Name __________________________

Homework 6

Worked with _______________

Consulted: _________________

1. For each of the language classes: *regular, context-free, recursively enumerable,* and *recursive*, and each operation: *complement, union, intersection, concatenation,* and *closure, indicate whether the class is closed under that operation by filling in the corresponding spot in the following table with "yes" or "no".

	regular	context-free	recursively enumerable	recursive
complement				
union				
intersection				
concatenation				
*-closure				

#2. Suppose L_1, L_2, and L_3 are recursively enumerable languages over the alphabet Σ such that
(a) $L_i \cap L_j = \Phi$ when $i \neq j$; and
(b) $L1 \cup L2 \cup L3 = \Sigma^*$

Prove that L_1 is recursive. Indicate clearly how each of the hypotheses (a) and (b) are used in your argument.

#3. Suppose L is a recursive language. Prove that L^* is recursive.

#4. Complete each sentence with one of the answers:

- *regular*
- *context-free*
- *Recursive*
- *Recursively enumerable*
- *none of the above*

Make the strongest true assertion you can. In each case you should explain your answer by reference to the appropriate closure properties of the relevant language classes. (B^c *means the complement of B)*

(a) If A is regular and B is regular, then $A \cup B^c$ is:

(b) If A is context-free and B is regular, then $A \cup B^c$ is:
.

(c) If A is regular and B is context-free, then $A \cup B^c$ is:

(d) If A is recursive and B is recursive, then $A \cup B^c$ is: *recursive.*

(e) If A is recursive and B is Recursively enumerable, then $A \cup B^c$ is:

(f) If A is recursively enumerable and B is recursive then $A \cup B^c$ is: ***RE.***

(g) Show ~L are both r.e if and only if L is recursive

#5. Consider the following language:

HALT = $\{< M, w > \mid M$ is a Turing machine and $w \,\varepsilon\, L(M)\}$

Here < M,w > denotes some encoding of the pair consisting of a description of machine M and of input w, so that HALT is an encoding of the set of machine-input pairs such that the machine accepts the input. The specific details of the encoding itself are irrelevant to the questions which follow.

The theorem on the undecidability of the halting problem states that HALT is not decidable. For this problem, take that theorem as given.

(a) Is the language HALT recursively enumerable? Defend your answer.
(b) Is the complement of HALT recursively enumerable? Defend your answer.