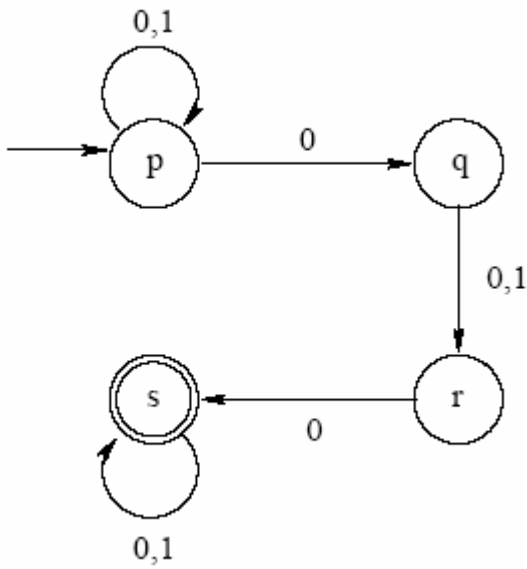


Name \_\_\_\_\_

**CS3133  
Homework #3**

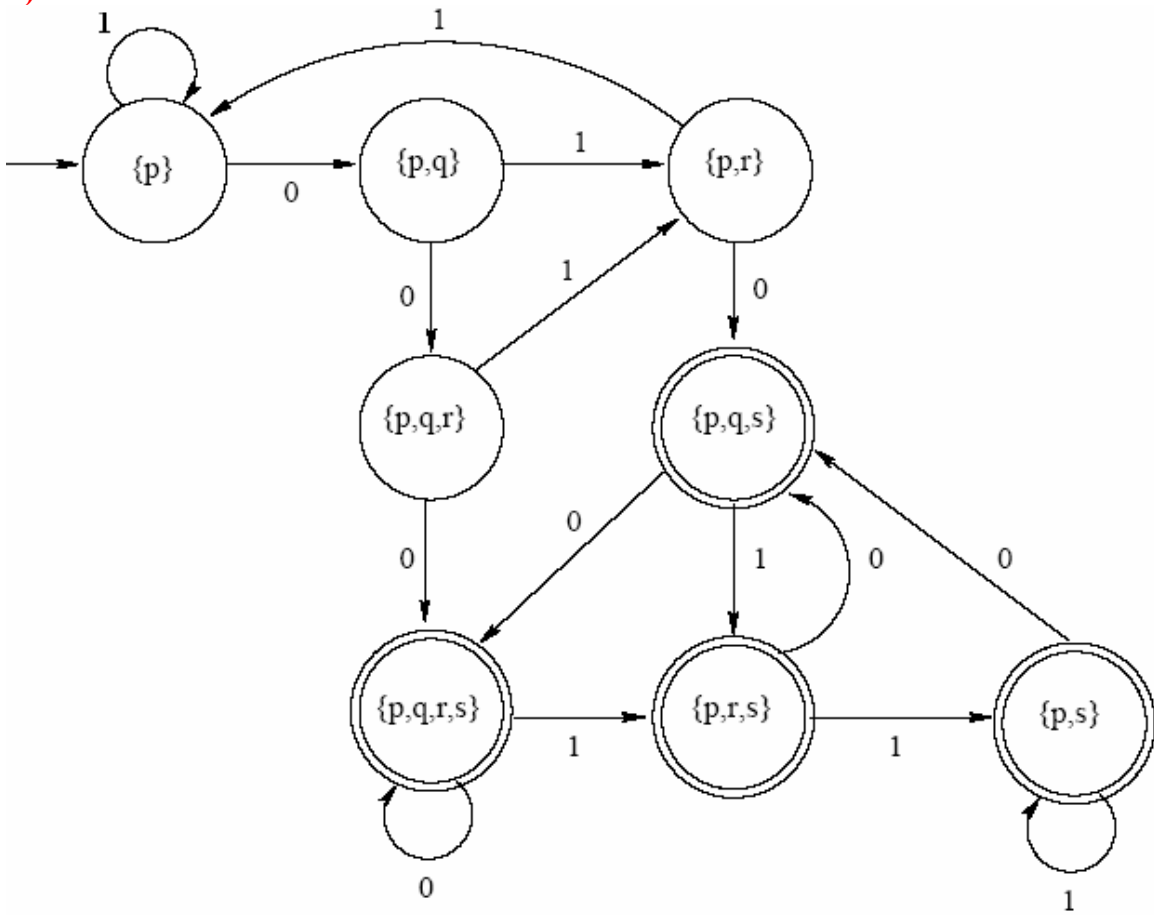
**People talked to and URL's consulted:**

#1. a) Use the subset construction to convert the following NFA to a DFA:

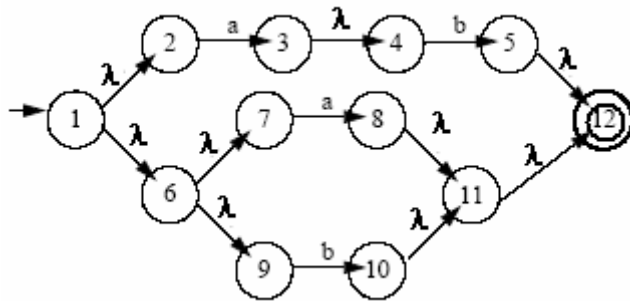


**Solution**

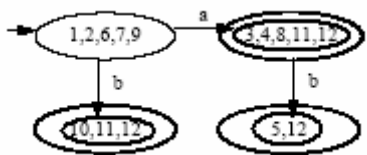
b)



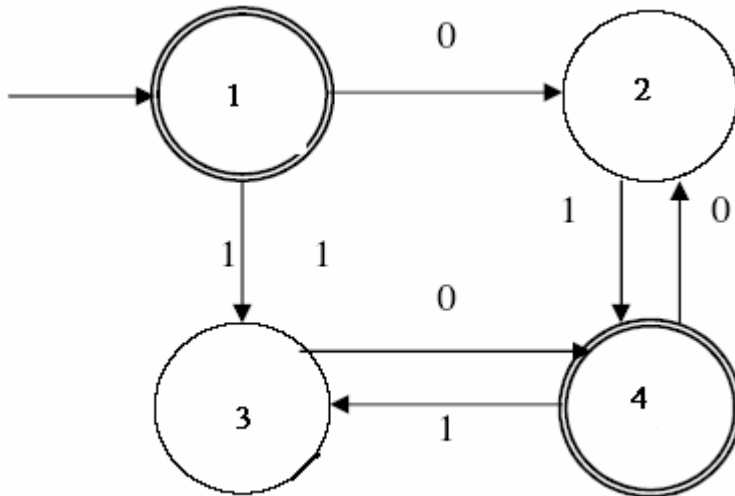
b) Use the subset construction to convert the following nfa to a dfa:



**Solution:**

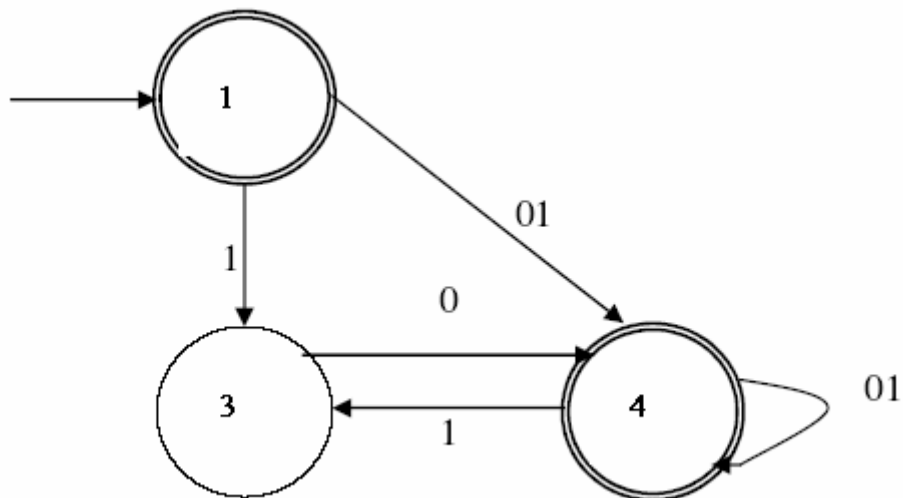


#2. Create the regular expression for the following automaton by eliminating states

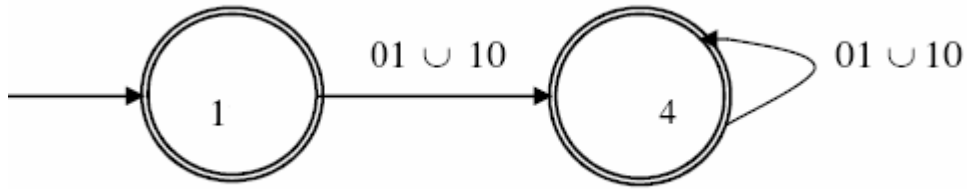


**Solution**

**Eliminating state 2:**



**Eliminating state 3:**



So  $L(M) = \lambda \cup (01 \cup 10) (01 \cup 10)^*$  or just  $(01 \cup 10)^*$

#3. a) Show that all finite languages are regular

If a language  $L$  is finite, we need just create a finite automaton for each of the strings in  $L$  and then union these automata (by creating a new start state and  $\lambda$  transitions into each of the automata).

b) Consider the operation  $g$  on three languages defined as:

$$g(L_1, L_2, L_3) = L_1 \cup L_2 - (L_3 \cap \sim L_1)$$

Show that regular languages are closed under the  $g$  operation.

Regular languages are closed under complement so  $\sim L_1$  is regular.  
 Regular languages are closed under intersection so  $(L_3 \cap \sim L_1)$  is regular.  
 Regular languages are closed under set difference so  $L_1 \cup L_2 - (L_3 \cap \sim L_1)$  is regular.

Therefore regular languages are closed under  $g$ .

#4. Show that it is decidable whether a regular language,  $L$ , is empty

Just look at the dfa for  $L$  and see if there is a path from the initial state to a final state.

#5. Use the pumping lemma to show the following language is not regular:

$$\{0^n 1^m 0^{n+m} \mid n, m \geq 0\}$$

If  $L$  were regular, then there is a dfa,  $M$ , with  $k$  states accepting  $L$ .

Pick  $z = 0^k 1^k 0^{2k}$

Then, since  $z \in L$  and  $|z| \geq k$ , by the pumping lemma:

$z = uv^i w$  with  $|uv| \leq k$ ,  $\text{length}(v) > 0$  and  $uv^i w$  is also in  $L$  for all  $i \geq 0$ .  
Thus  $uv$  and thus  $v$  are all 0's. Say  $v = 0^j$ .

When  $i = 0$ , we have  $0^{k-j} 1^k 0^{2k}$  which is not the right form for strings in  $L$ .  
Thus this string is not in  $L$  and  $L$  is not regular.