Name_______________________

Homework #2

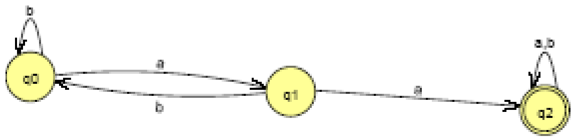
Solutions

1. Show the following languages are regular by creating finite automata with L = L(M)

a) Strings over {a,b} that contain 2 consecutive *a*'s
b) Strings over {a,b} that do not contain 2 consecutive *a*'s
c) The set of strings over {0,1} which contain the substring *00* and the substring *11*
d) The set of strings over {a,b} which do not contain the substring *ab.*

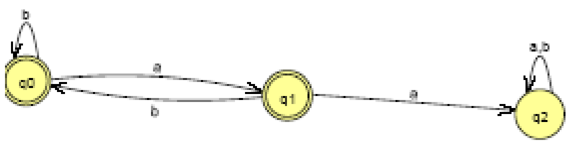
Show your answers in both table and graph form.

a) Strings over {a,b} that contain 2 consecutive *a*'s



	a	b
$>q_0$	q_1	q_0
q_1	q_2	q_0
$*q_2$	q_2	q_2

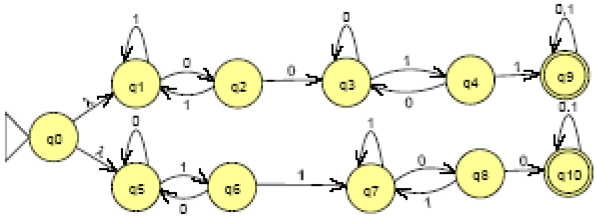
b) Strings over {a,b} that do not contain 2 consecutive *a*'s



	a	b
$>*q_0$	q_1	q_0
$*q_1$	q_2	q_0
q_2	q_2	q_2

c) The set of strings over {0,1} which contain the substring *00* and the substring *11*

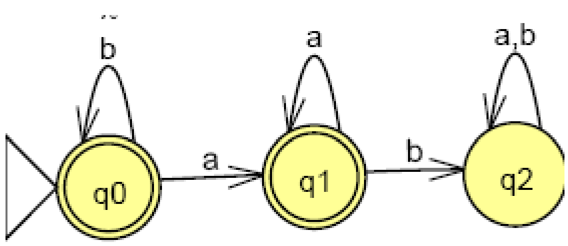
Problem doesn't say whether this must be a dfa and this is easier with an nfa:



	λ	0	1
>q_0	q_1 , q_5		
q_1		q_2	
q_2		q_3	q_1
q_3		q_3	q_4
q_4		q_3	q_9
q_5		q_5	q_6
q_6		q_5	q_7
q_7		q_8	q_7
q_8		q_{10}	q_7
*q_9		q_9	q_9
*q_{10}		q_{10}	q_{10}

d) The set of strings over {a,b} which do not contain the substring *ab*.

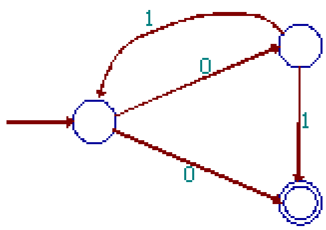
Similar to parts a and b, I will first create a fa that does accept *a b* and then I will reverse the final and the nonfinal states:



	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_2	q_2

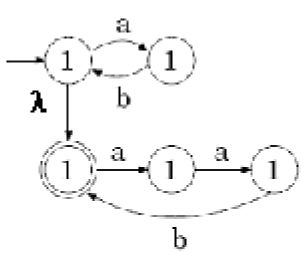
#2. Create an NFA (with λ transitions) for all strings over {0, 1, 2} that are missing at least one symbol. For example, *00010*, *1221*,and *222* are all in L while *221012* is not in L

a)



L(M) = Alternating 0's and 1's (including none) that begin with a 0
(01)* (01 U 0)

b)



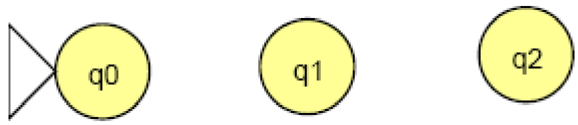
0 or more *ab*'s followed optionally by 0 or more *aab*'s
(ab)* (aab)*

#3. a) Given an NFA with several final states, show how to convert it into one with exactly one start state and exactly one final state.

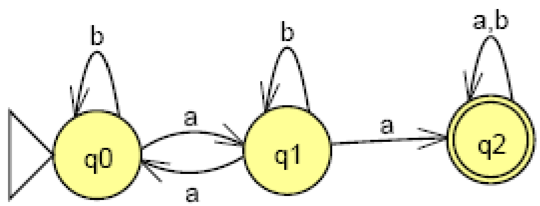
Create a new initial state and a λ-transition from it to all the original start states
Create a new final state and a λ-transition from all the original final states (which mark to no longer be final) to this new final state

b) Suppose an NFA with k states accepts at least one string. Show that it accepts a string of length k-1 or less.

Look at a fa with 3 states:



No matter how you draw the transitions or which states are final states, to process a string of length *3* means you visited a state twice. For example:



accepts the string of length 3: aba

But just by not visiting the revisited state (q_1), this will accept *aa* (of length 2)

In general, if a string of length k is accepted by a fa with k states, it visits (at least) 1 state twice. By not visiting this state the 2nd time (e.g., don't take the loop), we can accept a string with 1 fewer symbol, i.e, of length k – 1.

#5. Let L be a regular language. Show that the language consisting of all strings not in L is also regular.

If L is regular, there is a dfa, M, such that L = L(M), that is, M accepts L. If we create a new finite automaton, M', by reversing final and non-final states, we will accept what M didn't and reject what M accepted; that is, C(L) = L(M')