Homework #8
Solutions

#1 <u>True or False</u>

a) Regular languages are recursive TRUE FALSE
b) Context free languages are recursively enumerable (r.e.) TRUE FALSE
c) Recursive languages are r.e TRUE FALSE
d) R.e. languages are recursive TRUE FALSE

#2. a) Show computations with 000111 and 101 on the following Turing Machine

State	├	0	1	X	Y	⊔
q_0	(q_0, ├,R)	(q_1,X,R)	-	-	(q_3,Y,R)	
q_1		(q_1,0,R)	(q_2,Y,L)	-	(q_1,Y,R)	-
q_2		(q_2,0,L)	-	(q_0,X,R)	(q_2,Y,L)	-
q_3		-	-	-	(q_3,Y,R)	(q_4, ⊔,R)
q_4		-	-	-	-	-

<u>000111</u>

q_0 ├000111 ⊔…
├ $q_0$000111 ⊔…
├X$q_1$00111
├X0$q_1$0111
├X00$q_1$111
├X0$q_2$0Y11
├X$q_2$00Y11
├q_2X00Y11
├X$q_0$00Y11
├XX$q_1$0Y11
├XX0q_1Y11
├XX0Y$q_1$11
├XX0q_2YY1
├XX$q_2$0YY1
├Xq_2X0YY1
├XX$q_0$0YY1
├XXXq_1YY1
├XXXYq_1Y1
├XXXYY$q_1$1
├XXXYq_2YY
├XXXq_2YYY
├XXq_2XYYY
├XXXq_0YYY
├XXXYq_3YY

<u>101</u>

q_0 ├101
├ $q_0$101
dead end!

$XXXYYq_3Y$
$XXXYYYq_3$
$XXXYYYBq_4$
halt
q_4 is a final state

b) What is L(M) (you'll have to guess)

"Looks like" $\{0^n1^n \mid n > 0\}$

#3. Construct a Turing Machine to compute $\{w\, w^R \mid w\ \varepsilon\ \{0,1\}^*$

a) Show pseudo-code that describes how the TM operates

```
If symbol is 0
   write X
   enter a "branch" that iterates to the 1st Blank,X,Y
   if last symbol is 0
      go back to the beginning, repeat
(Similar for 1)
(Accept if read X or Y at 1st step)
```

b) Create the actual transitions

State	$\vdash$	0	1	X	Y	$\sqcup$
$\rightarrow q_0$	$(q_0,\vdash,R)$	(q_1,X,R)	(q_4,Y,R)	(q_7,X,L)	(q_7,Y,L)	—
q_1		$(q_1,0,R)$	$(q_1,1,R)$	(q_2,X,L)	(q_2,Y,L)	$(q_2,\sqcup,L)$
q_2		(q_3,X,L)	—	—	—	—
q_3		$(q_3,0,L)$	$(q_3,1,L)$	(q_0,X,R)	—	—
q_4		$(q_4,0,R)$	$(q_4,1,R)$	(q_5,X,L)	(q_5,Y,L)	$(q_5,\sqcup,L)$
q_5		—	(q_6,Y,L)	—	—	—
q_6		$(q_6,0,L)$	$(q_6,1,L)$	—	(q_0,Y,R)	—
q_7	$(q_8,\vdash,R)$	—	—	(q_7,X,L)	(q_7,Y,L)	—
$*q_8$	—	—	—	—	—	—

c) Show your TM processing (i) 1001, (ii) 101 and (iii) 110

$q_0 \vdash 1001 \sqcup \rightarrow \vdash q_0 1001 \sqcup \rightarrow \vdash Yq_4 001 \sqcup \rightarrow \vdash Y0q_4 01 \sqcup \rightarrow \vdash Y00q_4 1 \sqcup$
$\rightarrow \vdash Y001q_4 \sqcup \rightarrow \vdash Y00q_5 1 \sqcup \rightarrow \vdash Y0q_6 0Y \sqcup \rightarrow \vdash Yq_6 00Y \sqcup \rightarrow \vdash q_6 Y00Y \sqcup$
$\rightarrow \vdash Yq_0 00Y \sqcup \rightarrow \vdash YXq_1 0Y \sqcup \rightarrow \vdash YX0q_1 Y \sqcup \rightarrow \vdash YXq_2 0Y \sqcup \rightarrow \vdash Yq_3 XXY \sqcup$
$\rightarrow \vdash YXq_0 XY \sqcup \rightarrow \vdash Yq_7 XXY \sqcup \rightarrow \vdash q_7 YXXY \sqcup \rightarrow q_7 \vdash YXXY \sqcup$

$q_0 \vdash 101 \sqcup \rightarrow \vdash q_0 101 \sqcup \rightarrow \vdash Yq_4 01 \sqcup \rightarrow \vdash Y0q_4 1 \sqcup \rightarrow \vdash Y01q_4 \sqcup$

➔ **├Y0q_5** 1 ⊔ ➔ **├Y$q_6$0Y** ⊔ ➔ **├q_6Y0Y** ⊔ ➔ **├Y$q_0$0Y** ⊔
➔ **├YXq_1Y** ⊔➔ **├Yq_2XY** ⊔

110 is similar

#4. Show that r.e. languages are closed under union and intersection.

union
Assume L_1 and L_2 are recursively enumerable. We can consider two single tape machines M_1 and M_2, which accept L_1 and L_1 respectively. We define M as a single tape TM with three tracks. Track 1 will hold the input. M will simulate M_1 using track 2. If M_1 halts in an accepting configuration then M accepts; otherwise M moves the tape head back to the left end and starts simulating M_2 or track 3. If M_2 accepts the input string then the string is accepted by M.

intersection
Assume L_1 and L_2 are recursively enumerable. We can consider two single tape machines M_1 and M_2, which accept L_1 and L_1 respectively. We define M as a single tape TM with three tracks. Track 1 will hold the input. M will simulates M_1 using track 2. If M_1 halts in an accepting configuration M moves the tape head back to the left end and starts simulating M_2 or track 3. If M_2 also accepts the input string then the string is accepted by M.