Name________________________

Homework #2

People I worked with and URL's of sites I visited:

#1. Convert to Chomsky Normal Form. Please follow the steps even if you can "see" the answer:

a) the expression grammar, G:
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid a$

Recursive Start

$E' \rightarrow E$
$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (E) \mid a$

No λ productions

Chain Rules

$F \rightarrow (E) \mid a$ ok
Change $T \rightarrow T * F \mid F$ to $T \rightarrow T * F \mid (E) \mid a$
Change $E \rightarrow E + T \mid T$ to $E \rightarrow E + T \mid T * F \mid (E) \mid a$
Change $E' \rightarrow E$ to $E' \rightarrow E + T \mid T * F \mid (E) \mid a$

So have:
$E' \rightarrow E + T \mid T * F \mid (E) \mid a$
$E \rightarrow E + T \mid T * F \mid (E) \mid a$
$T \rightarrow T * F \mid (E) \mid a$
$F \rightarrow (E) \mid a$

Useless

1. **All productions produce terminal strings**
2. **All symbols reachable from S**

Chomsky Normal Form

Introduce T_a, $T_($, $T_)$, T_+, T_*:

$E' \rightarrow E\ T_+\ T$
$E' \rightarrow T\ T_*\ F$

$E' \rightarrow T_(E\ T_)$
$E' \rightarrow a$
$E \rightarrow E\ T_+\ T$
$E \rightarrow T\ T_*\ F$
$E \rightarrow T_(E\ T_)$
$E \rightarrow a$
$T \rightarrow T\ T_*\ F$
$T \rightarrow T_(E\ T_)$
$T \rightarrow a$
$F \rightarrow T_(E\ T_)$
$F \rightarrow a$
$T_a \rightarrow a$
$T_(\rightarrow ($
$T_) \rightarrow)$
$T_+ \rightarrow +$
$T_* \rightarrow *$

Introduce Intermediate variables: V_1, V_2, V_3, V_4, V_5:

$E' \rightarrow T\ V_1$
$V_1 \rightarrow E\ T_)$
$E' \rightarrow a$
$E \rightarrow E\ V_2$
$V_2 \rightarrow T_+\ T$
$E \rightarrow T\ V_3$
$V_3 \rightarrow T_*\ F$
$T \rightarrow T_(V_4$
$E \rightarrow a$
$V_4 \rightarrow E\ T_)$
$T \rightarrow a$
$F \rightarrow T_(V_5$
$V_5 \rightarrow E\ T_)$
$F \rightarrow a$
$T_a \rightarrow a$
$T_(\rightarrow ($
$T_) \rightarrow)$
$T_+ \rightarrow +$
$T_* \rightarrow *$

b) $S \rightarrow A \mid A\ B\ a \mid A\ b\ A$
$A \rightarrow A\ a \mid \lambda$
$B \rightarrow B\ b \mid B\ C$
$C \rightarrow C\ B \mid C\ A \mid b\ B$

Recursive Start
none

Remove λ Productions

Null = {A, S}

$C \rightarrow C\ B \mid C\ A \mid b\ B$
$B \rightarrow B\ b \mid B\ C$
$A \rightarrow A\ a \mid a$
$S \rightarrow A \mid A\ B\ a \mid A\ b\ A \mid B\ a \mid b\ A \mid A\ b \mid b \mid \lambda$

or
$S \rightarrow A \mid A\ B\ a \mid A\ b\ A \mid B\ a \mid b\ A \mid A\ b \mid b \mid \lambda$
$A \rightarrow A\ a \mid a$
$B \rightarrow B\ b \mid B\ C$
$C \rightarrow C\ B \mid C\ A \mid b\ B$

Remove chain rules

$S \rightarrow A\ a \mid a \mid A\ B\ a \mid A\ b\ A \mid B\ a \mid b\ A \mid A\ b \mid b \mid \lambda$
$A \rightarrow A\ a \mid a$
$B \rightarrow B\ b \mid B\ C$
$C \rightarrow C\ B \mid C\ A \mid b\ B$

Remove useless

Term = {A, S}

so have:

$S \rightarrow A\ a \mid a \mid A\ b\ A \mid b\ A \mid A\ b \mid b \mid \lambda$
$A \rightarrow A\ a \mid a$

Reach = {S, A}
so above grammar is ok.

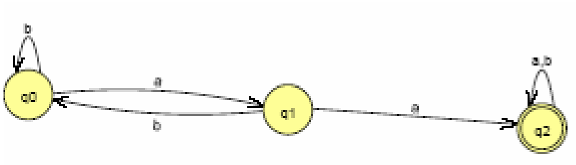
Chomsky Normal Form

Introduce new variables: T_a, T_b
$S \rightarrow A\ T_a \mid a \mid A\ T_b\ A \mid T_b\ A \mid A\ T_b \mid b \mid \lambda$
$A \rightarrow A\ T_a \mid a$
$T_a \rightarrow a$
$T_b \rightarrow b$

Introduce new variables: V_1
$S \rightarrow A\ T_a \mid a \mid A\ V_1 \mid T_b\ A \mid A\ T_b \mid b \mid \lambda$
$V_1 \rightarrow T_b\ A$
$A \rightarrow A\ T_a \mid a$
$T_a \rightarrow a$
$T_b \rightarrow b$

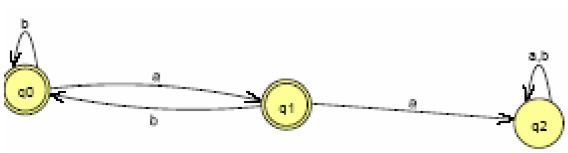
#2. Show the following languages are regular by creating finite automata with L = L(M)

a) Strings over {a,b} that contain 2 consecutive *a*'s



	a	**b**
>q_0	q_1	q_0
q_1	q_2	q_0
*q_2	q_2	q_2

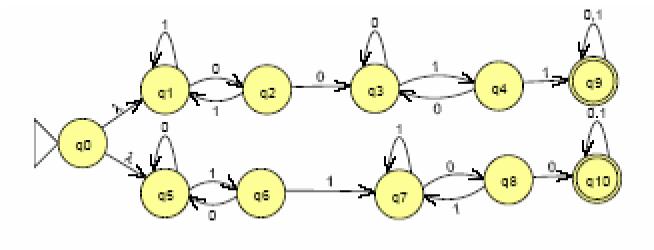
b) Strings over {a,b} that do not contain 2 consecutive *a*'s



	a	**b**
>*q_0	q_1	q_0
*q_1	q_2	q_0
q_2	q_2	q_2

c) The set of strings over {0,1} which contain the substring *00* and the substring *11*

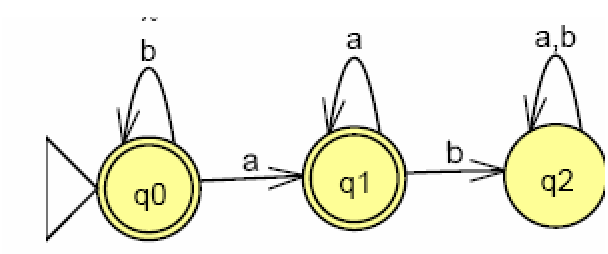
Problem doesn't say whether this must be a dfa and this is easier with an nfa:



	λ	0	1
>q_0	q_1 , q_5		
q_1		q_2	
q_2		q_3	q_1
q_3		q_3	q_4
q_4		q_3	q_9
q_5		q_5	q_6
q_6		q_5	q_7
q_7		q_8	q_7
q_8		q_{10}	q_7
*q_9		q_9	q_9
*q_{10}		q_{10}	q_{10}

d) The set of strings over {a,b} which do not contain the substring *ab*.

Similar to parts a and b, I will first create a fa that does accept *a b* and then I will reverse the final and the nonfinal states:

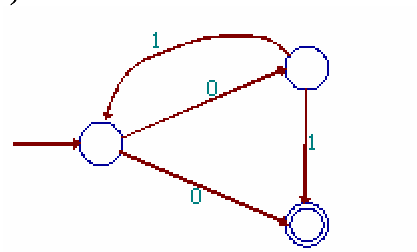


	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_2	q_2

Show your answers in both table and graph form.

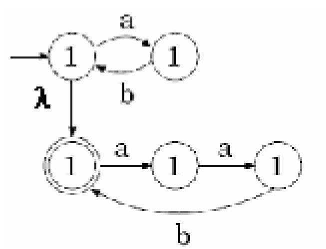
#3. Describe L(M) for the following nfa's: a) in words and b) as a regular expression

a)



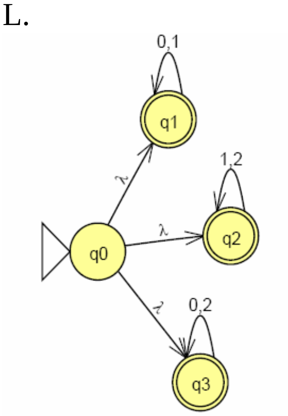
L(M) = Alternating 0's and 1's (including none) that begin with a 0
(01)* (01 U 0)

b)



0 or more *ab*'s followed optionally by 0 or more *aab*'s
(ab)* (aab)*

#4. Create an NFA (with λ transitions) for all strings over {0, 1, 2} that are missing at least one symbol. For example, *00010*, *1221*,and *222* are all in L while *221012* is not in L.

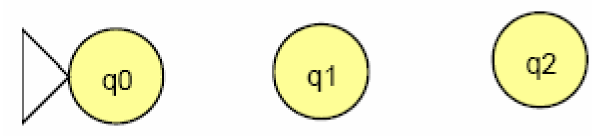


#5. a) Given an NFA with several final states, show how to convert it into one with exactly one start state and exactly one final state.

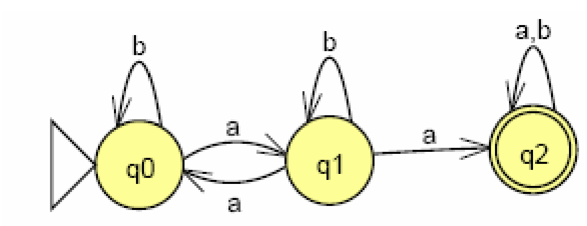
Create a new initial state and a λ-transition from it to all the original start states
Create a new final state and a λ-transition from all the original final states (which mark to no longer be final) to this new final state

b) Suppose an NFA with k states accepts at least one string. Show that it accepts a string of length k-1 or less.

Look at a fa with 3 states:



No matter how you draw the transitions or which states are final states, to process a string of length *k* means you visited a state twice. For example:



accepts the string of length 3: aba

But just by not visiting the revisited state (q_1), this will accept *aa* (of length 2)

In general, if a string of length k is accepted by a fa with k states, it visits (at least) 1 state twice. By not visiting this state the 2nd time (e.g., don't take the loop), we can accept a string with 1 fewer symbol, i.e, of length k – 1.

c) Let L be a regular language. Show that the language consisting of all strings not in L is also regular.

If L is regular, there is a dfa, M, such that L = L(M), that is, M accepts L. If we create a new finite automaton, M', by reversing final and non-final states, we will accept what M didn't and reject what M accepted; that is, C(L) = L(M')