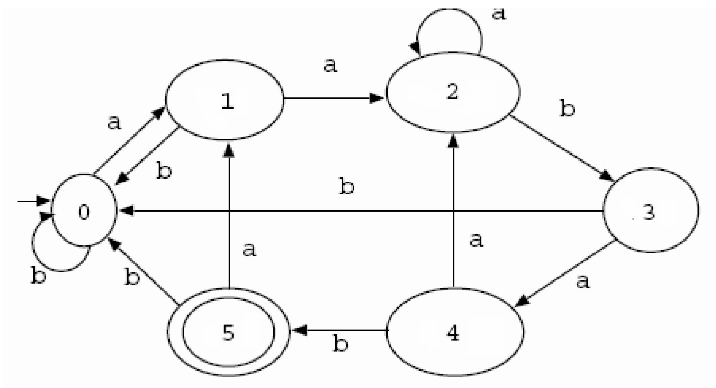
$(xy)^R$	$= (x\,(wa))^R$	where $y = wa$, $|w| = n$,
	$= ((xw)\,a)^R$	associativity
	$= a(xw)^R$	def'n of reversal
	$= a(w^R x^R)$	induction hypothesis
	$= (aw^R)x^R$	associativity
	$= (wa)^R x^R$	definition of reversal
	$= y^R x^R$	substitution of y for wa

#4. Disprove: All WPI computer science professors are men.

Proof by counterexample (me)

DFA's

#5. What set of strings does the following automaton accept?



Strings of *a*'s and *b*'s that end in *a a b a b:*
(a + b)* a a b a b

#6. Create a DFA that accepts an odd number of *a*'s