CS4123 THEORY OF COMPUTATION. B97 **EXAM 1**
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Name: ____

Instructions

- Answer all 5 problems. Each problem is worth 20 points.
- Show your work
- Justify your answers
- Use the space provided to write your answers
- Write your name on each page
- ASK if you have any questions

Problem 1. (20 points)

Let $\Sigma = \{a, b\}$. Construct the state–transition diagram of a deterministic Turing machine that decides the following language:

$$L = \{w | w \in \Sigma^\star \text{ and the number of occurrences of the letter "a" in } w \text{ is a multiple of 3 } \}$$

Make ALL transitions explicit in the diagram. (Note: Remember that 0 is a multiple of 3.)

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Problem 2. (20 points)

Consider the function:

$$g(n) := \text{"the number of values } m \text{ such that } m \text{ is prime and } m \leq n\text{"}.$$

Examples:

g(4) = 2	since there are two primes ≤ 4	namely 2 and 3
g(1) = 0	since there are no primes ≤ 1	
g(13) = 6	since there are six primes ≤ 13	namely 2, 3, 5, 7, 11, and 13

Show that the function g is primitive recursive.
Any functions used in your solution and that were not shown to be primitive recursive in class or one of the handouts must be shown to be primitive recursive as part of your solution.

Hints:

- Use the primitive recursive function $prime(n) = \begin{cases} 1, & \text{if } n \text{ is prime} \\ 0, & \text{otherwise} \end{cases}$

  (you do not need to define it).

- Define g using the following template:

  $g(0) =$

  $g(k+1) =$

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Problem 3. (20 points)

Let Σ be an alphabet and let $\mathcal{S}$ be the set of all semi–decidable languages over Σ:

$$\mathcal{S} = \{L | L \text{ is a semi–decidable language over alphabet } \Sigma\}$$

3.1 (14 points) Prove that $\mathcal{S}$ is closed under intersection. That is, if L_1 and L_2 are semi–decidable languages over Σ, so is $L_1 \cap L_2$.

3.2 (6 points) Is $\mathcal{S}$ closed under complementation? That is, is the complement $\bar{L} = \Sigma^\star - L$ of every semi–decidable languages over Σ also semi–decidable? Explain your answer. (You do not need to write formal proofs.)

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Problem 4. (20 points)

Let Σ be an alphabet. Show that every semi–decidable language L over Σ for which there is a Turing machine M that enumerates the strings in L in length–increasing fashion (i.e. M enumerates all strings of length n in L before enumerating strings of length $n+1$ in L, for all $n \geq 0$) is decidable.

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Problem 5. (20 points)

Let Σ be an alphabet. Show that the set

$$ALL_{DFA} = \{\langle A\rangle | A \text{ is a DFA and } L(A) = \Sigma^\star\}$$

is decidable.